

An Analysis of the Discourse of Written Reports of Investigative Work in GCSE Mathematics

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Abstract

The focus of this study is students' written reports of mathematical investigations carried out for examination at 16+. These coursework texts are produced within a discourse of 'investigation' that involves the students, their teachers and an official, practical and professional literature. This discourse has been examined through analysis of written and oral texts produced by the different groups.

A method of analysis of mathematical texts has been developed, based on Halliday's functional grammar, using techniques of critical discourse analysis. This takes into account the ways in which mathematics, mathematical activity and the relationships between writer, reader and subject matter are constructed in the texts. The method was applied to a set of students' written reports of investigations, revealing some variety in the types of text and in the ideational and interpersonal functions served by the texts.

The fact that coursework texts are examined by the student's teacher is a significant aspect of the context of their production. The assessment process was therefore investigated through interviews with mathematics teachers reading and assessing student texts. Tensions were identified between the stated aims of investigative work, the values of the assessment process and those of the traditional practices of mathematics and school mathematics. These tensions were manifested in the teachers' readings and assessments of the student texts and were resolved in various ways by different teachers. Textual features significant to the teachers' readings were identified and described although the teachers themselves generally appeared unable to describe explicitly the forms they would value highly. Teachers' responses to unusual or erroneous aspects were also explored. Variations in teachers' readings indicated that students' texts cannot be taken as transparent representations of their thinking.

Table of Contents

Acknowledgements	7
1 Introduction	9
Background	
2 The characteristics of mathematical texts	13
3 Learning to write mathematical text	27
4 The assessment of investigations and teacher assessment	35
Analysing mathematical texts	
5 Tools for analysis of mathematical texts	51
Study of students' GCSE texts	
6 The study of GCSE coursework texts	73
7 The coursework tasks	87
8 Case study analyses of GCSE coursework texts	101
Study of teachers reading GCSE texts	
9 Investigating teachers reading coursework: methodology	115
10 Comparisons between teachers reading the same texts	131
Themes arising from teachers' readings	
11 Teachers' identification of and response to features of students' texts	149
12 'Algebra' in coursework	181
13 Dealing with error - is 'Practical' different?	197
14 Teachers' responses to 'unusual' features of students' work	207
15 Conclusions	221
Bibliography	253
Appendix 1 Grade descriptions for assessing coursework	269
Appendix 2 Coursework tasks and performance indicators	271
Appendix 3 Glossary of linguistic terms	281
Appendix 4 Extracts of an academic mathematics text	283
Appendix 5 Case study student texts and analyses	285
Appendix 6 Extracts of generalisations from 'Inner Triangles' texts	351
Appendix 7 Steven's additional coursework texts	355
Appendix 8 Teacher background questionnaire	371
Appendix 9 Summary of teacher background information	372
Appendix 10 Teacher interview schedule	373
Appendix 11 Comparison of teachers reading the same texts	375
Appendix 12 Harry reading Sandra's naturalistic diagrams	385
Appendix 13 Extracts of 'coherent' sections of student texts	387
Appendix 14 Expression of generalisation	391
Appendix 15 The discourse of 'investigation'	395

Tables

Table 6.1	The students submitting GCSE coursework texts	74
Table 6.2	Number of texts on each task	74
Table 6.3	Variables used in analysis of 'Inner Triangles' texts	80
Table 6.4	Qualities of 'Inner Triangles' texts chosen for case studies	82
Table 6.5	Variables used in analysis of 'Topples' texts	83
Table 6.6	Qualities of 'Topples' texts chosen for case studies	84
Table 8.1.1	Clive: Summary analysis of 'Inner Triangles' text	102
Table 8.1.2	Steven: Summary analysis of 'Inner Triangles' text	103
Table 8.1.3	Richard: Summary analysis of 'Inner Triangles' text	104
Table 8.2.1	Steven: Summary analysis of 'Topples' text	106
Table 8.2.2	Ellen: Summary analysis of 'Topples' text	107
Table 8.2.3	Sandra: Summary analysis of 'Topples' text	108
Table 11.1	Identification of problem statements as copied from the question paper or composed in the student's own words	160
Table 11.2	Teachers' examples of statements of processes	164
Table 12.1	Algebra in the 'Inner Triangles' texts	182
Table 12.2	Algebra in the 'Topples' texts	183
Table 12.3	Algebra in the 'Inner Triangles' extracts	183
Table 13.1	Errors identified in 'Inner Triangles'	198
Table 13.2	Errors identified in 'Topples'	198
Table 14.1	'Different' features in 'Inner Triangles' texts	208
Table 14.2	'Different' features in 'Topples' texts	209
Table 14.3	'Different' features in 'Inner Triangles' extracts	209
Table A5.1	Actors and Processes: Clive - 'Inner Triangles'	292
Table A5.2	Actors and processes: Steven - 'Inner Triangles'	302
Table A5.3	Actors and processes: Steven - 'Topples'	319
Table A5.4	Actors and processes: Ellen - 'Topples'	333
Table A5.5	Actors and processes: Sandra - 'Topples'	345
Table A5.6	Adjusted Actors and processes: Sandra - 'Topples'	346

Figures

Figure 4.1	Forms of mathematical communication expected of students by the London examination board	41
Figure 10.1	Steven's first formula and Steven's 'inverse' formula	143
Figure 11.1	Two dimensional table vs. one dimensional table	154

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1 Introduction

Since the introduction of the GCSE in 1988, examinations at 16+ in mathematics in England and Wales have included a coursework component involving investigative, practical and extended tasks to be completed by students in class or at home and to be assessed by the students' own teachers. This innovation was intended to enable assessment of those parts of mathematical activity that are not readily assessed by traditional timed examinations, in particular problem solving processes and communication skills. At the same time, it was intended that the assessment should encourage 'good classroom practice'. The coursework component became compulsory in 1991, involving many mathematics teachers for the first time both in implementing investigative work in their classrooms and in assessing such work for examination purposes. The requirement for teacher assessment of students' mathematical processes as defined by Attainment Target 1 of the Mathematics National Curriculum, 'Using and Applying Mathematics', has similar implications for teachers at primary as well as at secondary level. The introduction of coursework, intended to influence the mathematics curriculum, has raised a number of issues for teachers and for students as they have attempted to come to terms with the changes it has entailed in their practices.

An important part of the practice of coursework is the production of a written report of the student's investigative work. This written report forms the main object of assessment and must, therefore, provide evidence of the student's problem solving processes as well as reporting the results of the investigation. At the same time as moving the focus of their activity from mathematical 'content' to 'processes', students and teachers have thus had to come to terms with the production and assessment of extended pieces of written work in an area of the curriculum in which traditionally very little writing has taken place. This has caused some concern for mathematics teachers who have perceived that many students may not provide adequate evidence of their problem solving activity, either because they do not realise what should be written down (MacNamara & Roper, 1992b) or because they lack the skills with written language that would enable them to provide such evidence (Ball & Ball, 1990). The training and previous experience of most mathematics teachers has not provided them with strategies for helping their students to acquire such skills or, indeed, with explicit knowledge of the forms of mathematical writing that might adequately meet the requirements of the coursework examination process. While the characteristics of what may be judged to be 'appropriate' writing are established in practice and may to a large extent be implicitly agreed by teachers, without a more explicit description of the genre it may not be successfully communicable to those students who do not already share the assumptions and resources of their teachers.

In recent years some attention has been paid to the introduction of writing activities into the mathematics curriculum (e.g. Connolly & Vilardi, 1989). The focus of this attention, however, has been almost exclusively on the possible benefits that writing may have for students' learning of mathematics. The writing itself largely appears to have been taken to be non-problematic and it has been assumed that students either already possess the necessary knowledge and skills or will develop them spontaneously. This assumption is not warranted (Kress, 1990) and, particularly in a context in which the written product is used to form an assessment of mathematical achievement, may disadvantage some groups of students.

The writing and assessing of mathematics coursework takes place in a 'gate-keeping' situation. It has particular significance for the writer because the judgements made about her on the basis of her writing will contribute to her success or failure, not only in this localised context, but also in future access to further education and to employment. It is thus an important site for research as the consequences of a mismatch between the writing produced by the student and the teachers' expectations about 'appropriate' writing are socially significant. In the context of job interviews, Fairclough suggests that:

if an interviewee gives what is felt to be a poor or weak or irrelevant answer to a question, this is likely to be put down to her lack of the requisite knowledge or experience, her uncooperativeness, and so forth; the possibility of miscommunication because of differences in discursal conventions rarely suggests itself. (1989 : p.48)

Similarly in the coursework context, language that does not match the teacher's expectations about the genre is likely to lead to negative judgements about the student's mathematical competence.

In order to understand what may be judged to be 'appropriate' forms of language within the coursework context it is necessary to consider not only the texts themselves but also their use in the assessment process and the practices and values of secondary school mathematics which contribute to their production and evaluation. The aim of this study, therefore, is to create a description of the genre of written reports of mathematical investigations produced for coursework examination by analysing the texts of the reports and by examining the interactions between writers, readers, subject matter, text and the discourses within which these are situated.

To describe the written reports themselves, a method of text analysis has been developed, using tools derived from Halliday's (1985) functional grammar and drawing on interpretative techniques which take account of the social context in which the texts are situated (Hodge & Kress, 1993, Fairclough, 1989). In addition, the nature of the mathematics register has made it necessary to develop ways of describing and interpreting non-linguistic forms such as algebraic symbolism and diagrams. The method of analysis relates the forms of language used in a mathematical text to the following main questions:

- How are mathematics and mathematical activity portrayed in the text?

- What relationships are constructed in the text between writer, reader and subject matter?
- What is the text trying to achieve (i.e. telling a story, presenting an argument, etc.)?

This makes it possible to consider the relationships between the forms of the text and the possible meanings and values ascribed to it within the discourses of mathematics, school mathematics and 'investigation'.

The central object of study is the coursework produced in response to investigative tasks set by the London examination board (LEAG, 1991) and submitted in the summer of 1991. Analyses have been carried out of two of these tasks and of a sample of students' texts responding to them. One of these tasks may be considered a 'pure' investigation while the other is 'practical' in that it involves manipulation of physical apparatus. By itself, however, the text analysis can provide only a description of the form of the text and speculation, however strongly grounded theoretically, about how the form might relate to the meanings constructed by a potential reader. It is, therefore, complemented by a study of teachers' reading and assessment practices. Task-based interviews were conducted with experienced secondary mathematics teachers during which they read and assessed students' coursework texts. The main questions addressed in analysing these interviews are:

- What features of students' texts do teachers treat as important?
- What meanings do teachers construct from the coursework texts?
- What relationships are there between the form of coursework texts and teachers' evaluations of students' achievement?

In addition, close attention to the ways in which teachers make sense of the students' texts provides insight into the process of assessment and the tensions that teachers may experience between their various roles as teachers and as examiners. An attempt is made to articulate the teachers' readings and assessment practices both with the analyses of the students' texts themselves and with the values espoused within the discourse of 'investigation'.

By constructing a more thorough understanding of the ways in which mathematics coursework texts are written and read and by gaining knowledge of the forms of writing that are acceptable and successful within the genre, the possibility may be created of developing ways of helping students to gain greater control over their writing and thus to become more effective communicators of mathematics as well as producers of coursework that will be more highly valued by their assessors.

The chapters in the next section present the background to the study, starting with issues related to students' writing. Chapter 2 considers the nature of mathematical text and the features of mathematical written language which have been identified in the literature. In

chapter 3 aspects affecting students' development of writing in mathematics are addressed. Chapter 4 turns to the context within which the students' writing considered in this study takes place. The practices of writing and reading reports of mathematical investigations are situated within a discourse which influences the ways in which teachers and students may act and the values they may espouse. This discourse is described and the ways in which such investigation reports may be assessed and the research into teachers' assessment of their students' work are reviewed.

The method of text analysis used throughout the study is detailed in chapter 5, the grammatical tools used and the principles by which features of mathematical texts are identified as significant are described. I then turn to the students' coursework texts themselves. A sample of three texts on each of two tasks was chosen in order to provide a range of types of linguistic features. The basis of the selection of this sample of texts is described in chapter 6. In order to interpret these texts, it is necessary to be aware of the characteristics of the tasks that the student-writers were responding to; an analysis of these tasks is presented in chapter 7 and in chapter 8 an analysis of each of the selected texts is provided.

Having described the students' texts, the teachers' reading and assessment of the texts is then addressed. In chapter 9 I discuss the methodology for investigating teachers' reading and assessment practices through the use of interviews; the sample of teachers interviewed and the method of conducting the interviews are described. Two case studies of pairs of teachers reading the same texts compare and contrast their reading practices in chapter 10.

The features of students' texts which were identified and commented upon by teachers are described in chapter 11 and the values ascribed to various forms are discussed. Teachers responses to the students' use of algebraic symbolism are analysed separately in chapter 12. Given the assessment context within which the study is situated, the identification of mismatches between the students' texts and the teachers' expectations is significant throughout the analysis of the interviews with teachers. Teachers' readings and responses to two specific aspects of mismatch are considered separately in the next two chapters: in chapter 13 those parts of the students' texts which teachers perceived to be incorrect, and in chapter 14 those features which, while not seen as errors, were nevertheless identified as unusual or unexpected.

Finally, in chapter 15 I present the conclusions of the study and discuss its implications.

2 The characteristics of mathematical texts

Unlike most of those who have written about the nature of mathematical language in the context of mathematics education (see, for example, reviews by Austin & Howson (1979) and Ellerton & Clements (1991)), my interest within this study is not the relationship between the use of language in the mathematics classroom and the quality of students' learning of mathematics. My aim is rather to produce a description of the nature of the language, specifically the written language, produced by students and accepted by teachers as 'appropriate' within the discourse of mathematical investigations. In doing this, one area of concern is the extent to which the texts produced may be considered to use 'mathematical language'. In this chapter, therefore, I review the literature on the nature of mathematical language in order to identify the linguistic features that may characterise mathematical texts.

To claim to describe 'the language of mathematics' is to ascribe a unity to the field that is justified only in the broadest terms. Just as there are a number of varying social practices that may be labelled as mathematics (including academic, school, recreational, etc.) there are a variety of genres of text that may be called mathematical. Within any mathematical practice some texts will be considered to be appropriate to the practice and others will not. The linguistic features that contribute to a text's acceptance within a particular mathematical context include its vocabulary and symbolic content. They also include its grammatical structure and the forms of argumentation used. The focus of the present study is those features characterising texts considered appropriate to the particular genre of reports of investigative work in school mathematics. While some aspects of such texts may be unique to this genre it seems likely that they should also share some characteristics of other texts that are considered to be mathematical. Moreover, they are produced within the wider discourse of school mathematics and as such are likely to be specifically influenced by the characteristics of other mathematical texts experienced by students and their teachers both in the school and beyond. The relationship of this genre to other types of writing by mathematicians is itself of interest, particularly in the light of claims that involvement in investigative work is similar to the activity of research in mathematics. In this chapter, therefore, I seek to establish a background of knowledge of the characteristics of those mathematical texts which have been considered in the literature. This knowledge is necessarily limited to a relatively small selection of genres, although it may be possible to make some general statements about characteristics common to a range of mathematical genres.

A number of authors have provided descriptions of general features of mathematical texts. Notably, Halliday's contribution to the 1974 UNESCO symposium on Linguistics and Mathematics Education introduced the use of the concept of a mathematical 'register'¹ to discussions about language in the mathematics education context and provided an overview of some of the grammatical characteristics of such a register. This has been elaborated from a mathematics education perspective by Pimm (1987). The nature of a mathematical register is discussed in section 2.1. I then turn to the two types of mathematical text whose features have been considered in more specific terms: the academic text and the school mathematics text book (although neither of these types of text are entirely homogeneous). The writing demanded of students in the traditional mathematics classroom is then considered. Finally, a number of research studies which have investigated the forms of written language used by students are considered.

2.1 The mathematical register in general

As has been argued above, there is some difficulty in producing a comprehensive definition and description of what constitutes 'mathematical language'. Nevertheless, many authors discussing this topic have treated it as an identifiable entity. The characteristics that have been identified as part of mathematical language are likely to contribute towards making a text containing them likely to be seen as 'mathematics'. In the context of the present study, in the discourse of students' reports of investigative work, it is of importance that a student's text should be recognised as mathematics by a teacher-assessor. In this section, therefore, those characteristics that may contribute to such a recognition are reviewed.

Some attempts to characterise mathematical language have focused almost entirely on its symbolic system (e.g. Erynck, 1992) or on the specialist vocabulary used to name specifically mathematical objects and concepts (e.g. Otterburn & Nicholson, 1976), usually from the point of view that these aspects cause difficulties for students. Apart from the recognition that some symbols and terms are used either exclusively or in unusual ways in mathematical contexts, a full characterisation of the nature of mathematical lexis is beyond the scope of the present study; discussion of its form and derivation may be found in Halliday (1974) and Pimm (1987). While symbolism and specialist vocabulary are perhaps the most obviously visible aspects of many mathematical texts, they are clearly inadequate to provide a full description of the nature of mathematical texts. Mathematical texts do not on the whole

¹Halliday defines 'register' as:

a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings . . . including the styles of meaning and modes of argument (Halliday, 1974: p.65)

consist only of strings of symbols or of naming things; rather, they are, like other academic texts, rhetorical in nature, addressing and attempting to persuade a reader (Hansen, 1988, Ernest, 1993b). It is, therefore, necessary to look beyond the level of vocabulary at the syntax of the text and at the structures which serve to construct mathematical arguments.

Discussions of mathematical language often attempt to describe it as a set of additions to some more basic or 'natural' form of language. Kane, for example, describes "mathematical english" as a mixture of "ordinary english" and "various brands of highly stylized formal symbol systems" (1968: p.296). This definition is not, however, entirely adequate, as Kane himself demonstrates when he attempts to elaborate it. The non-symbolic "ordinary" component also has specifically mathematical aspects which bring into question its "ordinary" nature:

For example, phrases such as "if and only if, if ... then, A or B" are direct translations
from the sentential calculus (ibid.)

This demonstrates some of the difficulties brought about by considering mathematical language as formed by an augmentation of a basic form of 'ordinary' or 'natural' language. The 'ordinary' component must itself be transformed in order to express mathematical meanings. As well as specialist vocabulary this may involve the creation of new grammatical structures or "the bringing into prominence of structures which already existed but were rather specialized or rare . . . like . . . 'the sum of the series to n terms'" (Halliday, 1974: p.67). While such developments are also characteristic of the development of the language of other specialist domains, Pimm (1987) suggests that the extent to which this occurs in mathematics is such as to be qualitatively different.

In addition to specialist vocabulary and structures, other features that have been identified as characteristic of much mathematical text include its "density and conciseness . . . which tend to concentrate the reader's attention on the correctness of what was written rather than on its richness of meaning" (Austin & Howson, 1979: p.174). Halliday & Martin point out that scientific texts in general have a high 'lexical density', that is a high ratio of 'content' words to 'grammatical' words (1993: p.76).

It must, however, be recognised that there is substantial diversity between the forms of language that are used in different mathematical contexts, and it is not clear that the idea of a single mathematical register is sufficient to cope with the variation of functions and meanings to be found, for example, in a primary school text book and in an academic research paper. Not only does the subject matter vary but the modes of argument used in different domains of mathematical activity are likely to differ substantially (Richards, 1991).

Rather than attempting to characterise a unified field of mathematical language, I feel it is more appropriate to consider mathematical English in the way that Halliday & Martin define

scientific English - as a semiotic space with diatypic variation and diachronic evolution (i.e. varying both between a number of different practices and over time) which is:

by and large . . . a recognizable category, and any speaker of English for whom it falls within the domain of experience knows it when he sees it or hears it (1993: p.54)

A text may thus be identified as mathematical if it is identified as such by a reader whose experience qualifies them to make such a judgement. While the presence of symbols, specialist vocabulary and grammatical structures and a high level of density and conciseness may serve to make it likely that a text will be identified as mathematical by such a reader, to be judged a 'good' or 'appropriate' text within a particular mathematical practice the text is likely to have to conform to a number of other characteristics.

2.2 Academic mathematics texts

Given the claim made by the advocates of investigative work in school mathematics that the processes involved in investigative activity are similar to those used by professional mathematicians (see Appendix 15), the academic research report may be seen as the 'adult' equivalent of the investigation report. (This is not a claim that the two types of report resemble each other closely, merely that it is of interest to compare and contrast them.)

While the literature reveals relatively little analysis of mathematical academic writing, mathematicians have offered advice to each other about the forms their writing might take, focusing in particular on reading difficulties for both novices and experts perceived to be caused by the excessive use of symbols, the construction of proofs and an impersonal style. A formal, impersonal style, including an absence of reference to human activity, is one aspect that mathematical writing appears to share with many other academic areas, in particular with writing in the sciences. In this section, the nature of academic mathematics texts is considered, taking into account the views of mathematicians themselves and research concerned with scientific academic texts in general. Examples from one academic mathematics text (Dye, 1991) will be used to illustrate the ways in which some of the characteristics are manifested in a mathematical context².

2.2.1 The professional's 'common sense' view: mathematics is its symbol system

For students and new entrants to the scientific and technical academy and professions there is a considerable body of publications providing advice on how to write in scientific and technical genres. In mathematics the field is less substantial; it is possible that this difference is in part due to the smaller lay readership for technical and academic mathematical texts but

²Two extracts from this text may be found in Appendix 4.

it may also be due to a common perception among mathematicians that the only significantly meaningful part of a mathematics text resides in the symbol system. Since producing 'correct mathematics' may be seen as equivalent to producing a correct sequence of symbols, the mathematical writer's task is merely to record the content without any need to pay separate attention to the form of the language in which it is recorded. Tobias characterises the mathematicians' and mathematics teachers' attitude that 'spoken language' or any non-symbolic elaboration that provides context "is a temporary scaffolding to be discarded as soon as the new code is mastered" (1989: p.49).

The frequently stated aphorism that 'mathematics is a language' is associated with this identification of mathematics with its symbol system (Rotman, 1993). To take an extreme example, Ervynck's (1992) paper entitled *Mathematics as a Foreign Language* deals almost exclusively with the nature of mathematical symbols and their internal syntax. Non-symbolic elements are described merely as "connectives" (p.226). While advice to academic mathematics authors (e.g. Gillman, 1987; Knuth et al., 1989; Steenrod et al., 1973; London Mathematical Society, cited by Pimm, 1987: p.121) recommends the use of 'natural language', this is intended merely to make the text easier to read rather than to contribute to its meaning and may be seen to be "semantically empty" (Roe, 1977: p.11). Mathematics educators, writing from a psychological point of view, tend to recognise that the symbolic system is embedded in 'natural language' (e.g. Kaput, 1987; Ernest, 1987) although this does not necessarily mean that their discussion of mathematical language takes this into account.

A detailed examination of the lexis and syntax of the symbol system itself is beyond the scope of this study as, although much of school mathematics may consist of the repetitive manipulation of symbols (Ernest, 1993a), the range of symbols used up to GCSE level is restricted almost entirely to those of arithmetic and elementary algebra and the quantity of symbolic activity involved in most investigative work is extremely limited. Nevertheless, the extent of the identification of mathematics with its symbol system must be significant in its effect on readers' interpretations of mathematical texts and must, therefore, be taken into account in the development of a method of analysing such texts (see chapter 5).

2.2.2 Formality and the place of human beings in the text

It is accepted as common knowledge that both scientific and mathematical texts are impersonal and formal. Clearly, the symbolic content discussed above contributes to this, but there are also a number of relevant characteristics of the 'natural' language. While there has been little detailed consideration of this aspect of mathematical texts, it has been an area of substantial study in scientific writing. Strube's (1989) attempt to define what constitutes a formal style with specific reference to physics textbooks identifies a major aspect as compliance with conventions and rules that "give writing authority and objective validity in its own sphere" (p.292). Some of the rules Strube identifies seem to apply more widely to

academic writing in many fields, in particular the use of a *distant authorial voice*, manifested in the use of the passive voice and the absence of direct reference to the author (apart from an impersonal “we”) or the reader.

One source of the “distant authorial voice” is the use of nominal rather than verbal expressions. Halliday (1966; 1974) notes the extensive use of nominalisations in both scientific and mathematical texts. In particular, long nominal constructions are very common in mathematics (Halliday & Martin, 1993). This form of grammatical metaphor, by transforming an action into an object, clearly contributes to the ‘formality’ of the text by obscuring any human agency involved in the action. At the same time, however, “packaging a complex phenomenon into a single semiotic entity” (Halliday & Martin, 1993: p.60) also contributes to the kinds of meanings that can be expressed in the text; the nominal expression can itself become an actor in the text and can be the cause or effect of other phenomena. Thus Dye (1991) uses statements such as:

The demand that the first vertex of (13) is polar to the other two gives
 $Aj + Bj^2 - Cj^3 = Aj^2 - Bj^3 + Cj = 0.$

in which the use of nominalisation alienates the reader from the source of the “demand”. The necessity is represented as an abstract entity whose independent existence has material consequences. Moreover, nominalisations such as *stabilizer*, *permutation*, and *discriminant* are used extensively within this paper. The ability to represent processes as objects and hence to operate on the process-objects themselves is part of the power of mathematics, at the same time it increases the impersonal effect, strengthening the impression that it is these process-objects that are the active participants in mathematics rather than the human mathematicians.

It is clear that an impersonal style is an accepted convention in much academic writing, particularly in the sciences, and analysis of the linguistic features contributing to this style (e.g. passive voice, use of personal pronouns and choice of tenses) is a major focus of research in this area (e.g. Tarone et al., 1981; Malcolm, 1987; Sutton, 1989; Master, 1991; Harvey, 1992). The extent of the use of passive or active constructions may, however, vary according to individual taste or contemporary fashion (Sutton, 1989) and may differ between different genres (e.g. text book, popular article, research article) and between different branches of science. Tarone et al. (1981) point out that most quantitative studies showing extensive use of the passive do not differentiate between texts in different genres or in different fields of scientific endeavour. Their own study showed that the authors of astrophysics research papers used the passive voice where established, standard procedures had been followed but used *we* + active voice “to indicate points in the logical development of the argument where they have made a unique procedural choice” (p.195). Thus, while the passive might be seen to be the default form of expression for describing methods used, at critical points in the development of their work the authors claim personal responsibility.

Similar observations about the presence of the author are made by Bazerman (1981), who notes in his analysis of a scientific paper that:

all the uses of the first person are to indicate intellectual activities: statement making . . . , making assumptions . . . , criticizing statements . . . , and placing knowledge claims within other intellectual frameworks. (1981: p.367)

The differences identified by Tarone et al. between the quantitative findings of their own and other studies illustrate the danger of assuming that features of one type of scientific writing will also occur in other fields or genres. However, Swales (1985) points out that the context of such decisions is likely to be relevant to understanding choices of voice in other fields.

The extensive use of the personal pronoun *we* by teachers in the mathematics classroom is discussed by Pimm (1984; 1987) and Rounds (1987). Its use in written texts is not addressed to the same extent but Gillman's (1987) advice to mathematical writers that *we* means "you and the reader" (p.11) appears oversimplistic in the light of the evidence of how the pronoun is used in other forms of academic writing. It seems likely that academic mathematics texts would to some extent resemble those in the sciences in their use of passive constructions and personal pronouns. This may be illustrated by some examples from Dye (1991) in which the passive is used to refer to previously established results:

It was shown in [5] that $PO_3(9)$ acts on a set of 12 hexagons . . .

while the first person plural (apparently referring to the single author) is used to state new definitions:

For us, a hexagon is a set of six points, no three collinear, in $PG(2,K)$: we call the six points the vertices of the hexagon, and their 15 joins in pairs its edges.

and in other contexts, such as "Operating by V we see that . . .", may indeed comply with Gillman's advice to include the reader.

A human presence does, however, intrude into mathematical text in a way that is not so characteristic of academic writing in other sciences through the use of imperatives, conjuring up a human actor (Rotman, 1988). The imperative is also a characteristic component of school text books, although the nature of the relationship between author and reader thus constructed is rather different.

2.2.3 Forms of mathematical argument

As Hansen's (1988) and Bazerman's (1981) comparative studies of academic writing produced within different disciplines point out, the forms of argument used relate to the standards and the epistemologies of the disciplines concerned. In academic mathematics high value is placed on deductive reasoning as a means of both 'discovering' knowledge and providing its warrant.

Alibert & Thomas (1991) remark on the linear nature of the traditional presentation of proofs. This is questioned by other analyses which reveal references backwards to previously

established results (Roe, 1977) and structuring using both linguistic and paralinguistic signals (Konior, 1993). One characteristic of the proofs contained in Dye (1991) is the occurrence of strings of statements thematising both the fact that an act of reasoning is occurring (i.e. starting with words such as *hence* or *but*) and the previously established facts which act as the bases for the deductions:

*But by (4), Hence, by (16), $B=C$. Then, by (4), (15), $A=C(p-j)=C$.
Hence, . . .*

There may, however, be differences between the forms of argument used in different domains within mathematics. Knuth's (1985) analysis of a set of texts from various fields suggests that a number of different forms of reasoning were being used. Although Knuth does not make explicit reference to linguistic features of these forms of reasoning it seems likely that there would be identifiable differences between them.

2.3 School mathematics texts

Most of those authors who have examined the language of school mathematics text books have focused on the difficulties that students may have in "getting the meaning off the page" (Shuard & Rothery, 1984: p.1). My concern here, however, is not with the student-reader's understanding of the mathematical meanings intended by a text book's author but with the characteristics of the genre of mathematics text book. Although school mathematics texts have on the whole very different functions from those of reports of investigative work, both in relation to their subject matter and in the relationship between author and reader, they nevertheless form a very large part of most school students' experience of mathematical text. Characteristics of their language are likely, therefore, to influence student writing and teachers' readings of student writing.

Like academic texts, school texts contain a symbolic element, although the range of symbols is likely to be more limited. In addition, most school mathematics texts contain a substantial graphic element, including "tables, graphs, diagrams, plans and maps, pictorial illustrations" (Shuard & Rothery, 1984: p.45). The extent to which these elements may be considered to be 'mathematical', however, is likely to vary. Indeed, Shuard & Rothery themselves make a distinction between those graphic elements that may be considered "essential" to the meaning of the text and those which are merely "decorative" (p.47). While it is clear that even the "decorative" elements do contribute to the meanings that readers may construct from the text³, this distinction suggests that teacher-readers are unlikely to recognise as mathematical those graphic features which they see as serving a decorative function.

³See Kress & van Leeuwen (1990) for discussion of the contribution of illustrations to the meanings of a social studies text book.

Ernest's "criteria of rhetorical style which apply to school mathematics":

- Use a restricted technical language and standard notation
- Use spare, minimal overall forms of expression
- Use certain forms of spatial organisation of symbols, figures and text on the page
- Avoid deixis (pronouns or spatio-temporal locators)
- Employ standard methods of computation, transformation or proof (1993b: p.8)

he claims, serve to "depersonalise" the discourse. While some of these criteria echo other empirically based descriptions of school mathematics texts, the claim that the school mathematics text book is impersonal disregards one very characteristic feature that serves to involve the student-reader in the text: the 'question'. Questions and instructions apparently intended to involve the student as an active participant occur not only in those sections of text books providing practice exercises or testing student knowledge but also in sections which are attempting to impart new knowledge (Shuard & Rothery, 1984). This includes the use of rhetorical questions and, in some types of texts, series of exercises intended to lead to the 'discovery' of a new fact or generalisation (van Dormolen, 1986). Indeed,

very few examples of exposition by straight explanation are to be found other than in advanced texts. (Shuard & Rothery, 1984: p.11)

Such dominance of instructions for student action, together with a proliferation of worked examples, clearly contributes to a procedural emphasis in much of school mathematics. A repetitive structure is also characteristic of some parts of school mathematics texts, in particular those parts which Shuard & Rothery (1984) label "examples and exercises"; the extent of this type of structure is probably not found in other genres.

Even within the domain of secondary school text books there are major differences in the forms of language used, varying with the age of the students, with their supposed 'ability' (Dowling, 1991), and with the particular type of pedagogic relationship between author and reader (van Dormolen, 1986; Fauvel, 1991). Indeed, one of the problems in producing this review of the language of school mathematics texts is the fact that most analyses have either focused on extremely limited samples of texts or, apart for some discussion of symbolism and vocabulary, have focused on differences between texts and what makes a particular text easy or hard to read rather than on what differences there might be between mathematics texts and those of other subjects. I cannot claim to have completely characterised the genre of school mathematics texts but merely to have identified a number of features of that genre that may be of relevance in the present study.

2.4 Student writing in the traditional mathematics classroom

Before the general introduction of assessment by coursework brought about by the advent of GCSE, major studies of the writing taking place in different curriculum areas in secondary schools in the United Kingdom found very little writing taking place in mathematics classrooms. In England and Wales Britton et al. (1975) and Martin et al. (1976) found too little writing in mathematics for it to be analysed separately. Similarly in Scotland, Rogers &

MacDonald (1985) reported that in some cases no written work had been done in mathematics during the period of their investigation. Spencer et al. (1983), perhaps using a different definition of 'writing', found that large numbers of pupils had produced some written work in mathematics during the week of their survey. However, only 10% of pupils had written anything in their own words (while 72% had copied work) and these had only written an average of 6.2 lines of writing, none producing 'extended' writing over a page in length.

Surveys in secondary schools in the United States (Pearce and Davison, 1988) and Australia (Swinson & Partridge, 1992; Marks & Mousley, 1990) suggest that, in spite of the greater influence of the Writing-to-Learn movement in the professional literature in these two countries, neither the quantity nor the variety of writing use in the mathematics classroom was substantial. Even where some of the teachers involved were committed to the idea of increasing opportunities for students to use language in the classroom, the activities they employed only encouraged writing in a 'recount' genre and did not help students to develop a wider and more sophisticated range of writing skills (Marks & Mousley, 1990).

The limited nature of the secondary school student's experience of producing written text in the traditional mathematics classroom is reflected in the lack of attention to this in the literature. Even Pimm's (1987) extensive discussion of the language of the mathematics classroom restricts its consideration of students' written production almost exclusively to the recording of generalisations and the nature of mathematical symbolism, nevertheless representing a broader view of students' legitimate mathematical writing activity than that characterised by, for example, Laborde (1990) and Bauersfeld (1992) as being entirely circumscribed by the technical format prescribed by text book or teacher. This lack of attention to the form of students' written production may be related both to the limited and tightly prescribed nature of much of school mathematics and to the identification of mathematics with its symbol system discussed in section 2.2.1 above. If it is assumed that the meaning resides only in the symbols then only the 'correctness' of the symbolic formulation needs to be considered; non-symbolic aspects are merely ritual or are relevant only in the affective domain.

Ernest is unusual in considering the rhetoric of the traditional, repetitive type of school mathematics writing, pointing out that, while the text may appear to present the processes gone through in achieving the answer, this rhetoric is in fact only a "rational reconstruction" (1993b: p.9). In his analysis of a single example of the production of a solution and written answer to a trigonometry problem (1993a), he identifies a number of features of the student's writing process which relate to what he calls "the rhetorical requirements of classroom written mathematical language" (p.242), including a transformational sequence leading to a labelled answer. While the task discussed by Ernest is clearly very different from the investigative tasks considered in this study, it seems likely that students will be influenced by their

knowledge of the expectations of writing related to other aspects of school mathematics. It will be relevant when analysing students' written reports of investigations to identify those features which might mark the texts as 'school mathematics', including the repetition of sequences of symbolic transformations and the labelling of answers.

2.5 Analyses of student writing as a research tool

In spite of the growing body of literature and curriculum developments advocating writing as a means of learning mathematics, only a small number of studies have analysed the writing actually produced by the students involved in the course of evaluating the developments. A number of studies have used analysis of student texts within cognitive research into students' understanding. The features of student writing identified by these studies and the interpretations offered will be considered in relation to their relevance to the present study. Studies which have claimed improvements in student writing through innovative writing activities are discussed in chapter 3.

Even in a non-traditional context, motivated by the idea that writing may assist learning, the influence of traditional forms and school text books may be seen. Shield & Swinson (1994) found predominately procedural writing produced in response to a demand for 'expository' writing and suggest that this result was probably caused by the overwhelmingly procedural view of mathematics found in much school mathematics teaching and text books. It clearly cannot be assumed that students will share a teacher's or a researcher's view of the nature of the writing task they are undertaking.

The writing produced in a curriculum development involving mathematical journals in an Australian secondary school is analysed by Waywood and Clarke (Clarke et al., 1993; Waywood, 1992a; 1992b; 1993; 1994). Three categories of text produced within the journals are labelled 'recount', 'summary' and 'dialogue', defined by the organisation of the text. The three types of text are seen, both by the researchers and by the teachers involved in assessing the students' work, as a hierarchy with the dialogue being the most highly valued form. Journal writing is clearly different from the writing of reports of investigative work but the categories of 'recount' and 'dialogue' are also likely to be found in such reports, although their roles within the text, and hence the interpretation of their significance, may be rather different. The recount is likely to appear as a narrative of the problem solving process, while dialogue, with its acknowledgement of the possibility of an alternative point of view, may be found in an argument supporting the student's conclusions. The finding that the dialogue was valued highly by the teachers in Waywood and Clarke's study is particularly interesting. In spite of the researchers' association of type of text with orientation towards mathematical knowledge, they present no evidence of whether the teachers involved shared this interpretation or of the reasons for the value they placed on dialogue.

In the context of investigative work in mathematics at primary level, a discussion by Billington & Evans (1987), illustrated by children's texts, suggests that the language used strongly influenced the authors' evaluation of the students' cognitive processes. Impersonal style or the use of narrative forms were taken as evidence of a student's skills in problem representation and solution.

In much research in mathematics education it is assumed that there is a simple transparent relationship between the student's intention, the text she produces and the researcher's interpretation of that text; there has, therefore, been little systematic consideration of the forms of language used in student texts and the relationships these may have with the student's thinking. The main exceptions to this have been in the area of logical reasoning and proof, perhaps because this is an aspect of school mathematics which is difficult to express entirely in symbolic form⁴. A full review of this area is beyond the scope of the present study, particularly as most of it is located within the French curriculum and has little connection to the sort of mathematical activity or writing expected of students within the present UK system. However, a number of studies have indicated the ways in which linguistic features of texts, especially features which indicate the author's personal involvement, may be interpreted by those involved in mathematics education as signals of particular types of cognitive activity. The interpretation of such features depends strongly on the context. For example, while it might appear that Balacheff's (1987) insistence on the symbolic and the impersonal as an indicator of a focus on the general rather than the particular is in opposition to Duval's (1989) interpretation of personal language as a sign of a higher level of engagement with proof, an examination of the types of personal processes involved suggests that the two positions are compatible. Whereas Balacheff's examples show the student writers to be performing material actions such as calculating and drawing examples, Duval's are performing mental actions, reflecting on their state of knowledge rather than on describing a series of events. Moreover, Coquin-Viennot's (1989) study of written explanations showed differences in the degree of personal implication in texts produced in a variety of social settings. Although the primary purpose of these studies was not to describe the forms of language used by students in writing about mathematics, their descriptions of students' writing and their use of linguistic features as signals of particular types of mathematical thinking help to identify features of student's writing that may be interpreted as significant and as mathematical by readers. In particular, the explicit presence of the writer in the text is clearly influential in the way in which the readings of the text and of various aspects of the writer's understanding and competence are constructed.

⁴Although Balacheff (1987) identifies use of a strictly symbolic language as the highest level of proof, he admits that in practice there is normally recourse to a mixture of 'natural' and symbolic language.

While acknowledging the wide variety of forms of language that may be found in texts that may be recognised as arising within mathematical practices, I have attempted to identify features that may serve to make it likely that a particular text will be seen to be mathematical. In the context of the present study, it is of interest to consider the extent to which students' mathematics coursework texts share the characteristics of texts from within other mathematical practices, in particular the high status texts of academic research mathematics and of more traditional school mathematics, and the extent to which teacher-assessors may appear to consider such features to be appropriate to the coursework genre.

3 Learning to write mathematical text

Recent curriculum developments related to the introduction of investigative work into mathematics classrooms have focused attention on students' writing in two ways. Firstly, some of those advocating investigative ways of working have simultaneously advocated writing as part of the investigative process, claiming that writing can help students in their learning of mathematics, in particular in supporting moves towards the symbolisation of generalisations (e.g. James & Mason, 1982; Mason, 1987) but also in supporting reflection and the development of problem solving processes (e.g. Mason et al., 1985; Ollerton & Hewitt, 1989; Whitworth, 1988). Secondly, the introduction of assessment by coursework at GCSE, including the assessment of reports of investigative work, has ensured that the great majority of mathematics students and teachers in secondary schools have had to be involved in the production of extended writing, in many cases for the first time. In the coursework context, although the use of writing may be justified by reference to its learning benefits, the main purpose of the written report of investigative work is as a means by which students' achievement may be assessed. It is, therefore, of importance to the student that the quality and style of their writing should be such that their teacher-readers will evaluate their achievement highly. As Kress (1990) argues, it is not necessarily the case that all students will develop the linguistic skills needed to produce such writing within the particular genre without help.

As was seen in chapter 2, however, studies have consistently revealed very limited writing taking place in mathematics classrooms. Even since the introduction of coursework, there has been remarkably little attention paid in the UK to writing in mathematics. The National Writing Project, which coincided with the beginning of GCSE, identified the writing of coursework reports in mathematics as one of the areas in which work needed to be done (White, 1991), but, although two reports of work related to mathematics are to be found in the project newsletters (National Writing Project, 1987; 1989), there is no evidence, for example in professional journals, that the project succeeded in disseminating its work more widely among mathematics teachers. In spite of increasing interest in the role of oral language in mathematics education, little attention has been paid to the role of writing. A review of perspectives and current issues related to language in mathematics education (Durkin & Shire (eds.), 1991) contains papers by a selection of mainly UK authors; students' production of written language is dealt with in the context of the writing of numerals and the problems encountered by dyslexic children but the only mention of more extended writing is made by Pimm whose contribution suggests briefly that "oral reporting back" on investigative work can help in preparing for "coursework write-ups" (1991: p.22).

Nevertheless it appears to be widely recognised that some students are not successful at producing extended writing and that the requirement to write may prevent them from doing the mathematics, or at least prevent them from being assessed to have done the mathematics (e.g. Ball & Ball, 1990). A particular difficulty with non-narrative writing is noted by Waywood in the context of journal writing:

What is often seen in journals is the start of a dialogue which is cut off in mid-flight because the students haven't the control over language, or their thinking, or the material, to carry the dialogue through. (1992a: p.38)

Waywood's difficulty in identifying which of the suggested factors is lacking indicates the difficulty of making a clear distinction between the form and the content of a text. In a context in which texts are to be used to assess mathematical problem solving, it is of importance to the students that they should not be judged to lack control of their thinking or of the material because of their lack of control over the language. This concern has not, however, been substantially addressed. Where classroom experiences of investigative work are described or analysed, writing appears merely as a background activity that does not require specific attention. Other authors, concerned with the difficulty that students have in writing a complete record of their investigative processes for assessment purposes, merely advocate that the importance of the written record should be lessened and other means of communication valued more (e.g. Bloomfield, 1987; MacNamara & Roper, 1992a; 1992b). The concern that students' writing skills may not be adequate to represent their problem solving activity has been accompanied by little attempt to enhance the writing itself.

3.1 Natural development of mathematical writing

Even where attention that has been paid to the use of writing to learn mathematics¹ there is relatively little work considering how students may learn to write mathematically. General exhortations to include opportunities for writing in the curriculum tend to assume that the means of communication will develop naturally². Thus the influential NCTM Standards (1989), while discussing in some detail the power of language as a tool for learning, merely suggests that its development:

¹There is a growing body of literature concerned with claims about the value of writing as a support to learning in mathematics, particularly in the United States and Australia (e.g. Connolly & Vilardi, 1989; Borasi & Rose, 1989; Borasi & Siegel, 1994; Miller, 1992a, 1992b; Powell & Ramnauth, 1992; Waywood, 1992a)

²Much of the Writing-to-Learn movement in all areas of the curriculum is rooted theoretically in Britton's theory of language development which prioritises expressive forms of writing in the early stages of learning, claiming that other forms will develop from these (Britton et al., 1975). This appears to have been widely interpreted to mean that the development is natural and spontaneous.

is best accomplished in problem situations in which students have an opportunity to read, write, and discuss ideas in which the use of the language of mathematics becomes natural. (p.6)

This assumption of natural language development is common to many descriptions of programmes, particularly those involving younger children, which claim to develop mathematical communication (e.g. Mumme & Shepherd, 1990; Carton, 1990; Wilde, 1991; Greenes et al., 1992). McIntosh (1991) even seeks to encourage mathematics teachers to include writing-to-learn among their teaching techniques by stating categorically "We don't need to teach writing" (p.423).

Consequently, little attention ~~has been~~ paid to the various conventional forms of mathematical writing or to ways in which students may be helped to attain them. This approach to learning and language development has been criticised by, among others, Williams (1977) and Sheeran & Barnes (1991) who (although from different perspectives) argue that the distinctive forms of language used in different disciplines are closely linked to the distinctive ways of looking at the world and structuring knowledge that are characteristic of those disciplines. If students are to be required to produce writing in mathematics, it cannot be assumed that they will naturally develop forms which are conventionally appropriate or which will display 'mathematical thinking'.

Where there is discussion of how teachers might help school students to develop their use of mathematical language the method is usually aligned explicitly or implicitly with Graves' (1983) model of 'process' learning of writing, involving the first steps of prewriting, drafting and revising, often after peer or teacher review of a first draft. The use of peer reading of student writing³ and 'conferencing' as part of the drafting and revising process is found in a number of studies (National Writing Project, 1989; Havens, 1989; Keith, 1989; Gopen & Smith, 1989; Duncan, 1989; Hoffman & Powell, 1992) which claim consequent improvements in the quality of writing produced. Another technique that is suggested to help students to develop their writing processes is the modelling of writing behaviour by the teacher (Wilde, 1991; Richards, 1990).

Marks & Mousley (1990; Mousley & Marks, 1991) criticise this 'process' approach for the limited experience of different genres of mathematical writing that it is likely to provide for children. They refer to critics of the way in which 'process writing' has been implemented in language education, who have observed that it gives rise to predominantly narrative writing, and claim that "Writing is an *unnatural* act: it needs to be learned" (Reid, 1988, cited in Marks

³The use of peer reading during the drafting and revising process is also related to the idea that having a real audience will improve the quality of writing. The issue of audience for writing is discussed in section 3.3 below.

& Mousley, 1990: p.134). They argue that, in order to develop mathematical literacy, children need to learn a wide range of the types of writing used in mathematics and recommend that students should be led “to development of an explicit understanding of the role of language in specific mathematical contexts” (p.133) through critical reading of alternative models of writing by adults, including their teacher, and by their fellow students.

The most deliberate attention to teaching students to acquire mathematical writing skills has been at college level in the United States (e.g. Snow, 1989; Gopen & Smith, 1989; Paik & Norris, 1984; Price, 1989). Although some authors simultaneously use the rhetoric of Writing-to-Learn, there is greater concern with the students' need as future professionals for technical writing skills. The descriptions provided of the methods used to help students to achieve better mathematical writing range from drafting and revising with teacher comments (Paik & Norris, 1984; Snow, 1989) to Price's (1989) provision of fully detailed “guidelines” with rules for good writing. All these authors claim major improvements in student writing during their teaching programmes although little evidence is generally offered.

3.2 Evidence of development in mathematical writing

Waywood (1994), Powell & López (1989) and Powell & Ramnauth (1992) report ‘improvements’ in student writing in long term programmes involving mathematics journal writing. The finding that the quality of students' writing (as defined by the teachers and researchers involved) developed over time points to the possibility that, with extensive experience and feedback from teachers and from peers, students come to learn the features of the genre that will be valued by their teachers. Driscoll & Powell (1992) describe writing arising from more discrete writing activities in which students wrote in small groups. Whereas the first drafts included features such as the use of additional personal knowledge and personal involvement in the text, the final text produced by the group is described as:

clear direct and concise . . . without personal references . . . resembles the level of abstractness and precision that opinion would have us believe is the interpretive starting point of readers' response to transactional text. (p.262)

It would appear that such peer group feedback and editing provided these mature students with a context which prompted them to use some of the characteristic features of formal mathematical texts.

While any development in the cases discussed above seems to arise from the sort of feedback and editing associated with the ‘process’ approach to writing, few programmes have paid deliberate attention to ‘appropriate’ forms of language. Exceptions include Penniman (1991) and Gopen & Smith (1989). Penniman describes a programme of action research in which primary children were involved in explicit discussion of the forms of graphs and their effectiveness in communicating information; this is claimed to have led to development of

more consistent use of conventional forms and apparently better appreciation of the meanings communicated. Gopen and Smith (1989) provide evidence that drawing undergraduate students' attention to the language of their reports of computer-based investigations through asking questions such as 'what actions are taking place?', 'what is new and important?', 'who is the agent?' was effective in developing the students' grasp of the genre required by the course.

The questions used by Gopen & Smith, by paying attention to the transitivity system (the actions and actors) and the thematic structure (identifying what is new) address the way in which the nature of mathematical activity is constructed within the text as well as the type of argument that is constructed. Similar questions will be addressed in the analysis of students' texts in the present study. However, Gopen & Smith use these analytic tools in a normative rather than a descriptive way. For example, their theoretical position leads them to value more highly those texts in which the agency is clear; they mention in particular their dislike of the use of nominalisations and their wish to see the student-author's actions made explicit through the use of personal pronouns. It is not certain that similar qualities will be valued by other teachers, particularly where the context of the writing is different.

One aspect of mathematical writing which has received some attention in the UK is that of the use of algebraic symbolism, although the focus has normally been on the development of algebraic thinking rather than on the notation itself. James & Mason's (1982) sequence of seeing-saying-recording, may be seen as an approach that, while valuing children's own forms of writing, simultaneously sees them as leading, with teacher support, to conventional mathematical forms. It includes a role for the teacher in assisting the development of more conventional forms by intervening "to plant the seeds of helpful language patterns and recording devices" (p.256). This focus on the development of symbolisation, however, does not address the production of other forms of writing required within the investigation report.

3.3 'Audience'

While the student's own teacher is the actual reader of the coursework text, consideration of the teacher's own state of knowledge about the subject matter may not help the student to produce writing that will be evaluated as effective. It is sometimes suggested that students may be helped to write more effectively by addressing imaginary audiences such as "your friend in Australia" (School Mathematics Project, 1989: p.8) or someone who "is intelligent but knows nothing about your assignment" (Bull, 1990: p.105)⁴. In this section, research and issues related to audience in school writing are considered.

⁴Bull (1990) is a guide for students doing coursework. It is discussed in Appendix 15.

In their discussions of the audiences for school writing, Britton et al. (1975) categorise the various audiences that children may write for in school in terms of the relationship expressed in the writing between the writer and the reader: self; teacher (subdivided into: child to trusted adult; pupil to teacher, general; pupil to teacher, particular relationship; pupil to examiner); wider audience, known (subdivided into: expert to known layman; child to peer group; group member to working group); unknown audience; virtual named audience; no discernible audience. Both Britton et al. and Applebee's (1984) study found that student writing is most frequently addressed to the teacher-as-examiner.

These categorisations, however, while providing a useful framework for thinking about school writing, are not as straightforward as they might seem; there are problems both in the application and in the theoretical basis of the concept of *audience*⁵ used in these studies. Both the studies mentioned and most others concerned with audience in education have adopted an apparently naive view of audience which fails to take into account the dichotomy between the audience *addressed* by writers, the "actual readers external to their texts", and the audience that writers *invoke* "within their texts, teaching their readers through textual cues how to relate to and read a given text" (Willey, 1990: p.26). Studies of the practices of writers suggest that they may create an imaginary reader (Odell, 1985) or manipulate their relationship with their reader by deliberately breaking the usual conventions of the discourse (Faigley, 1985).

3.3.1 The teacher-reader

As identified above, the audience for the vast majority of school writing is the student's teacher. Purves identifies a multiplicity of reading roles that a teacher may adopt, ranging from "common reader" through, among others, "proof reader", "gatekeeper", "critic" and "diagnostician". The student-writer:

must learn to deal with all these kinds of readers, know something of what the concerns of each might be, and know that a writing is not simply to or for *an* audience, but that the text is read variously by different people for different purposes, but also variously by the same reader. (1984: p.265; original italics)

While Purves suggests that any or even all of these roles may be adopted in reading a single text, most studies of school writing have focused primarily on the teacher's role as a judge of student writing. A crucial problem with writing that is addressed to a teacher-as-examiner is that the teacher-as-examiner is "not personally involved in the topic" (Redd-Boyd & Slater, 1989). She is an "overhearer" (Bell, 1984) rather than the rhetorical audience of the text; her role is to judge whether correct information has been included and whether the writing is persuasive, rather than to be informed or persuaded herself. As Kinneavy (1971) points out,

⁵Rather than using italics or inverted commas every time I use the word 'audience', I would ask the reader to bear in mind throughout that this concept is not unproblematic

this is an artificial context for writing where the student is expected to write informatively to a reader who is already completely informed about the topic. This may cause difficulties for writers in deciding what and how to write. It also may not be the best context for developing writing or subject skills. As Applebee says, "the teacher-as-examiner can be a very undemanding audience" (1984: p.3) because she is looking for "evidence" within the text and may construct the "argument" for herself. Thus in some cases such a teacher may read a text which contains specific desired pieces of information without noticing that it lacks textual or logical coherence. Adaptation to the needs of such an audience does not necessarily produce forms of writing that would otherwise be considered of high quality (Spencer et al., 1983).

3.3.2 Specifying non-teacher audiences

The main feature of the school writing tasks that Applebee (1984) categorises as for a "wider audience" is that they instruct the student writer to address an audience other than the teacher; this audience may be real or imaginary, specific or general but in almost all cases the child knows that the teacher will be the most important (and often the only) reader. In fact, Spencer et al. (1983) found that many children did not consider that being assigned an imaginary audience was helpful, suggesting that they recognised that the teacher was the main addressee. Butt suggests an alternative interpretation of this, based on his attempt to introduce "audience-centred" writing tasks into geography classes:

It was obvious that some children would have been much happier writing for the normal restricted audience of the teacher assessor as this required far less thought, originality and effort. (1991: p.76)

As Long (1990) points out, the writing teacher's advice "don't forget your audience" is superfluous because students are always aware of their audience - the teacher. Gilbert's (1989) case studies of high school writers showed that following such teacher advice rigidly was not necessarily the best way to get good marks; the most successful student "deliberately ignored" some instructions (p.132), predicting instead what the teacher *really* wanted.

Herrington's (1985) study of writing in undergraduate chemical engineering courses in which hypothetical audiences were specified showed not only that perceptions about the identity of the audience differed between teachers and students but also that perceptions of the characteristics of the audience differed, particularly perceptions of the amount of knowledge about the subject matter. Differences in perceptions about the amount of information that needs to be included are likely to lead to negative evaluations of student writing by the teacher, both where the teacher perceives the amount of information included to be too little and where it is perceived to be too much. Herrington argues that it might be more helpful to students to be honest about the purposes of school writing rather than to provide mixed messages about imaginary audiences.

The complexity of the relationship between student-writer, teacher-reader and assigned audience is also demonstrated by studies considering the effects of assigning audiences on the writing produced (e.g. Crowhurst & Piché, 1979; Donin et al., 1992; Hayes et al., 1990; Prentice, 1980; Redd-Boyd & Slater, 1989; Rubin & Piché, 1979; Quick, 1983). Factors which may play a part in the amount of adaptation made for a specified audience include the students' level of socio-cognitive development (Hayes et al., 1990), their knowledge of the subject matter (Prentice, 1980), their relationship with the supposed audience (Prentice, 1980; Butt, 1991), as well as the extent to which they genuinely adopt the assigned audience (Redd-Boyd & Slater, 1989).

In a mathematical context, the evidence of Guillerault & Laborde (1982; 1986) suggests that calling up a hypothetical reader seems unlikely to help the writer to identify what might conventionally be considered to be ambiguities. Moreover, Schubauer-Leoni et al. (1989) show that, in the classroom, children (aged 8-9 years) still use the conventions of formal mathematical notation when asked by the teacher to write "so that the other children would understand" or when asked to write by and for a non-teacher adult, although outside the classroom context "their written solutions are more heterogeneous in nature using natural language, illustrative drawings etc." (p.675). It would seem that the institutional setting is likely to have a stronger effect on students' perceptions of the task of writing coursework, the content they choose to include, and the forms of writing they choose to use than any exhortation to imagine a non-teacher audience.

In this chapter a general lack of attention to the form of writing in mathematics classrooms has been identified. It has been argued that an assumption that mathematical forms of writing will develop naturally is unwarranted and that there are problems related to the question of 'audience in school writing, both when the audience is unambiguously the teacher and when a non-teacher audience is specified. This review leads to the conclusion that deliberate attention to forms of language may be a more effective way of supporting learners. Hence one of the aims of the present study is to identify those forms which may be considered 'appropriate' in the context of reports of investigations.

4 The assessment of investigations and teacher assessment

The reports of investigations written by students cannot be analysed without considering the social context within which they are embedded. One of the most important aspects of this context is the fact that the reports are used as assessment instruments in an external examination. This affects, in particular, the relationship between the student-authors and their teacher-readers. The GCSE examination is clearly a 'high-stakes' assessment both for students, whose future education and job prospects may be affected, and for their teachers, whose professional reputations are at least partially dependent on the performance of their students. At the same time, the introduction of investigative work into the mathematics curriculum has aims that may be in conflict with those of the assessment process.

In this chapter, the characteristics of the discourse of investigative work are summarised and methods of assessing investigative work are discussed in the context of similar new developments in assessment internationally which aim to reflect, if not lead, curriculum development in mathematics education. The relationship between the values of the discourse of investigative work and its assessment is discussed. In particular, the assessment of divergent and creative work and the assessment of the use of mathematical forms of communication are considered.

One of the crucial features of coursework is the fact that it is assessed by teachers themselves rather than by external examiners. As assessors of a coursework text, teachers assign value to the whole text and to particular features of the text. These values not only serve to assign a grade to an individual student's text and hence to the student herself but also influence the teacher's behaviour in the classroom and, to the extent that they are communicated to students, the writing of the texts themselves. An investigation of the ways in which teachers read students' texts and interpret and assign values to them is thus an important part of the present study. In this chapter, therefore, issues related to teacher assessment are considered, including the question of its consistency and the small amount of research into teachers' assessment practices. As a major focus of the present study is the role of the form of the student's written text, the research on the relationship between the form and teachers' evaluations of writing is reviewed.

4.1 The discourse of 'investigation'

The texts that will be analysed in this study were written in response to tasks labelled as 'investigations'. Such tasks are located within a discourse which influences the participants' understanding of the characteristics of the tasks themselves and of students' ideal responses to them. An awareness of this discourse is essential in order to interpret the significance of features of students' texts and to understand teachers' readings of the texts. Appendix 15

contains a monograph presenting an analysis of the discourse of 'investigation' through examination of a number of key publications drawn from official curriculum documents, guidance for teachers and students and the professional literature. The main points of this analysis are summarised here.

There is substantial agreement about the intended properties of the ideal 'investigation':

- it is essentially mathematical and involves students in activities similar to those engaged in by professional mathematicians;
- its learning objectives are predominantly (but not exclusively) related to process rather than content;
- it is exploratory, creative and empowering for students and may have multiple valid outcomes;
- it is part of 'good classroom practice', and, hence,
- it ought to be included in assessment programmes.

These properties form the rhetorical basis for curriculum and assessment development involving investigative work, including the introduction of assessment by coursework at GCSE. There are, however, a number of areas of tension within the discourse, arising particularly strongly in the context of assessment:

A way of working or a special kind of task? While the rhetoric of official and professional publications suggests that investigative ways of working are an everyday part of mathematics teaching and learning, the requirement to assess investigative work at GCSE (DES, 1985) has made it necessary to distinguish particular types of task that will satisfy the assessment requirements. While GCSE regulations demand that both practical and investigative work should be assessed, the distinction between these is not clear.

Any length or an extended task? Again, the rhetoric suggests that an investigation may be of any length and may even be a brief episode within an otherwise non-investigative lesson. However, the value placed on students exploring and asking their own questions has led to the idea of 'extending' tasks set by the teacher and hence to lengthy tasks. In the assessment context, creating an 'extension' has become one of the indicators of high attainment.

Creative mathematics or standardised tasks? In spite of the value placed on creativity, institutionalisation has led to the use of stereotyped tasks and routine procedures, including the data-pattern-generalisation (Wells, 1993) inductive algorithm. It is largely recognised, at least in the professional literature, that such routines do not share the characteristics of investigation listed above.

Process or content or both? There is an inherent difficulty in separating process from content. Although assessment schemes may list only process related criteria, it is clear from examining practical guides for teachers (Pirie, 1988) and students (Bull, 1990) that content related issues such as accuracy and the 'difficulty' of the mathematics used, in particular the presence of 'algebra', are also relevant in assessing investigations.

While there is a perception that students find writing reports of investigations difficult, there appear to be no linguistic means within the discourse to describe the desired characteristics of student texts. These characteristics are implicitly defined through the use of examples. The only advice explicitly provided for students, however, relates to the presentation of results and the overall presentation; no support is provided for the communication of processes, which are supposedly the main objective of investigative work and the assessment of coursework.

4.2 New developments in assessment

Although the particular form of the introduction of investigative tasks and their assessment in the UK has some unique features, there has recently been considerable international concern with the development of new modes of assessment in mathematics, including assessment of extended projects and of problem solving skills (e.g. Charles & Silver, 1989; Houston, 1993; Lesh & Lamon, 1992; Niss, 1993). In particular, 'performance' or 'authentic' assessment is being discussed: that is, assessment which supposedly either reflects 'good classroom practice' or actually assesses the learning that takes place during everyday classroom activity, often involving teachers directly in the assessment process. This concern is not only in mathematics but in all areas of the curriculum. In the UK it has been manifested in the development of GCSE coursework, the National Curriculum assessment procedures and instruments, and a variety of vocational education initiatives (Torrance, 1995a). While general interest in assessment may have arisen from political concern with 'standards' and 'accountability' (Eisner, 1993), the concern of many mathematics educators with 'authentic' assessment has been related to the belief, stated very clearly by Burkhardt (1988) and by Ridgway & Schoenfeld (1994), that the success of curriculum development depends on the use of assessment that reflects and supports the aims of the curriculum. This belief has been an explicitly stated motivation in the development of mathematics coursework assessment in the UK at A-level (Sulke, 1990) and undergraduate level (Haines, 1991) as well as at GCSE (ULEAC, 1993). In considering assessment led curriculum innovation such as GCSE coursework, it is necessary to consider the extent to which the assessment-led nature of the innovation may distort its curriculum focused aims.

The development of such new methods of assessment in mathematics has been largely "intuitive" (Collis, 1992), and, Ruthven (1994) argues, inadequately theorised assessment

methods and practices thus come to define the curriculum. There are a number of tensions and contradictions between the espoused aims of curriculum developments and the consequences of assessment practices. For example, Galbraith (1993) argues that the constructivist views of the nature of learning and problem solving that have driven curriculum reform are incompatible with the view that the assessment of open-ended problems can provide a measure of problem solving ability. Moreover, he rejects the generally accepted idea that external assessment requirements should be used to influence the curriculum on the grounds that, viewed from a critical perspective, it is

ultimately disempowering to teachers in impeding the growth of full professional responsibility, and to students in making their choices and interests irrelevant. (p.82)

This brings into question the ideals of openness and 'empowerment' inherent in the discourse of investigation (section 4.1 and Appendix 15).

4.2.1 Methods of assessing investigative work

As teacher assessment of coursework at GCSE was about to be introduced, Foxman et al. (1986) reported a debate comparing different approaches to the assessment of investigative work. They identified three approaches which they labelled: 'Mathematical process criteria', 'Judgement on autonomous merit' and 'Simple focusing categories'. These parallel approaches to assessing problem solving identified by Lester & Kroll (1990). Most of the GCSE examination boards initially developed generic 'mathematical process criteria' reflecting heuristic models of problem solving and making no distinction between the criteria to be applied to different tasks.¹ William (1994) identifies two major problems with such assessment schemes. Firstly, they take no account of the difficulty of the mathematics involved in a given problem; for example, the criterion "formulates general rules" (ULEAC, 1993) makes no distinction between the formulation of a simple linear rule and one involving much more complex relationships. Secondly, the heuristic model behind the criteria defines a progression through the task; where a student's work diverges from the defined progression it becomes difficult to assess. In consequence:

it appears that teachers have 'played safe', and used only coursework tasks that conform to the model of progression and the particular calibration implicit in the generic descriptors. (William, 1994: p.53)

One of the consequences of such 'playing safe', as Wolf points out, has been the increasing preponderance of the use of stereotyped 'investigations' over 'practical' tasks in order to "offer able pupils greater opportunities to display the type of work required to obtain high grades" (1990: p.141).

¹The set of criteria (formulated into 'grade descriptions' of work at three different levels) used by the London examination board, whose tasks are used in the present study, may be found in appendix 1.

In response to the difficulties that teachers experienced in applying such generic criteria, the London examination board also issue task specific “performance indicators” exemplifying the typical outcomes of a given task for students at each grade level.² In spite of statements from the examination board that alternative outcomes may be equally valid and should be assessed as such, William, drawing on his experience of work with teachers during the development of the Graded Achievement in Mathematics Scheme, claims that:

by delineating particular ‘canonical’ responses, the task-specific schemes appear to lead teachers to direct students towards approaches that yield more easily ‘assessable’ responses. (1994: p.54)

The characteristics of the assessment scheme are thus in conflict with some of the stated aims of the curriculum development they are supposed to support, in particular the aims of ‘empowering’ students by valuing individual, alternative and creative responses.

4.2.2 Assessing creativity

While the discourse of investigations values diversity and creativity, these are not traditionally valued by assessment practices in mathematics (Galbraith, 1993) and there are problems for teachers in attempting to come to terms with them. Before the introduction of GCSE coursework, Haylock suggested that mathematics teachers’ attempts to promote systematic approaches had produced the “dull, predictable and narrow” responses that children provided to his questions seeking to measure divergent production (1985: p.550) while the tension between the values placed on accuracy and on creativity “sets up a conflict in the minds of the assessor” (p.548). As Lester & Kroll (1990) point out, having reviewed a number of alternative schemes for assessing students’ problem solving behaviour, none adequately measure a student’s “willingness to take risks” (p.69).

Indeed, valuing and encouraging risk taking seem antithetical to the desire for reliability in teacher assessments. Ahmed & Bufton (undated), addressing teachers beginning to use investigative work, recognise this tension but do not provide guidance as to how acceptance of a variety of responses to “open ended starters” should be achieved and valued. In reporting their project on the development of mathematical modelling skills, Tanner & Jones (1994) deny that variety of outcomes is a serious problem for assessors, claiming that:

Good solutions are “reliably recognizable” (Scriven, 1980). Mathematicians work to a set of assumptions, often unstated, related to generality, economy and elegance. Teaching students mathematics must include acculturation into these assumptions. (p.422)

Although the authors recognise that there are differences between the traditional culture of the mathematics classroom and the mathematical culture they describe, they again do not address the question of the acculturation of the teachers themselves, perhaps assuming that

²The performance indicators for the tasks used in this study (LEAG, 1991) are in appendix 2

this is merely a question of will on their part. What teachers 'reliably recognise' as good solutions must be constrained by their knowledge and experience of the examination system and of school mathematics as well as by their own beliefs about mathematics (which do not necessarily coincide with the assumptions listed by Tanner & Jones).

While the valuing of creativity is a relatively new development in school mathematics, it has a longer tradition in some other subject areas. Here too, however, its assessment is not unproblematic, as may be seen in considering a report by Dixon & Brown (1985) on teachers' and examiners' ratings and comments on the 'response to literature' aspect of 'A' level English students' coursework and examination scripts. While the value placed on originality (here labelled "personal involvement" and "imagination") is made explicit in the report, students were also praised for showing understanding of the 'intention' of the author whose work they were discussing. This suggests that only those 'personal' responses which coincided with the interpretations of the assessor would be valued. Clearly, as Haylock (1987) points out, 'appropriateness' is also required in order to validate creative production.

In the context of GCSE, a nationally recognised and externally moderated examination, it is not surprising that teachers should be concerned that their assessments should be seen to be consistent with those of others. This concern has given rise to the deliberate construction of consensus through the publication of performance indicators and assessment schemes and through in-service training and school level collaboration. This process may, however, serve more to ensure that teachers advise students towards standard responses than to help teachers to know how to value creative ones; as Pike & Murray (1991) are pleased to report: "problems with rogue strands, off beat ideas etc. . . soon disappear" (p.33) when teachers have been involved in such in-service activities. While there is evidence that supports the view that mathematics teachers are remarkably consistent in their assessments of coursework (see section 4.3.1 below), there does not appear to have been any investigation either of the extent of 'original' content in students' coursework texts or of teachers' assessment practices when faced with work that they perceive as unusual.

4.2.3 The assessment of 'mathematical communication'

Although the procedures and sets of criteria for the assessment of investigative work vary between the various GCSE examination boards, all of them include some assessment of 'communication'. The actual nature of the forms of communication is specified to varying extents by the different boards. Figure 4.1 below shows the parts of the London board's grade descriptions that refer to the forms of communication expected to be used by students; this is not untypical of the examination boards.

Figure 4.1

Forms of mathematical communication expected of students by the London examination board (extracts from LEAG, 1989: p.61)

At grade F:	Produces some sketches and graphs and, where appropriate, computer output. Able to make limited use of mathematical terms.
At grade C:	Uses an adequate range of mathematical language and symbols, including appropriate visual forms and, where appropriate, computer output. Uses some mathematical words relevant to the task and is generally familiar with the vocabulary of Level I.
At grade A:	Where appropriate, makes use of symbols when generalising. Selects the most appropriate methods for communicating results. Makes effective use of a range of mathematical language and notation, diagrams, charts and, where appropriate, computer output

The picture of the nature of mathematical communication constructed by such assessment criteria is a collection of discrete components: vocabulary, algebraic symbolism, forms of visual representation such as tables, graphs, etc. While it is clear from other criteria that extended writing is expected, the characteristics of such writing are not specified. As was seen in chapter 2, this does not provide an adequate description; in particular it fails to take account of higher level characteristics of a mathematical text, for example, the ways in which arguments are formed. This omission is consistent with the finding (Spencer, 1983; Langer & Applebee, 1987) that teachers in curriculum areas other than English tend to associate the language of their subject only with its specialist vocabulary rather than with its stylistic aspects. Such a lack of explicit knowledge of the range of features that characterise mathematical forms of communication within the investigation report genre does not necessarily mean that teachers are unable to recognise examples and judge them to be 'appropriate'. In the context of assessment of students' journal writing, Waywood (1994) claims that teachers reading students' mathematical journals were able 'intuitively' to distinguish between different types of writing. Appeal to teacher intuition, however, while contributing to the likelihood that teacher assessment of communication in mathematics will be consistent, does not provide any help for the student who does not share such intuition.

4.3 Teacher assessment

In spite of the recent increased international interest in 'authentic' or 'performance' assessment, including assessment carried out by teachers, there has been very little research into its practical implications (Gipps, 1992; Torrance, 1992). Since the introduction of the National Curriculum, there has been some increase, particularly at the primary level. This appears to have been motivated largely by the novelty of the introduction of 'high stakes' assessment at this level and by a perception that such assessment is likely to be problematic for primary teachers. As Torrance (1995b) points out in discussing the introduction of GCSE,

secondary teachers were already involved in some aspects of examination practice; the extension of their involvement to include a greater degree of teacher assessment may have been accommodated more easily. The process of assessment itself appears generally not to be seen as problematic at the secondary level, although concern has been expressed about the integration (or lack of integration) of assessment tasks into the classroom (Torrance, 1995b; Scott, 1991).

4.3.1 The creation of consensus in assessment of investigative work

There has been some concern expressed about the validity of current methods of assessing investigative work in mathematics, particularly about the extent to which assessment of the written product alone provides a true and full record of the student's mathematical achievement (Bloomfield, 1987; MacNamara & Roper, 1992a, 1992b; Tanner & Jones, 1994)). In practice, however, more attention has been paid to the question of reliability. The measure of the success of a scheme for teacher assessment appears to be taken to be the extent to which teachers arrive at the same judgement and the achievement of consensus among assessors is taken as a sign that the judgements themselves are in some sense 'objective' (as for example in Hoge & Colardarci's (1989) review of research into the 'accuracy' of teacher assessment) or that the assessment scheme "reflects those values of the intellectual community from which the tasks were derived" (Haines & Izard, 1994: p.379). Such reliability does not, however, guarantee that an objective statement of those values is possible.

Evidence of the success of the training of teachers to assess mathematics coursework consistently is largely anecdotal (e.g. Banwell, 1987; Pike & Murray, 1991; Wiliam, 1994). Gill's (1993) account of work with student teachers suggests that, through working in groups, even inexperienced assessors can very rapidly come to close agreement about the level of investigative and practical work. Apart from the consensus achieved through the communal development of 'standards', the official process of moderation by the external authority of the examination boards must also have contributed to the development of teachers' assessment practices (Roper & MacNamara, 1993). Teacher assessment in English has a much longer tradition, using similar methods to induct teachers into the formation of holistic judgements of students' work, and here there is considerable evidence of consistency (e.g. Britton et al., 1966; Cooper, 1977).

In spite of the 'objective' appearance of the use of an assessment scheme based on either generic criteria or task-specific performance indicators, the practical application of the scheme relies on the existence of a consensus among the assessors about their meaning. Gill (1993) and Ruthven (1987, 1995) suggest that this consensus is based on the application of a general construct of 'level' or 'ability' rather than on the use of detailed criteria. Roper & MacNamara (1993) found that the constructs of 'level' of secondary and of primary teachers

differed and suggest that these are determined more by group membership than by any 'objective' meaning that might be attached to the criteria.

An examination of the wording of the generic criteria issued by the London examination board (LEAG, 1989; see appendix 1) reveals one significant feature to be the frequent use of subjective qualifiers. In the description of a grade A candidate, "clear" and "appropriate" each occur four times with "relevant" occurring twice; for grade C again "appropriate" is used several times together with "clear" and "adequate", while for grade F the candidate's work is described as "fairly clear" and "not always relevant". The application of such terms can only be understood in a relative way - dependent on the social context and the particular audience reading the student's work. Their undefined character supports the contention that the criteria describe a general construct of what constitutes a good piece of work or a poor piece of work rather than defining its separate properties. As Cherryholmes (1988) points out, the use of such 'transcendental signifiers', by reserving any determination of their meaning to those in authority (teachers and examiners), excludes some students from access to the means of fulfilling the criteria. One of the aims of the present study is to develop a clearer understanding of what such terms might mean in practice.

4.3.2 Research on teachers' assessment practices

While the concern with reliability reviewed in the previous section focuses largely on the outcomes of teachers' assessment activity, the concern of the present study is with the nature of the activity itself - the ways in which teachers make sense of students' texts rather than the grades they allocate to them. Very little detailed consideration has been paid to teachers' assessment practices. Indeed, this was one of the issues raised by Gipps (1992) and Torrance (1992) in their call for research to provide foundations for the implementation of the National Curriculum assessment procedures. Radnor & Shaw (1995) note that publications related to the moderation of GCSE coursework have not drawn upon "detailed fieldwork with teachers and schools" (p.127).

The development of the assessment of GCSE coursework and of the National Curriculum Attainment Target 1 in mathematics appears to have been based on the premise that the assessment of mathematical processes, while not necessarily easy, is not problematic. Filer's (1993) ethnographic case study of teacher assessment of a comparable component of the English curriculum suggests, however, that there are problems related to teacher assessment of students' processes. Filer argues that the teacher's strong expectations about the 'knowledge content' of the writing led to her accepting deviations from the stereotyped content only from those children who were capable of writing clearly and independently. It was thus impossible for her to assess criteria such as 'imaginativeness of expression' for those children whose technical skills were less, in spite of the fact that such skills were supposedly assessed separately. When considering this analysis in relation to the

assessment of mathematics coursework, it suggests that both 'content' and the technical aspects of the writing may affect teachers' assessments of a student's level of achievement in the 'processes' that are the official subject of the assessment. In particular, deviations from the expected response may be less likely to be valued when the student has poor language skills or is perceived to be of 'low ability'.

A Belgian study of teacher behaviour, while set in a context very different from that involved in investigative work, raises some issues that are of interest in considering teachers' assessment practices in general. Rapaille's (1986) analysis of high school teachers' assessment processes makes use of a model of teacher assessment behaviour that involves the use of a "norm product" for a given task as a standard against which to compare students' texts. While the unique nature of Rapaille's "norm product" appears inappropriate in a context in which alternative products may be equally valid and creativity is valued, the idea of a norm or "ideal text" (Miller, 1982, cited in Gilbert, 1989) may be relevant in considering mathematics teachers' practice in respect to investigations. The characteristics of such a text need to be explored.

Rapaille analyses a teacher's treatment of those components of students' answers which differed from the norm. Some differences were observed between the same teacher's treatment of similar divergent components of answers by different pupils and Rapaille's analysis of the teacher's commentary suggests that such differences are the result of 'external' factors, including the influence of the student's answers to previous questions.

The influence of 'external' factors on the teacher's treatment of student deviations from the norm is also remarked by Broadfoot (1995) in her study of primary teachers' administration of the National Curriculum Standard Attainment Tasks (SATs). As well as expressing concern about the lack of standardisation in the context in which the tasks were carried out and in the amount of help provided, Broadfoot suggests that teachers interpreted the criteria inconsistently in order to make the results match their previous assessments of individual children. One explanation for this practice of reinterpreting the criteria to fit the child's perceived level is teachers' wish "to do the best for their own candidates" (Radnor & Shaw, 1995: p141). Several authors comment on the tensions that this wish creates for teachers involved in 'high stakes' assessment procedures, both in the administration and in the assessment of the work. Scott (1991) and Paechter (1995) report variations in practice between teachers and between schools in the amount of support provided to secondary school pupils while carrying out school-based assessment tasks. Paechter remarks on the conflicts that the teachers studied experienced between their roles as teacher and as examiner, in particular the divergence between the concept of being 'fair' to the pupils and that of administering a 'fair' test. Similarly, Radnor & Shaw (1995), report the tensions that teachers experience between what are labelled "outsider" and "insider" perspectives (p.137).

as they moderate other teachers' assessments and attempt to justify their own. While the scope of the present study does not enable detailed consideration of the classroom context within which mathematics coursework is produced, these conflicting roles and divergent concepts of 'fairness' are likely also to play a role in teachers' terminal assessment practices.

In studying teachers' assessment practice, it cannot be assumed that teachers will all use the same methods or share the same attitudes towards the assessment process. In a study of primary teachers' assessment of their pupils' National Curriculum levels, McCallum et al. (1995) analysed teachers' self-reports of their practice to categorise three different types of teacher-assessors. The very different procedures involved in coursework assessment at GCSE and more general cultural differences between primary and secondary schools and teachers mean that the specific categories defined by McCallum et al. are unlikely to be applicable in the present study. Nevertheless, their analysis points to the importance of recognising the complexity of teachers' assessment practices and the possibility of substantial differences both between individuals and between groups of teachers.

4.3.3 The relationship between form and content in teacher assessment

While the ability to use 'appropriate' forms of mathematical communication is one of the criteria applied in the assessment of coursework, it is generally assumed that it is both possible and desirable to apply the other criteria (related to, for example, problem solving strategies or the choice and accurate use of mathematical techniques) independently of the quality of the writing. This separation of 'form' from 'content' is based on an assumption that writing is transparent in conveying the writer's meanings to the reader and that the meanings that are so conveyed are concerned only with the explicit subject matter of the assessment task. Such an assumption, however, is challenged by theories of communication which take into account the social context of the interaction between writer, reader and text and by research into teachers' assessment of student writing.

The relationship between the forms of language used and the meanings that are read from a text is discussed by Hodge & Kress (1993), who stress that language conveys messages about the social situation and, drawing on Bernstein, that different social classes have differentiated access to certain forms of language. Readers (and listeners) make judgements about writers (and speakers), mediated through expectations of class, which become judgements about intelligence, character, grasp of subject matter etc. Kress (1990) exemplifies the effect of this in an educational context through his analysis of two economics essays awarded very different marks by the teacher. While both cover the same areas of 'content' without error, the language of one shows less control over conventional forms of academic argument and is thus assessed to show less control of the subject matter.

Sociological and sociolinguistic studies of the discourse of classrooms (e.g. Mehan, 1979; Cazden, 1988; Edwards & Mercer, 1987) have indicated the influence of the form of children's language on the teacher's acceptance of the correctness, value and appropriateness of the utterance. These studies have been primarily concerned with oral discourse but the conclusion that, in order to be judged positively, students' responses need to be both "academically correct and interactionally appropriate" (Mehan, 1979: p.133) is equally applicable to written discourse. The concept of 'ground rules' is adopted by Sheeran & Barnes (1991) in their study of writing in school to describe the set of teacher expectations that students must fulfil in order to be judged to be successful both in their control of language and in their grasp of the subject matter. Such ground rules vary both between curriculum areas and between different contexts and tasks within a given subject area. As Sheeran & Barnes point out, the ground rules for school writing are not always made explicitly available to students.

Much of the research into the ways in which teachers assess written work has focused on teachers of English. While the interests of English teachers in relation to the assessment of student writing are clearly different from those of teachers of other subjects, the findings do point to the difficulty of separating judgements of content from those related to form, even for those teachers who might be expected to have more sophisticated knowledge about language. The research findings are not, however, consistent in discerning the relative weights given by teachers to the various aspects of the texts. For example, Harris (1977), Stewart & Grobe (1979), Stewart & Leaman (1983), Cavallero (1991) and Freedman (1979a) report various contradictory findings about the weights accorded by teachers to mechanical accuracy, sentence structure, organisation, vocabulary and content. Similarly apparently contradictory and inconclusive results have been reported in relation to the influence of handwriting (Briggs, 1980; Massey, 1983; Soloff, 1973).

Investigations of the effects of stylistic factors associated with academic writing show similarly complex results. While Hake & Williams (1981) suggest that the "syntactic complexity" of a nominal style causes more favourable judgements about content and organisation, more general indicators of complexity (Stewart & Grobe, 1979; Stewart & Leaman, 1983) do not appear to have similar effects. Anderson's (1988) study of the reading of academic scientific articles found that papers which had been rewritten in "simpler" language were judged to be reporting less valuable research. In a school setting, in contrast, Gilbert (1989), citing a study by Freedman (1979b), notes that teachers "reacted badly" to aspects of essays, such as the use of nominalisations, which served to distance the author from the text or to express authority and confidence (p.51). Each of these studies is based within a particular social context with its accompanying conventions and relationships between writers and readers. While their findings may be taken as an indication that the use of a nominal style or other

complex syntactic forms is likely to be influential in the ways in which teachers assess a student's text, the precise nature of that influence (both the value and the meanings associated with the various forms) is likely to vary between contexts.

The importance of considering the context in which the text is read and judged is clear in a study by Hayes et al. (1992) which found that groups of teachers and of students formed different opinions about the 'personality' of the writers of essays. While Hayes et al. do not analyse the particular features of the texts associated with the various character traits, their study points to the potential significance to the assessment process of interpersonal aspects of texts as well as to a possible mismatch between teacher and student expectations and perceptions of those interpersonal aspects.

These studies have all been concerned with 'general purpose' writing in which a holistic judgement of the quality of the essay has been made. The specific 'content' of the writing in such a context, while clearly contributing to teachers' judgements, is probably not as great a concern as it would be in writing based within a curriculum area. Wade & Wood's (1979) study of the assessment of written work in science, however, suggests that features of the form of students' writing, including neatness, technical accuracy and the use of cohesive devices to create a coherent text, also contribute to teachers' evaluation of the writers' level of scientific understanding. Spear's (1984, 1989) studies of science teachers suggest that features such as neatness, presentation and length of the text may be interpreted and valued differently when they occur in boys' or in girls' texts. While the scope of the present study does not directly address gender differences in texts or in teachers' assessments, Spear's findings again indicate the complexity of the contextual factors that must be taken into account when interpreting teachers' readings of student texts.

Very little attention has been paid in the literature to mathematics teachers' assessment of students' written work, perhaps because of dominant assumption that answers in mathematics are unambiguously right or wrong. One exception, looking at students' work in a traditional style of 'problem solving' in the US, is a study by Flener & Reedy (1990) which found that some teachers were unwilling to accept answers that were expressed in unconventional forms. New developments in assessment which involve more open problems have not, on the whole, addressed the issue of the more diverse responses from students. This neglect is questioned by Collis (1992), who, in critiquing new approaches to assessment in the US, provides examples of students' writing in response to an open question in the innovative California Assessment Program. Many students had responded with effective answers in a 'non-mathematical' genre but the assessment criteria only allowed credit to be given to those students who had used technical mathematical terminology. Collis identifies a mismatch between the student expectations (arising from the wording of the problem) and the expectations of the examiners and suggests that the type of response expected by the

examiners should be indicated more clearly to the students. This critique, while recognising that different styles of writing may be produced by students and may be equally effective or acceptable in some contexts, nevertheless still assumes that a more precise statement of the problem or a different set of assessment criteria would ensure that valid assessments of students' mathematical understanding would be achieved.

The research discussed in this section has identified a number of linguistic features that may be influential in teachers' judgements of students' written work and which will be considered in the analysis of student texts in the present study. In particular, in addition to technical accuracy, interpersonal aspects of the texts and those features such as nominalisation and impersonal constructions that are typical of adult academic writing appear to be significant, although the nature of their effects is context dependent. The students' use of conventional mathematical terminology and other forms of representation is also likely to be of interest.

4.4 Conclusions

Consideration of the research discussed in this chapter leads to a rejection of naive views of direct relationships between assessment and the curriculum and between assessment and student achievement. In spite of the claims made for the curricular aims of coursework assessment in relation to the encouragement of investigative ways of working, there are particular problems in reconciling the values placed on creativity and diversity within the discourse of investigation with the values and practices of assessment. The questions raised for the present study by the issue of 'creativity' are: what features do teachers identify as creative in students' texts, what values do they place on such features, and how do they reconcile creativity with the more traditional values of assessment in mathematics.

There is some evidence to suggest that teachers can achieve substantial consensus in their assessment of students' coursework texts. In spite of the official listing of criteria which give the assessment procedure the appearance of objectivity, it appears likely that teachers form their judgements on the basis of general constructs of 'level' or 'ability' or by comparing a student's text with some norm or ideal. The basis upon which teachers make their judgements of students' texts is one of the foci of the present study and the approaches to assessment identified in this chapter will inform the analysis. It has been pointed out that teachers experience some conflict between their roles as teachers and their roles as examiners in the administration and moderation of school-based assessment. The effects of such conflicts between roles will also be considered.

The largely 'intuitive' nature of judgements formed on the basis of such constructs or norms is likely to disadvantage some groups of students. The 'intuition' is formed within a social context in which teachers come to share certain expectations of students' work. These

expectations are, however, shared through practice rather than through explicit statements; even where criteria are applied, the use of 'appropriateness' as an assessment criterion reserves knowledge of the meaning of the criterion to those who already know, without providing guidance for the student who does not already share the teacher's expectations. The present study seeks to elaborate the notion of 'appropriateness' in relation to the assessment of investigative work. A particular focus is the nature of 'appropriate' forms of mathematical communication, as the model of the nature of mathematical language explicitly acknowledged by the examination boards and by teachers is largely restricted to a set of specialist vocabulary and visual forms which does not provide an adequate description.

The 'common sense' view of assessment assumes that it is possible to identify unambiguously and hence evaluate objectively the object of the assessment. There are, however, difficulties in disentangling the content of a text from its form or, where (as in mathematics coursework) the object of assessment is the student's processes, the processes used from the content knowledge and skills. These difficulties are not solely practical, although there is substantial evidence that teachers' assessments are influenced by features of student's work that are not the explicitly stated object of assessment; there are also logical problems in attempting to separate the form of a text from its meaning. The aspects of the language of student's texts that have been identified as influencing teachers' assessments of the value of the text as a whole include interpersonal aspects and the use of features of conventional academic writing such as nominalisations as well as mechanical aspects such as spelling and handwriting. The context-dependent nature of the reading of such features must, however, be recognised; what is valued in one context may be deemed inappropriate in another. These aspects will be considered in developing the analysis of coursework texts and in exploring teachers' reading and assessment practices in the present study.

5 Tools for analysis of mathematical texts

The aim of my analysis of mathematical texts is not so much to create descriptions of the nature of mathematical writing as to provide a means of identifying and interpreting features of mathematical texts that are of significance to the mathematical and social meanings that may be constructed from them by their readers. The linguistic and non-linguistic tools to be used in the analysis of the mathematics coursework texts considered in this study are detailed in this chapter, together with a discussion of the means of applying and interpreting them. The set of tools and the analytic method were developed recursively through consideration of the literature and of a range of mathematical texts.

The main linguistic tools used are based on Halliday's (1985) functional grammar and may be used to describe any verbal text. The analysis of non-verbal parts of texts is less fully developed in the literature; my main source has been the work of Kress & van Leeuwen (1990; 1993a; 1993b) but the set of tools for the analysis of visual text is probably less generally applicable. The interpretation of features identified by use of all these tools, however, must be made within the particular context in which the text occurs. An important aspect of the context for the texts considered here is their identity as mathematical texts. While some linguistic features may be identified as typical of or even unique to mathematical discourse (see chapter 2), the diversity of contexts and purposes of mathematical writing means that a diversity of language may also be expected. In the assessment context in which the students' writing I am considering takes place, this diversity is of importance in so far as it may be related to teachers' judgements about the mathematical activity portrayed. Throughout this chapter, possible interpretations of linguistic features are illustrated by extracts from a variety of types of text: coursework texts from the sample used in the present study (see chapter 6), reports of investigations produced by Year 9 students during an earlier study (Morgan, unpublished; 1992a), and an academic mathematics research paper (Dye, 1991; see Appendix 4). The consideration of academic writing in mathematics allows examination of the extent to which the characteristics of mature mathematical writing are found (and considered to be appropriate) in students' writing in the light of the claim that one of the purposes of investigative work is to enable children to engage in activity that is like that of 'real' mathematicians (see Appendix 15).

In developing a method of analysing mathematical texts I have been guided by the beliefs that:

- The forms of language used and the meanings for writers and readers are interdependent; a difference in form is accompanied by a change in meanings.
- There is no necessary simple correspondence between a piece of text and the meanings its various readers construct. Rather, the meanings will vary according to the positions

that the reader takes and the discourse within which the text is read. As Kress (1989) argues, the text itself constructs an “ideal reader” by providing a reading position from which the text is unproblematic and “natural” (p.36), but readers do not necessarily take up the “ideal” position and may resist the text by interpreting it in a different discourse (p.43).

- Any analysis must take account of the situation of texts within their contexts of production and interpretation. In the case of this study, students’ mathematical texts must be considered within the context of the discourses of school mathematics and assessment practices, while the texts produced by adult mathematicians are situated within the discourse of the academic research community of mathematicians.

I am making a distinction here between the terms *text* and *discourse*, following Hodge and Kress (1988), although in the literature these terms are often used interchangeably and with a number of different interpretations. The term *text* will be used to refer to a piece of written or spoken language that has “a socially ascribed unity” (p.6) such as an article, a conversation, a piece of coursework, an interview. Such texts are often labelled as discourse by those linguists and others who are concerned with the study of language at this level as opposed to at the level of the sentence or below (e.g. Sinclair & Coulthard, 1975). The term *discourse* here, however, will be used to refer to the wider set of social and linguistic practices within which the text is situated. Discourse refers to “the social process in which texts are embedded, while text is the concrete object produced in discourse” (Hodge & Kress, 1988: p.6).

My method of analysis of mathematical texts makes use of Halliday’s systemic-functional linguistics, starting from the premise that all texts perform functions which contribute to the texts’ meanings and that, in general, all texts perform the following macro-functions:

- (i) the *ideational* or *experiential* function which expresses “the categories of one’s experience of the world” (Halliday, 1973: p.38) and one’s interpretation of that experience. In the mathematical texts under consideration the relevant aspects expressed thus will include the nature of mathematics and of mathematical activity, including the structure of logic and relationships within mathematics.
- (ii) the *interpersonal* function which expresses social and personal relations between the author and others, “including all forms of the speaker’s intrusion into the speech situation and the speech act” (p.41). The expression of the author’s authority within the discipline of mathematics and the community of mathematicians is of interest here, as is the relationship between author and reader(s) in the context of assessment or of academic discourse.
- (iii) the *textual* function which makes language “operationally relevant” in its context and “distinguishes a living message from a mere entry in a grammar or a dictionary” (p.42). In

the discourse of academic mathematics, logical argument and especially proof are highly valued. It is of interest to discern how and to what extent this is manifested in texts produced within the academic community and by children.

The way in which these functions are performed is through the choices of particular forms made by the users of language. By choosing one form rather than another, a speaker or writer changes what is said or written and thus changes the functions performed by the text. There is no one-to-one correspondence between a piece of text and a particular function. Rather, every text fulfils all three macro-functions. The separation of the three macro-functions, while providing a useful analytical tool, does not take account of interrelations among the three. For example, if a section of text has textual features which identify it as an argument, the fact that it has been included in a supposedly mathematical text suggests that mathematics itself is, at least in part, 'about' making arguments (ideational/experiential) and that the reader is expected or expecting to be persuaded by the writer's argument or at least impressed by its presence (interpersonal).

In order to analyse a text, it is necessary to have some means of describing its features and a systematic way of making sense of them. Halliday's functional grammar (1985) provides such a means of description of verbal texts, categorising linguistic features and identifying their roles in contributing to the ideational, interpersonal and textual functions of the text as a whole. While Halliday's grammar provides a useful starting point for the analysis of the texts, it is a linguistic theory and hence may be applied only to texts consisting of verbal language. A functional analysis (although not necessarily a grammatical one) may, however, also be applied to non-verbal texts and parts of texts, in particular to the types of non-verbal features such as diagrams, tables and algebraic expressions frequently found in mathematical texts as well as to visual features such as colour, underlining, etc. This theoretical structure, while it may indicate the general nature of the functions fulfilled by the identified features, does not, however, enable one to identify their specific nature. In order to do this it is essential to relate them to the context within which the text is situated. In the words of Potter and Wetherell:

The analysis of function thus cannot be seen as a simple matter of categorising pieces of speech, it depends upon the analyst 'reading' the context. (1987: p.33)

Potter and Wetherell, however, reject the systematic study of the language of the text in favour of only "reading the context". Their work, therefore, suffers from a lack of a clearly defined and reproducible method for identifying what is significant. In the present study, the use of linguistic tools provides an unambiguous description of the text; Halliday's analysis of the functions of the various parts of clauses identifies the areas likely to be affected by choices made by the producer of the text; the analyst must then interpret the functions of these choices through 'reading' the context. The 'reading' of the context in which this study takes place involves making use of knowledge about mathematics, the explicit conventions of

formal mathematical writing, the academic community, the school mathematics curriculum, social practices within the mathematics classroom, assessment practices etc.

In this chapter I will first outline the features that will be used as tools to analyse the verbal parts of the mathematical texts and the main principles of their application in the context of this study. The functions of features of the non-verbal parts of the texts will then be considered. It must be remembered that, where interpretations of extracts of text are offered, these do not attempt to discover the intentions of the author but the possible meanings that may be constructed by a reader within the given discourse.

5.1 Linguistic analysis of mathematical texts

The grammatical features related to each of the three macro-functions of language and their interpretation will be discussed separately although, as mentioned earlier, the functions themselves are not so easily separated.

5.1.1 The ideational function: presenting a picture of the nature of mathematics

The central question to be addressed by using the analytical tools discussed in this section is 'What is mathematics (as it appears in the text being analysed)?'. This general question, which makes the assumption that the text under consideration is in some sense about mathematics, includes the following more specific issues:

- What sort of events, activities and objects are considered to be mathematical?
- How is 'new' mathematics brought about (or created or discovered)?
- What is the role of human beings in mathematics?

The significance of answers to each of these questions must be considered in the light of existing differences and debates within mathematics and mathematics education. In particular, the absolutist/ social constructivist divide in the philosophical basis of mathematics (Ernest, 1991) will be considered, together with educational questions about the relative importance of mathematical processes or content matter.

In analysing the picture of mathematics and mathematical activity presented in a text, a significant role is played by examining the transitivity system, that is, the types of processes and the types of participants that are active in them. Halliday identifies six main types of processes¹: material, mental, relational, behavioural, existential and verbal, of which the first three types are the most common. At the coarsest level of analysis, the relative weightings of the different types of process are indicative of the nature of mathematical activity presented in

¹A glossary of grammatical terms is provided in Appendix 3.

the text. A high proportion of material processes may be interpreted as suggesting a mathematics that is constructed by doing; mental processes may suggest that mathematics is a pre-existing entity that is sensed (discovered) by mathematicians; relational processes present a picture of mathematics as a system of relationships between objects. Given the importance of the formation of generalisations in mathematics and the way in which this is used as a marker of a successful GCSE candidate, it is of particular interest to note whether a generalisation is expressed as a relation:

$$(TOP LENGTH + BOTTOM LENGTH) \times SLANT LENGTH = No. OF TRIANGLES$$

or as a material procedure:

If you add the top length and the bottom length, then multiply by the slant length, you get the number of unit triangles.

The procedural formulation is likely to be less highly valued than the relational one as it may be seen as representing an earlier stage of development of algebraic thinking².

Relational processes are very frequent in both mathematical and scientific academic writing (Halliday, 1985), particularly those which serve an identifying function (Huddleston, cited in Halliday, 1966). Stating identities is clearly of importance in mathematics and is frequently expressed by the use of an equals sign. The use of the equals sign is of interest in its own right in examining children's texts. While it frequently does play such an identifying role, there is also evidence that it is frequently used to play other roles. In particular, Kieran (1981) reports persistent use of an 'operator' concept of the equals sign by students at all levels which suggests that it is fulfilling a material rather than a relational role. Another common role is as a logical connective between statements, for example:

$$\begin{aligned} 5x + 3 &= 2x - 15 \\ &= 5x = 2x - 18 \end{aligned}$$

Such usages are likely to be considered to be mathematically incorrect by a secondary school teacher-assessor; a recent handbook for mathematics teachers (Backhouse et al, 1992) picks out use of the equals sign as a connective as its single "example of bad practice" (p.126) to illustrate writing that does not "make sense when read aloud".

In examining the picture of the nature of mathematics presented in a text it is clearly significant to ask not only what types of process take place but also what kinds of objects are participants in the text and hence what sorts of objects are the actors in mathematical processes or are

²Van Dormolen (1986) draws parallels between this distinction between procedural and relational thinking and Freudenthal's (1978) distinction of levels of language. However, whereas Freudenthal is clear that it is the *language* that is at a lower level in the procedural example, van Dormolen slips between referring to levels of language and levels of knowledge.

affected by these processes. As was noted in chapter 2, many discussions of mathematical language have focused on the naming of mathematical objects. The use of various kinds of 'specialist' vocabulary is relevant to the interpersonal aspects of a text and will be discussed below in section 5.1.2. In this section, however, my concern is with the types of objects themselves.

The extent of the use of nominalisations in mathematical text, transforming processes into objects such as *rotation*, *permutation* or *relation*, was noted in chapter 2. Such nominalisation has a number of effects on what it is possible to say with and about such process-objects. Firstly, it brings the process into immediate relation with another verb and hence, where this verb is relational, with other nominals; it allows the process to act as the theme of a clause (see section 5.1.3); it also allows the process to be presented as a cause or effect (Halliday, 1966). The power of the use of nominalised process-objects may be illustrated by this example taken from a piece of GCSE coursework³. The author starts by describing a number pattern:

As you can see the unit no. increases by two every time the top length increases by one.

He then is able to extend his generalisation to a wider range of situations:

*This can be done by using any slant no. but if you change this you may find that **the unit increases** may be different.*

By expressing the process of increasing as a nominal, he shifts the focus from the particular property of the original pattern to a more general relationship between a range of patterns:

*This time the **unit increase** is by 4 instead of 2.*

*On the next one when you increase the slant **it increases** to 6.*

A further consequence of the use of nominalisations that is pointed out by authors concerned with the social and ideological aspects of language use (Hodge & Kress, 1993; Fairclough, 1992a) is the obscuring of agency; the transformation of process into object removes the grammatical need to specify the actor in the process. In the context of mathematics, the use of, for example, *rotation* or *permutation* without any indication that these processes are actually performed by anyone fits in with an absolutist image of mathematics as a system that exists independently of human action. As Halliday and Martin (1993) point out, there is a difference between objectification and objectivity but, in the rationalisations for their practices provided by scientists and other academic writers, the two are often confused.

Another powerful feature of many mathematical texts is the symbolisation of mathematical objects which allows them not only to act and to be acted upon but also to be combined and manipulated to form new objects. As was discussed in chapter 2, it appears that, for many

³This example is taken from one of the case study texts whose analysis is presented in chapter 8.

people, the use of symbols is what characterises mathematical language and even mathematics itself; this has implications similar to those of the use of specialist vocabulary which will be discussed further in section 5.1.2. While I am not concerned here with the grammar of mathematical symbolism, its presence or absence is relevant to the image of mathematics that is presented in a text and, for some readers, the extent to which the text itself is considered to be 'mathematical'. (For further discussion of the role of symbolism see section 5.2.1.)

While mathematical symbols and the grammatical category of nominalisation are significant in affecting the type of mathematics and mathematical activity portrayed in a text, further significant types of participant may also be identified by reference to their role and importance in mathematics, although not necessarily by their grammatical form. These include:

- *human beings*, who may be further categorised as *specific* individuals (e.g. the author, the individual addressee, a named third party) or as *general* human participants (usually addressed as 'you' during the explanation of a general process);
- *basic* objects such as numbers or shapes and objects *derived* from these⁴ such as factors, products, lengths or areas;
- *relational* objects such as patterns or formulae;
- *representational* objects such as tables, diagrams or graphs.

Of particular interest is the place of human beings in the text and in the doing of mathematics: the extent of their presence and the sorts of processes in which they act. Is the main role of human beings to 'see' or 'discover' (perhaps suggesting a Platonist view of mathematics), or do they manipulate shapes and symbols (the main activity of pupils in the mathematics classroom)? The interpretations offered here, being out of context, can only be illustrative of the possible significance of different roles. As mentioned earlier, the use of nominalisations obscures the presence of human beings as agents in mathematical activity. A similar function is performed by the use of representational objects as actors in verbal processes, i.e. *the table shows that . . .* rather than *I have shown in the table that . . .*, which obscures the writer's presence as author as well as mathematician. The use of passive rather than active forms of verbs is a further way of obscuring agency that is much used in academic writing.

In considering the portrayal of mathematical activity, it is also important to determine how causal relationships are represented in the text: that is, what types of objects cause or are caused. Here again the presence or absence of humans as causal agents is significant in the extent to which mathematics is seen as an autonomous system. For example, in an academic

⁴It is not always possible to make a clear distinction between basic objects and objects derived from them as what is taken as 'basic' depends on the particular problem that forms the context.

mathematical paper previously established facts (labelled by numbers and hence further distanced from the activity which originally established them) are presented as causes of other facts without any intervening activity:

By (4), (6) the other Branchon point of the former edge is (1,-1,1).

In contrast, a Year 9 pupil's rough work shows mathematical facts and relationships to be dependent upon human action:

whenever there is one dot inside and you count up the perimeter and the area will be exactly half it

The importance of explanation and proof in mathematics is also to be seen in the frequency with which expressions of causality occur in a text. The ways in which such explanations, proofs and arguments may be constructed textually will be discussed in section 5.1.3 below.

In summary, the image of mathematics and mathematical activity presented in a text will be considered primarily through examination of the types of processes, in particular the uses made of the equals sign and the types of processes used in the expression of generalisations; the types of participants in these processes; the portrayal or suppression of agency through nominalisations, non-human actors, and non-active forms of verbs; the nature and extent of the expression of causal relationships.

5.1.2 The interpersonal function: the roles and relationships of the author and reader

Of concern here are not only the relationship between the author and her reader(s) but also the ways in which the author and the reader are constructed as individuals, distinguished by Fairclough (1992a) as the 'identity' function. In asking 'Who is the author of this text?', the areas of interest include her attitude and degree of authority towards mathematics and the particular mathematical task being undertaken. The analysis should also consider how the reader's relationships to mathematics and to the task are constructed within the text. This includes asking the question: 'Why is the constructed reader reading this text?' which may itself involve considering the relationship between author and reader. An important aspect of this relationship is its symmetry or asymmetry: to what extent are the participants 'equal' members of a community of mathematicians or is there greater authority ascribed to one or to the other? How intimate is their apparent relationship?

One of the most obvious ways in which interpersonal relationships are expressed in a text is through the use of personal pronouns. This has been remarked upon by authors concerned with ideological aspects of language use in general (Fairclough, 1989; Fowler & Kress, 1979) and by those specifically concerned with the nature of academic scientific writing (Tarone et al, 1981; Bazerman, 1981) and the language of mathematics education (Pimm, 1984;1987). The use of first person pronouns (I and we) may indicate the author's personal involvement with the activity portrayed in the text (see chapter 2). It may also indicate an expectation that

the reader will be interested in this personal aspect as, for example, a teacher might be concerned to know to what extent the mathematics in a piece of coursework was the product of work done by an individual pupil. In his 'Inner Triangles' coursework, a student⁵ introduced the original problem thus:

The problem that we were given was . . .

and uses the plural pronoun to refer to the group as a whole throughout the first part of his text. When he starts his 'extension', however, he claims individual ownership of both problem and solution :

For my extension I am going to . . .

In the assessment context in which this coursework is situated it is important that the author should make this personal claim because of the weight given not only to understanding and performing adequately, which could be achieved and evaluated in a group setting, but also to posing an appropriate, original extension question, which can only be done as an individual. In contrast, the academic mathematics paper used to illustrate this chapter uses the first person plural throughout in spite of the fact that it was written by a single author, claiming that:

We shall show that . . .

and thus suggesting that the author is not speaking alone but with the authority of a community of mathematicians that "guarantees the generalized transmissability of that discourse" (Greimas, 1990).

While such uses of the first person may draw attention to the activity and authority of the author, *we* may also be used in an inclusive way to imply that the reader is also actively involved in the doing of mathematics. For example, from the same academic paper:

We saw in section 2.2 that . . .

and

By Theorem 1 we may assume that H is H^ .*

gives the reader a share in the responsibility for constructing the argument. Not all readers, however, may be happy with accepting this responsibility; Pimm comments on similar uses of the first person plural in a mathematics text book:

The effect on me of reading this book was to emphasize that choices had been made, ostensibly on my behalf, without me being involved. The least that is required is my passive acquiescence in what follows. In accepting the provided goals and methods, I am persuaded to agree to the author's attempts to absorb me into the action. Am I therefore responsible in part, for what happens? (1987: pp.72-3)

⁵This example is taken from one of the case study texts whose analysis is presented in chapter 8.

The ways in which the second person pronoun is used are also of interest. Addressing the reader as *you* may indicate a claim to a relatively close relationship between author and reader or between reader and subject matter. For example, one boy wrote in his coursework:

On this grid you will notice that it has coloured boxes around the numbers.

By including the words *you will notice* it appears that the author is addressing an individual reader personally and directing her attention with a degree of authority; it also suggests that the reader ought to be interested in the details of the mathematics presented in the text. On the other hand, some uses of *you* appear to be attempts to provide expressions of general processes rather than being addressed to individual readers. This seems to be the case particularly where children are struggling both to formulate generalisations and to communicate them. For example, the generalisation by Year 9 students:

the area would be half of the perimeter if you add one to the area

contains a mixture of relational and procedural forms as well as a combination of a general relationship between two properties of a shape and an action by a human agent. Martin (1989) points out that 'mature' writers will use one form consistently; such lack of consistency of expression is thus likely to be interpreted negatively by a teacher-assessor as a lack of maturity or a mathematical deficiency.

While considering the significance of different ways of using personal pronouns it is also relevant to mention their absence. As mentioned in the previous section, constructions such as use of the passive voice obscure the presence of human beings in the text. This not only affects the picture of the nature of mathematical activity but also distances the author from the reader, setting up a formal relationship between them rather than an intimate one.

One characteristic of academic mathematics texts (and some school texts) is the conventional use of imperatives such as *consider, suppose, define, let x be . . .* Like the use of *we*, these implicate the reader, who is addressed implicitly by the imperative form, in the responsibility for the construction of the mathematical argument (Pimm, 1987). The use of imperatives and of other conventional and specialist vocabulary and constructions characteristic of mathematics (see chapter 2) marks an author's claim to be a member of the mathematical community which uses such specialist language and hence enables her to speak with an authoritative voice about mathematical subject matter. At the same time it constructs a reader who is also a member of the same community and is thus in some sense a colleague (although the nature of this relationship may vary according to the type of action demanded⁶). In academic writing

⁶Rotman (1988) draws a distinction between the roles constructed for the reader by the use of inclusive ('Let's go' although the 'Let's' may be only implicit) and exclusive ('Go') imperatives in mathematical writing. Inclusive imperatives, which Rotman identifies as mental processes like those above, are addressed to a "thinker" and "demand that speaker and hearer institute

this assumption of mutual membership of the mathematics community is to be expected. In the school coursework context, however, there are tensions between the need for the pupil to display her familiarity and facility with conventional mathematical language and the demand made by the assessment criteria to explain the processes she has gone through so that 'someone who knows no mathematics' can understand it (see chapter 9). A pupil who addresses the teacher-assessor with authority as a colleague may even be perceived as arrogant. When some of the children's texts used in the present study were read by teachers and researchers at a meeting of the British Society for Research into Learning Mathematics, several expressed negative reactions to the text of a student who had adopted an authoritative position in his writing.

An interpersonal feature of one child's work was commented upon negatively. He wrote:

'When I had finished writing out this table I had seen another pattern. Can you see it?'

This way of addressing the reader was seen as inappropriate. It is, however, typical of the way that children are themselves addressed either by the writers of mathematics text books and work cards or (usually orally) by their teachers. This boy seems to have identified and copied one of the features of the mathematical texts provided for him without realising that it might be considered inappropriate in his own writing.

(Morgan, 1992c: p.12)

Relations between author, reader and subject matter may also be seen in the modality: "indications of the degree of likelihood, probability, weight or authority the speaker attaches to the utterance" (Hodge & Kress, 1993: p.9). This may be expressed through use of modal auxiliary verbs (*must, will, could*, etc.), adverbs (*certainly, possibly*), or adjectives (e.g. *I am sure that . . .*) (Halliday, 1985). Expressions of certainty are particularly sensitive in the relationship between pupil and teacher-assessor where they may be interpreted as inappropriate claims to knowledge or authority.

In summary, the roles and relationships of author and reader will be considered primarily through: examination of the use of personal pronouns; the extent of specialist mathematical vocabulary and conventional forms of language such as imperatives; the expression of certainty and authority in the modality of clauses.

5.1.3 The textual function: the creation of a mathematical text

In this section, the way in which the text is constructed as a coherent, meaningful unity is considered: what sort of text is it? This will be addressed by examining internal features which contribute to the way in which the text is constructed as well as the overall structure of the text as a whole. Answers to this question, as mentioned above, also contribute to the ideational

and inhabit a common world or that they share some specific argued conviction about an item in such a world" (p.9). Exclusive imperatives are addressed to the reader as a "scribbler" who must perform some material action (*integrate, multiply, drop a perpendicular . . .*).

and interpersonal functions of the text. By constructing a particular kind of text as a part of mathematics the nature of the discourse of mathematics is implicated, as are the expectations of the participants about what constitutes appropriate writing within the given context.

By examining the types of theme that a writer has chosen to use, a picture can emerge of what sort of things the text as a whole is about. The *theme* of a clause is an indication of its main subject matter; in English it is not only the starting-point of the message but is also realised by being positioned at the start of the clause (Halliday, 1985: p.39). Vande Kopple (1991) citing Fries, for example, contrasts two descriptions of houses, one of which presents a picture of movement through the house with a progression of clauses whose themes refer to location, while the other orients the reader's attention on the house as a set of components by thematising the contents rather than their locations. Given the high status of deductive reasoning in the mathematics community, we might expect to find expressions of logical reasoning thematised, focusing the reader's attention on the progression of the argument. For example, the presence in a report of mathematical activity of a large number of themes expressing reasoning (e.g. *Hence, Therefore, By Theorem 1*, etc.) would serve to construct the text as a deductive argument. On the other hand, a predominance of temporal themes (e.g. *First, Next, Then*, etc.) would construct a story or report recounting what happened or, if used with imperatives, would construct an algorithm. These are clearly not the only alternatives that may arise in an analysis of the thematic progression of mathematical texts. They do, however, reflect important aspects that are valued in the assessment of mathematics coursework but which may be incompatible.

It is also worth examining the way in which reasoning is constructed in the text. Martin (1989) points out that reasoning can be expressed through the use of conjunctions (*because, so*), nouns (*the reason is . . .*), verbs (*X causes Y*) or prepositions (*by, because of*). It may also be expressed less explicitly through the juxtaposition of causally related statements. Such lack of explicitness is discussed by Swatton (1992) in the context of assessment of processes in science; he points out the difficulty in determining whether a hypothesis has been formulated (and hence assessing a specified criterion) unless explicit causal language is used. Martin (1989) claims that 'mature' writing tends to express causal relationships without using conjunctions, which are characteristic of what Altenberg describes as "the unpremeditated, rambling progression of conversational discourse" (1987: p.61). In the context of assessment this difference is significant because writing that uses forms of language characteristic of speech may be judged to be expressing less 'mature' forms of thought as well. This distinction between forms of reasoning in speech and in writing is, however, a general one which may not necessarily apply to mathematical discourse. Altenberg (1987) suggests that the order cause-result, because it is chronological, is more common in conversation while formal writing often reformulates this into the order result-cause. While it

seems that, for example, scientific explanation might favour the result-cause order, the status of deductive thinking in mathematics might lead one to expect a greater emphasis on reasoning from cause to result. Interpretation of the expression of reasoning in mathematical texts, therefore, requires further examination of the writing and reading of such texts; it cannot be assumed that this spoken-written distinction is generally valid in all areas of discourse.

As well as looking at internal characteristics of the text it is important to consider the overall structure of the text as a whole. Do various parts of the text fulfil different functions and how clearly are these defined? Such sections may be signalled by explicit labelling, by paragraphing or other lay-out devices, or only by changes in content matter or style. In interpreting an element of a text, readers are influenced by its position and by what they expect to see at that point in the text. If the structure is unclear or unconventional this is likely to affect the meanings that the reader ascribes to the text, her evaluation of it and of its author. Hon (1992), for example, in a report addressed to classroom teachers, describes the conventional structure of science investigation reports written by adults and suggests that where children omit some of the "more-or-less obligatory elements" (p.4) this is "the result of a misunderstanding of the exact nature of the writing task and the accompanying requirement of the reader/s" (p.11). Structures that may be used by children in mathematics investigation reports but which are unlikely to occur in adult academic writing are those that commonly occur in the everyday discourse of the mathematics classroom. These include, for example, a question-answer format that strictly follows a structure laid down by text book or worksheet or a lengthy listing of closely related examples demonstrating the same procedure repeatedly.

The way in which a mathematical text is constructed as a coherent whole will, in summary, be investigated by considering thematic progression, the ways in which reasoning is expressed and the overall structure of the text.

5.2 Non-verbal features of mathematical texts

Non-verbal features play an important part in most mathematical texts. In particular, the system of mathematical symbolism plays a crucial role in the activity of doing mathematics as well as being "one of the subject's most apparent and distinctive features" (Pimm, 1987: p.138). Algebraic symbolism will be considered separately, not only because it can be translated into words and read in a linear way similar to verbal text but because of the significance of its role within mathematics. In addition to symbols, Shuard & Rothery (1984), describing the language of school mathematics text books, identify a large amount of what they call "graphic language", including tables, graphs, diagrams, plans and maps, pictorial illustrations (p.45). These share the characteristic of non-linearity and cannot be unambiguously translated into words. Not only will the functions of these features be considered in themselves, but also the

ways in which they are related to the rest of the text. As well as these discrete features of the texts, the overall question of 'presentation' will be considered. This includes graphological aspects such as the spacing of the text and the use of colour and underlining.

5.2.1 The role of algebraic symbolism

By representing an object, quantity, action or relationship by a symbol in a mathematical text, it is declared to be 'mathematical' and thus of significance. At the same time symbolising is an act of abstraction allowing the writer and the reader to focus only on the formal properties of the symbol itself and allowing "manipulation to move faster and more seamlessly by blurring the distinction between symbol and object" (Pimm, 1987: p.139). Mathematics itself thus appears as a domain in which the main activity is manipulation of symbols rather than of concepts or 'real world' objects. Symbols are not, however, always manipulated within the text. In students' texts a symbolically expressed generalisation may stand alone as the 'answer', suggesting that the purpose of the task undertaken was to produce this expression. In such cases the symbols are the product of the mathematical activity rather than a tool to be used during the process and mathematical activity is directed towards forming an algebraic expression. While I have suggested that extensive manipulation of symbols and the presentation of symbolic expressions as products present a picture of mathematics as a primarily symbolic activity, it is important to consider to what extent the symbols are explicitly linked to their referents. The algebraic product may be 'translated' back into 'real world' terms thus maintaining a picture of mathematics as modelling a concrete reality.

Halmos (in Steenrod et al., 1973) points out one of the interpersonal effects of the use of symbols in mathematical text:

if it looks like computational hash, with a page full of symbols, it will have a
frightening, complicated aspect. (p.45)

Since he was advising writers of academic papers, this would suggest that it is not only non-mathematicians who have a negative emotional reaction to excessive use of symbols. The extent of their presence in a text is, however, one way in which the author claims authority as an 'expert' member of the community of mathematicians - one who is not frightened and does not consider the symbolism complicated. The claim to expert status is particularly strong where new notation is coined within the text to signify 'new' mathematics created by the author (although there is also the possibility that the type of new symbol chosen may be seen by a reader as inappropriate - a "category mistake" (Pimm, 1987: p.145)). As with the use of specialist vocabulary and conventional language discussed above (section 5.1.2), there is also an assumption that the reader will be a member of the same community, sharing the ability to interpret the symbolic language. The claim to expert status is, of course, dependent to some extent on the type of symbols used and on the context within which the writing is situated. The use of numerals by themselves would not be considered of particularly high

status in most areas of mathematics. In GCSE coursework, in particular, it appears that use of algebraic symbolism is expected in order to attain a grade of C or above (see Appendix 15; ULEAC, 1993) and is thus seen to mark off those students who are capable of progressing to higher mathematics from the rest.

5.2.2 Diagrams, tables and graphs

The tools for analysis of 'visual' forms of text are less fully developed in the literature than those for verbal text. The main source that has informed this section is the work of Kress & van Leeuwen (1990; 1993a; 1993b) who, working in a Hallidayan tradition, have attempted to construct a 'grammar' for visual forms of communication, identifying features of pictures and diagrams that serve ideational, interpersonal and textual functions. While Kress & van Leeuwen's grammar provides some useful tools and concepts, its empirical basis does not on the whole include mathematical texts and its application and adequacy in the case of mathematics has yet to be fully developed. The forms considered in this section are those found within the corpus of coursework texts; the development of a fully comprehensive means of describing and analysing graphic forms in mathematical texts in general has not been attempted

Although there are structural and functional differences between diagrams, tables and graphs, these elements all share the characteristic that their components are not readily translatable into a linear verbal form. The ways in which they are read and integrated into a whole text may thus be distinguished from verbal text and from algebraic symbolism. Much of this section refers to diagrams; many of the features of diagrams that are identified as significant to the analysis are, however, equally relevant to the consideration of graphs and of tables. Where graphs or tables have features that are not found in diagrams these are dealt with specifically. The general properties that are significant in the present study (in that there are identifiable differences within and between children's texts) include: 'neatness', labelling, the degree of naturalism, dynamic or static forms, and the way in which the diagram is integrated into the text.

'neatness'

Diagrams which are drawn with a ruler, with largely correct proportions or exact measurements, possibly with great detail and apparent care, positioned between the margins and parallel to the edges of the paper may be categorised as 'neat'. The prime function of 'neatness' is interpersonal; it indicates that the text is formal and that there is some distance in the relationship between author and reader. This distance may be physical, in that detail and clarity are necessary if there is no possibility of oral communication to supplement the diagram - diagrams drawn on the back of an envelope in the course of a personal conversation are unlikely to have many 'neat' qualities. The distance may also be a social one, the neatness being a mark of respect for the reader, or it may be an 'intellectual' distance, indicating the

degree to which the reader is constructed as sharing the resources needed to understand the diagram. In the case of coursework texts, where the reader is a teacher-examiner, neatness may serve as an indicator of the amount of care and effort that the student-author has expended on the task as a whole.

A neat diagram is thus clearly addressed to a possibly distant audience. A rough diagram (one which lacks most of the qualities of neatness listed above), on the other hand, either is addressed to an audience that is constructed as intimate (socially and intellectually) or appears as a 'private' diagram, drawn for the personal use of the author rather than as part of the communication with a reader. Of course, the fact that such a 'private' diagram has been included in a text that is intended to be read by others means that it does have a public communicative function. In the coursework context, this function is likely to be to demonstrate that the student/author has done the required 'investigation', trying out specific examples for herself before recording the results more formally elsewhere. The roughness of such diagrams may also be taken as an indicator that they are the original work of the author herself as copying would be likely to have been done with greater care.

The degree of neatness may also serve a textual function, indicating to the reader how she should make sense of the diagram's role within the structure of the whole text. As suggested above, rough diagrams, being private, may be read as background material which provides evidence of process but which does not contribute to the thread of argument. Just as they were drawn without much care or attention to detail, it is not necessary for the reader to pay much attention to their detailed properties. A diagram that is neat, on the other hand, may form an important part of the argument, serving as explanation or justification. The textual function of a diagram can also be inferred from the way in which it is integrated into the text (see below). Where there is a mismatch between the degree of neatness and the reader's perception of the textual function it is likely that a negative judgement will be formed by the reader. Thus, a very neat diagram or set of diagrams which is read as background 'working out' may be judged to be a 'waste of time', while a very rough diagram which is read as forming part of an explanation may lead the reader to judge the author to be lazy or careless.

labelling

One of the ways in which the functions of a diagram are indicated is through the form of labelling attached to it. Three forms of labelling are of particular significance in the context of mathematics coursework: question numbers, specific quantities or measurements and variable names⁷. A question number used as a label for a diagram, or indeed for any other

⁷I am using the term 'variable names' to refer both to symbolic labels, such as a conventional n or a more contextually meaningful $T.L.$, and to verbal labels such as *difference* or *top length*.

segment of text, indicates that it is the answer or part of the answer to a question specified by the set task. It is thus a sign that the student/author's activity in doing the mathematics is directed by an external authority rather than being autonomous or creative. Specific quantities or measurements marked on a diagram suggest that the diagram is a specific example, either drawn in the process of experimenting before forming a generalisation or given as a demonstration of the truth of a generalisation (obviously depending on the position of the example within the text as a whole). In both cases the drawing implies that the physical object portrayed in the diagram is real, at least in the sense that it can be counted or measured, and that it is the subject matter of the mathematical problem. In the case of those diagrams apparently drawn in the experimental phase, such labels play a role similar to that of 'neatness', indicating that they are being presented for the reader's attention as part of the main flow of the argument rather than as background 'rough' work.

In contrast, a diagram labelled with variable names focuses on the general, i.e. the features that the object represented has in common with other similar objects, rather than the particular quantities or measurements that identify the unique object represented. The subject matter of the mathematical text is thus more abstract; the relationships between the properties of the physical objects represented are more important than the properties themselves.

naturalism

The presence of diagrams in a mathematical text implies that the mathematics involved is, at least to some extent, about the concrete objects portrayed. Even abstract and general geometric objects are given a concrete and particular form by being represented in a diagram⁸. However, it is also relevant to consider the extent to which the representation is abstract and schematic or naturalistic. For example, a student writing about the investigation 'Frogs' might draw her diagrams with naturalistic pictures of frogs (as they appear in the SMILE computer programme and in most published introductions to the investigation) or she might choose to use abstract iconic representations such as circles or squares. The latter, more

In both cases the label is identifying a generally significant feature of the object represented in the diagram without quantifying it within the particular diagram.

⁸It is interesting to note that, in spite of the geometric nature of the subject matter of the academic article referred to earlier (Dye, 1991) and an explicit appeal to physical imagery contained in its introductory section, the article contains no diagrams. The generality and abstract nature of the argument are thus emphasised. Within academic mathematics there is dispute about the status of graphic representations with a general historic trend away from the visual towards symbolic, deductive modes of argument. This trend has been justified by a demand for greater rigour, based at least in part on the suggestion that the eye is fallible. Davis (1993), however, argues for higher status for the visual, claiming that there are "graphical displays . . . from which certain pure or applied mathematical conclusions can be derived almost by inspection" (p.336). While admitting that pictures can deceive (he uses a proof that all triangles are isosceles as an example), Davis is arguing within a paradigm that assumes that pictures are 'transparent'; the reader would not be deceived if the diagram were drawn 'properly'.

abstract approach is likely to be judged by the teacher-assessor to demonstrate a higher level of mathematical thinking:

just as schooling in writing and reading that impersonal, abstract, objective prose remains restricted to the higher echelons of education, so those impersonal, abstract and objective diagrams, too, remain a cultural capital to which not everyone will have access
(Kress & van Leeuwen, 1993a: p.23)

The point of view from which three dimensional objects are represented is one of the features of a diagram that contributes to its naturalism or abstractness. Where perspective is provided, the concrete reality of the object is emphasised. This highlights the practical nature of the activity involving the object. For example, a pile of rods for the 'Topples' task (LEAG, 1991; see Appendix 2) that is drawn showing perspective suggests through its naturalism that physical features of the rods such as the material they are made of may be significant to the problem, while the drawing without perspective focuses more unambiguously on the length of the rods as the only important variable. At the same time, the direct frontal angle "is the angle of 'this is how it works', 'this is how you use it', 'this is how you do it'." (Kress & van Leeuwen, 1993b: p.94); it involves the reader by providing instructions about how to construct the pile. In the case of mathematics GCSE coursework it seems unlikely that teacher/examiners would expect variables such as friction to be effectively taken into account by children. It is likely, therefore, that the less naturalistic frontal view would be considered more appropriate, particularly as it also fulfils the role of 'explaining how you did it'.

dynamic signs of activity

While some diagrams have what Kress & van Leeuwen (1993b) call an *analytical structure* which displays an object and its attributes (corresponding to a relational verbal statement, e.g. *This trapezium contains 12 inner triangles*), others have an *action structure* which suggests that a process is taking place or has taken place. Kress & van Leeuwen (1990) identify directional vectors within a diagram as the main indication of such processes as they connect the actor to the goal of the action. A mathematical diagram with such an action structure not only identifies the process as part of the subject matter of mathematics but also suggests that the process is independent of human activity as it involves only the objects represented in the diagram. This analysis, however, assumes that the participants and process are entirely contained within the diagram itself and does not take account of the possible role of the author or reader as actor upon the objects portrayed. While Kress & van Leeuwen do discuss the case of, for example, traffic signs, in which the reader is the elided actor addressed in the imperative, 'Go this way', the action represented in such signs is expected to take place in the 'real world' outside the diagram. In the context of a mathematical text it may be the diagram itself that is to be acted upon.

Diagrams, like symbolism, can act as tools in the activity of doing mathematics as well as acting as representations of the objects or products of mathematical activity. This role may be indicated within the diagram in the shape of marks which suggest the author's processes and guide the way in which the reader should actively make sense of the diagram. For example, arrows may not only indicate the route which the reader's gaze should take but may also show how the diagram might be moved or transformed in the reader's imagination. Dots or other tally marks suggest that counting has taken place or should be done by the reader. Within a table, arrows or other indicators may also be used to identify values that are to be combined or compared in some way; a conventional example of such a dynamic table is a table of differences used as a tool for analysing the structure of a number sequence. Such uses suggest both concrete objects and material action (manipulating the diagram itself either physically or in imagination or performing an operation such as counting, measuring or subtracting) to be the subject matter of mathematics. They also suggest that the reader's role is an active one, reconstructing the processes described by the author rather than merely receiving an account of their outcomes.

Different ways of representing data can also present different views of the nature of the subject matter. The use of a line graph rather than a list or table of values or a bar chart "creates something like a dynamic process (translatable as 'change', 'vary', 'grow', 'decrease' etc.)" (Kress & van Leeuwen, 1993b: p.71). The participants are themselves dynamic variables rather than discrete and static objects. A graph may thus be interpreted to make a general active statement that could be verbalised as *The area increases steadily*, while a table presenting the 'same' information makes a series of relational statements about specific instances: *The area when $b=1$ is 6; the area when $b=2$ is 8; the area when $b=3$ is 10.*

integration into the text

In interpreting the roles of diagrams, graphs and tables it is necessary to look at how they are integrated into the body of the text. What sort of verbal references (if any) are made to them and how are they positioned on the page? For example, integrating a table with the words *I put the results in a table* suggests a focus on the author's processes; *here is a table* suggests that the table is seen as a product in its own right or as a sign of the author's mathematical expertise; *in the table you can see* suggests an expectation that the reader will be actively involved in making sense of the mathematics. Where no verbal reference is made to the table this may suggest that its role in the text should be taken for granted as a standard component of the genre.

The layout of the page may also indicate the textual functions of different parts of the text. Kress (1993a) relates left-right ordering on the page to the 'given-new' structure of the information contained in it, "given the directionality of Western 'reading paths'" (p.12). Where, for example, a page is arranged with diagrams on the left hand side and a table of results on

the right, this gives priority to the diagrams as representations of what is given and unquestionable while the table is presented as an interpretation of these 'facts' and a creation of the author. An ordering from top to bottom of the page may be taken to suggest a development either temporally in the narrative (i.e. what happened first, second, etc.) or logically in the argument. For example, a hypothesis stated in words or symbolic form at the top of the page, followed below by a specific diagram labelled with measurements and a calculation suggests that the diagram plays the role of proof of the hypothesis while the opposite ordering suggests that the hypothesis has been formed on the basis of the evidence provided by the example represented in the diagram.

the choice to use a diagram, graph or table

One of the principles of the method of discourse analysis used in this study is that the choices that are made between alternative linguistic forms when constructing a text are 'motivated' rather than arbitrary (Kress, 1993b). This means that the choices are significant in analysing the functions performed by the text. So far choices between alternatives within a section of verbal text or within a diagram have been considered. It is also relevant to consider the choices made between different forms of representation. As one of the criteria against which students' coursework texts are assessed is that they should use 'appropriate' mathematical forms of communication, one of the prime functions of including diagrams, graphs, tables, or indeed algebraic symbolism is to demonstrate that such forms have been used and thus to fulfil this criterion. As with algebra (see section 5.2.1 above), there is a tension between the perceived value of graphic or tabular forms and the potential difficulty of interpreting them if they are not accompanied by verbal text.

5.2.3 'Presentation'

In contrasting spoken and written language, Halliday (1989) argues that writing does not have paralinguistic features such as intonation and gestures which form an important part of the context of speaking. I would suggest, however, that there are features of written language which play similar roles. In particular, the use of devices such as underlining, italics, colour, different sizes of letters, etc. "may have clear linguistic implications, perhaps related to the semantic structure of the utterance (as in advertising or newspaper articles) or even to its grammatical structure" (Crystal & Davy, 1969: p.17) and thus can provide guidance to the reader about the status and relative importance of the various parts of the text. In formal academic writing, such devices are generally used sparingly and with limited conventional meanings. They are to be found, however, in published texts intended for children and are extensively used in mathematics text books (Shuard and Rothery, 1984: p.20).

The use of colour, for example, may thus mark a text as being produced by or for a child, the colour being included either to entertain by creating visual interest or to help guide the reader by highlighting important parts of the text. By way of contrast, a piece of coursework that is

typed or word-processed appears to be claiming to be a formal 'adult' text, setting itself apart as different from the hand-written work of the everyday mathematics classroom. The presence of features such as title pages, lists of contents, etc. similarly suggest a claim to be a 'publication'. Such devices may, however, be interpreted by a teacher-assessor to be a 'waste of time', distracting from the more important mathematical content matter.

5.2.4 Summary of non-linguistic features

In looking at the use of algebraic symbolism within a mathematics text, the characteristics to be considered include: the role of symbolism as 'tool' or as 'answer'; the extent to which the symbols are explicitly linked to their referents; the use of new or unconventional symbols. Graphic entities may also play a role as tool or product, 'private' or 'public'; in addition, the abstract or naturalistic nature of representational diagrams should be considered. In all cases it is relevant to examine how symbolic and graphic elements are integrated into the text as a whole through the use of verbal language. Presentational features such as colour, underlining, title pages, etc. can play a role in suggesting what sort of text is being presented through using the conventions of different genres.

These non-linguistic features have been discussed separately in this chapter. When analysing a text, however, the ways in which the three meta-functions are fulfilled will be addressed by considering both linguistic and non-linguistic features.

5.3 Applying the analytical tools

In this chapter, linguistic and non-linguistic features of mathematical texts have been identified as contributing to the ideational, interpersonal and textual functions of the texts. The functions that have been considered are those related to mathematics and mathematical activity in the discourses of the community of academic mathematicians and of the secondary school mathematics classroom and assessment system; there are likely to be other areas that have not been considered. While I have attempted to suggest possible interpretations for the various choices of these features, it is only within the context of a complete text that the interpretation can be made, particularly as apparently contradictory features may coexist within a single text. Indeed, similar features may function differently in different contexts; for example, Dowling (1992) remarks on the different functions served by cartoons in mathematics text books intended for 'high ability' and 'low ability' students. The features that have been identified should be seen as things to examine in the texts being analysed and interpreted in the light of the suggestions made here, taking into account the complete text and the contexts of production and interpretation. This does not mean that there may not be further, unanticipated features that prove to be significant during the analysis.

It should be noted that there is no suggestion that the author of a text has made conscious decisions to choose to use particular features in order to perform the types of function considered in this analysis. Rather, the individual's positioning within a particular social structure and consequent understanding of the nature of the genre within which she is writing makes it 'natural' for her to make these choices because they appear 'appropriate' to the task she is undertaking. They may or may not appear similarly appropriate to a reader, depending on the discourse within which the reader is positioned. The analyst, however, must stand apart from making judgements of appropriateness as this is socially constructed and is indeed one of the ideological concepts that is to be 'demystified' by the analysis. While she must share in the "member's resources" (Fairclough, 1989) used by the participants to produce and interpret the text, the analyst must use these with self-consciousness in the light of "rational understanding of, and theories of, society" (p.167). To the extent that I have succeeded in achieving such self-consciousness, the resources used in the present analysis are made explicit in chapter 6.

6 The study of GCSE coursework texts

In order to study the nature of reports of mathematical investigations written by students, a sample of mathematics coursework texts was collected. The size of this sample and the length of the texts themselves were such that only a very gross analysis of the entire set was possible. Such a level of analysis would not have been able to address the aims of the study in relation to achieving detailed knowledge of the linguistic and non-verbal forms used in coursework texts. It was, therefore, necessary to select a small number from within the sample to be analysed in detail. Three texts displaying a variety of characteristics were selected from the sets of texts on each of two contrasting tasks: 'Inner Triangles' and 'Topples'. This smaller sub-sample was also used in interviews with teachers to investigate their reading practices and the relation between the language of the texts and the evaluations made of them. In this chapter, I describe the structure of the whole sample and the way in which a method of analysis was developed which enabled the selection of the sub-sample to be used as case studies.

6.1 The sample of coursework texts

The sample texts were written by students from a single year group within a mixed comprehensive school that used the LEAG GCSE syllabus with 50% coursework. This meant that each student was required to submit work on five tasks completed within the last two years of the course, in this case chosen from a selection provided by the examination board. (The two tasks considered in detail are analysed in chapter 7.) The complete submissions of thirteen students from one class were collected. This was the second set out of four within the year group; all had been entered for either the Intermediate or the Higher level of examination and the final grades achieved (taking into account both coursework and timed examination) ranged from F to B, the great majority of the class achieving grades E, D or C. The students whose work was collected represented the whole of this range of attainment. It was intended to include work from two boys and two girls at each of the main grade levels, but only one boy achieved a C. The grades B and F were each achieved only by a single boy; these sets of texts are also included in the sample. In addition, in order to include some texts from students who had achieved A grades, texts on some of the tasks from five students (all girls) from the top set in the same year group were also collected. These texts were available because they had been selected by the class teacher as exemplar materials to be shown to future students. The grades achieved are aggregates and do not necessarily correspond to the teachers' assessment of each individual text; this assessment, in the form of a mark out of twenty, was recorded on each text. The structure of the sample is summarised in Tables 6.1 and 6.2 below.

Table 6.1

The students submitting GCSE coursework texts

Grade	Male	Female	Comments
F	1		The only student in the class achieving this grade. He only submitted three of the five pieces of work.
E	2	2	One of the boys submitted only four pieces of work.
D	2	2	
C	1	2	Only one boy in the class achieved a C grade
B	1		The only student in the class achieving this grade.
A		5	All five tasks are only available for one of these students.

Table 6.2

Number of texts on each task

Topples	14
Inner Triangles	15
Passola	16
Symmetry Groups	15
Pendulum	5
Area Under Curves	12
Total	77

It is clear that the set of coursework texts to be used in this study does not form a representative sample. In particular, the fact that the student-authors were all at the same school and in most cases were taught by the same teacher means that there is likely to be a greater degree of similarity between the texts than might otherwise be the case. However, participation by both teachers and students in the public discourse of investigation and coursework (described in Appendix 15), together with the in-service training of teachers, the publication of exemplar materials, and the process of moderation both within schools and by the examination board, seem likely to serve to lessen major differences at the school level. At an early stage of the study, a set of texts on the same tasks from another school were also read and an exploratory analysis was made of a sample of texts from a number of schools that were made available by the examination board. Neither of these informal comparative samples suggests that the present sample is aberrant in any way that would affect the objectives of this study to produce a description of a range of features found in students' coursework texts and to explore what may be achieved by these features when the texts are read by teachers. Subsequent interviews with teachers from different schools reading a selection of the texts confirmed the unexceptional nature of the sample as all the teachers

appeared able to read and assess the texts in a routine way, even telling anecdotes about their own students suggesting that they had produced similar texts.

6.2 Stages in development of the analytic method

Because no method for analysing the coursework texts was immediately available, and because the characteristics of the texts that were likely to emerge from the analysis were not pre-determined, the development of an analytic method necessarily involved substantial interaction with the data itself. This development took place at the same time as the development of the linguistic tools described in chapter 5. In this section, the history of the development of the analysis is described, indicating the contributions of the emerging familiarity with the data and of the increasing application of linguistic theory. This leads to a rationale for the selection of the sample of texts to be used as case studies.

Clearly my identification of features of a text as significant derives from my own resources as a reader of coursework texts which lead me to notice and interpret in particular ways. These resources include those arising from my own experience as a practitioner working in the classroom, participating in the process of working with students and assessing their coursework, and alongside other mathematics teachers, participating in the moderation process and in in-service education as well as more informal joint activities. In addition to such practitioner resources, I also have resources arising from my academic experience in the field of mathematics education which bring me to notice features that relate to issues identified during my analysis of the discourse of investigation (Appendix 15) and to my knowledge of issues related to the nature of mathematical activity and of school mathematics (e.g. Ernest, 1991). Finally, my reading of linguistics and the literature of critical discourse analysis brings me to identify features that are likely to be significant because of their linguistic functions. My position in analysing the texts is as a “self-conscious insider” (Fairclough, 1989), sharing the resources of the participants in the discourse but with an awareness of their sources and a perspective that allows me to analyse my own reading. The description that follows of the development of the method of analysis used in this study seeks to make explicit the way in which I have read and analysed the coursework texts to the extent that another analyst sharing similar resources would derive similar analyses.

6.2.1 Exploratory familiarisation

The initial step towards developing an analysis of the students' texts involved familiarisation with the data set and identification of themes within the sample and issues related to their analysis. All the texts for each task were read through and notes were made about each text under four headings, commenting on the orientations presented in the text towards: the task itself, mathematics and mathematical activity, the nature of mathematical communication, and the reader. This first reading was done before a clear method of linguistic analysis of

mathematical texts had been developed. The headings, therefore, while significant in terms of the purposes of the study, are not clearly linked to Halliday's meta-functions which form the basis of the linguistic theory subsequently employed. For example, features noted under 'orientation towards the task' included both textual aspects (e.g. the structuring of the text around headings taken from the given question paper) and interpersonal aspects (e.g. the expression of the writer's lack of authority in statements such as "We were given the problem . . ."). On the other hand, orientation towards mathematics was more clearly related to the ideational meta-function, while orientation towards the reader related to the interpersonal. Orientation towards mathematical communication, while including textual aspects, also incorporated the way in which specifically mathematical features of the text, such as tables, diagrams and algebraic symbolism, are used and integrated into the text. These aspects perform both ideational and interpersonal meta-functions. The notes under one heading, therefore, were frequently linked with notes under one or more of the other headings, forming an unwieldy data structure.

Several points emerged from the practicalities of this first reading. Firstly, without a formal method of identifying and describing significant features of the texts, the attempt at analysis lacked consistency. Although some of the more obvious non-verbal features of the texts (such as tables and formulae) were readily identifiable and hence could be noted systematically, the linguistic tools were not yet clearly enough defined to identify significant verbal features in a similar way. Moreover, a method of justifying the interpretation of the significance of such identifiable features was not available. Before the analysis could proceed it was therefore necessary to develop the set of linguistic tools and the method of applying them in order to form an interpretation. The full details of the linguistic tools and their interpretation may be found in chapter 5.

Although this exploratory stage of the analysis revealed major differences between texts, a further issue that arose was the difficulty of creating unique categories into which individual texts would fall. While some of the texts had clearly describable orientations towards one or more of the aspects, many were mixed and inconsistent. This implied that any analysis would be unlikely to be able to provide clear-cut generalisations about groups of texts or simple descriptions of individual texts, but should aim to produce detailed descriptions of texts in order to characterise them adequately.

The major achievement of this exploratory analysis was, however, to assist in the development of a set of variables to form the basis of the characterisation of coursework texts. While the four focusing headings reflected the theoretical interests of the study at a general level, the notes that were made under each of the headings related features observed in the data to more specific issues derived from knowledge of the published discourse of investigations and coursework, from experience of the practice of mathematics

teachers involved with teaching and assessing coursework, including the qualities likely to be valued in school mathematics, and from the literature of critical discourse analysis in relation to other domains of discourse (e.g. Fairclough, 1989; Fowler & Kress, 1979). During the analytic process, some of the issues became more clearly defined and more significant because common or contrasting features were observed within the set of texts. The issues identified under each of the headings included:

the task, i.e. the picture of the type of task being undertaken by the writer

- *the extent to which the task is determined by the instruction sheet* (e.g. statement of questions copied from the sheet or claims by the writer to have set her own aims). This is related to the idea of 'ownership' of the task and of mathematics by the student - an idea which is often used within the discourse of investigation as a justification for this way of working (see Appendix 15).
- *a stress on products or on processes* (e.g. through the use of headings such as "formula" or "investigating"; underlining answers; including a narrative of 'what I did'). The tension between the value placed on content and that placed on process is an important issue in the discourse of investigation and coursework assessment (see chapter 4, Appendix 15).
- *the structure and overall presentation* as a 'published' work with front cover, contents page etc. or as a list of 'school maths' type exercises. Experience of teachers' practice suggests that, while 'presentation' is valued, it may also in some circumstances be seen as a 'waste of time'. Presentation as a list of exercises is likely to be in conflict with the ideas of 'exploration' and 'ownership' valued in the discourse of investigation (see Appendix 15).

mathematics, i.e. the picture of the nature of mathematics and of mathematical activity presented in the text

- *the presentation of formulae as procedures or as relations*. Formulae play an important part in coursework; each of the tasks included in this study expected students to seek general formulae. The procedural-relational distinction emerged strongly during the exploratory analysis as a variable that distinguished clearly between different texts. As within mathematics the relational form is usually valued over the procedural, this distinction appears likely to be important in the assessment context.
- *the ways in which generality is represented* through the use of algebraic symbolism or less abstract forms. Generality is clearly a highly valued aspect of mathematics and analysis of the coursework tasks shows that it is expected of higher attaining students.
- *the ways in which causality is represented*. As is the case with generality, causality (particularly in the form of deductive argument) is of importance within mathematics. The identification of causes and effects is a tool used in critical discourse analysis in considering the way in which the subject matter is constructed in the text. In particular,

what sort of participants may act as causal agents? In the dominant platonist paradigm of academic mathematics, it may be more likely that abstract mathematical objects be presented as causal agents than that human mathematicians should be seen to be causes.

- *the relative status of concrete objects and manipulations, measurements or calculations based on them and of relational objects such as patterns or formulae.* One of the basic tools of critical discourse analysis is the examination of the types of objects present in the text. On applying this to the coursework texts during the exploratory phase it emerged that some authors focused on the basic objects forming the data for the problem and on calculations or measurements based on these objects, while others focused on relationships, patterns and formulae arising from the data. This distinction seems significant, given the different values ascribed to the concrete and the abstract within mathematics.
- *who or what is active in doing or creating.* Again, the identification of the active participants is a basic tool of critical discourse analysis in constructing a picture of the nature of the subject matter. The question of how mathematical knowledge is created is a fundamental issue in the philosophy of mathematics, centring on the question of whether it is the discovery of independently existing facts or the result of active human invention. This question also relates to the ideas of student activity and 'ownership' within the discourse of investigation.

mathematical communication

- *the ways in which devices such as tables, diagrams and algebraic symbolism are or are not used.* The use of such devices is recommended by teachers and by the examination boards. However, this recommendation is not generally accompanied by any advice about how they might be used. One of the issues motivating the present study is the lack of explicit knowledge available to students and teachers about what may be considered 'appropriate' uses of both verbal and non-verbal communication; examination of the ways in which they may be used is therefore of interest. The use of algebraic symbolism is of particular interest because of its high status within school mathematics.
- *the ways in which devices such as tables, diagrams and algebraic symbolism are integrated into the text as a whole..* Given the recommendation to use these devices, do they form part of a coherent whole text or are they presented as independent, self-sufficient means of communication or as displays of the student's ability to produce them.
- *the type of text, in particular, the use of narrative and/or logical argument.* Again, one of the pieces of advice provided for students by teachers and by published guides is to present a narrative of 'what I did'. At the same time, however, the construction of deductive argument is highly valued in mathematics.

reader, i.e. the picture of the writer's expectations of her reader and of the relationship between writer and audience, including the writer's degree of authority

- *direct addresses to the reader,*
- *expressions of personal feelings and attitudes, including confidence, emotion, uncertainty,*
- *the use of impersonal and formal language.*

All of these aspects are of importance in critical discourse analysis. They also relate to the idea of student 'ownership' of the mathematical activity. In the assessment context within which these texts were written, the extent to which the reader accepts the 'ideal' relationship constructed by the text is likely to be important in her evaluation of the coursework.

6.2.2 Selection of a sample of 'Inner Triangles' texts

Having identified the variables described above and the linguistic tools described in chapter 5 for systematic identification and interpretation of significant features of the texts a method for comparing and contrasting sets of students' texts was developed in order to be able to choose a small number of texts with contrasting features to be analysed in greater detail as case studies. This was carried out first on the entire set of texts on the task 'Inner Triangles'.

In order to be able to compare the features of the whole set of texts a matrix was created in which brief notes were entered for each text under an expanded set of headings derived from those described in the previous section: participants, causality, authority, attitude, reader, formality, structure, text type, tables, diagrams, algebra/ generalisation. These notes were made by re-reading the original texts rather than by reorganising the notes from the original focused reading. This was done because it was felt that, in the time that had elapsed between the two analyses, significant linguistic features related to the areas of interest in the texts had been more clearly identified and information about all of these features had not been systematically recorded during the first reading. Notes from the two readings were, however, subsequently compared and were found to be compatible.

Each column of the matrix was then scanned to identify common themes of qualities which could be used to compare and differentiate between texts. These qualities were then coded as shown in Table 6.3. The qualities coded are neither mutually exclusive nor exhaustive; a single text may have more than one quality under the same heading and some texts did not have entries under every heading. For example, the only quality that was coded under the heading 'structure' was the presence of a repetitive structure which was both easily identifiable and distinctive of a small group of texts. Of course, other texts also had structure but these were neither simple to describe nor obviously common to several texts. A further point to note is that different parts of a single text may even display qualities which appear to be contradictory. For example, five of the fifteen texts contained both impersonal and

Table 6.3

Variables used in analysis of 'Inner Triangles' texts

heading	qualities	notes and examples
participants	first person inanimate actors passive	<i>The numbers increase</i> <i>The bottom length was doubled</i>
causality	human action mathematical relationships formulae	<i>Whenever you increase the top length or the slant length the number always goes up by the same amount</i> <i>My formula is . . . because a triangle is like a trapezium.</i> i.e. using the formula to predict future empirical results
authority	self external	<i>I decided . . .</i> <i>The problem we were given . . .</i>
attitude	personally involved evaluative impersonal	expressing interest, surprise, etc. commenting on the difficulty of the problem, the worth of the work, etc.
reader	assessor trusted shared community	e.g. a major focus on providing answers to the given questions or on displaying prowess admitting weaknesses involving the reader, e.g. through the use of an inclusive <i>we</i> or an expectation that she will share an interest in the mathematics
formality	impersonal informal	e.g. including 'speech-like' aspects such as: <i>This is just a one off.</i> or <i>It really comes in handy!</i>
structure	repetitive	e.g. a pattern of results table, followed by formula, followed by example, repeated several times for different shapes
text type	narrative argument list of results	<i>In a very short time we found the formula.</i> <i>This shows that . . .</i>
algebra/generalisation	procedural relational formula as solution algebraic manipulation	e.g. appearing as the final statement in the text or emphasised by colour or underlining only present in a small number of texts

(The headings of tables and diagrams were not coded at this stage as it was felt that the features of these had not yet been described clearly enough to be categorised in this way.)

informal elements while two texts contained both procedural and relational forms of generalisations.

A reduced matrix containing only the codings was then constructed to assist in making a choice of texts to be analysed in greater detail. This method of data reduction, while allowing a crude comparison to be made between lengthy texts, nevertheless obscures the complexity of the texts and many interesting features. On the basis of this comparison, three of the texts were chosen for more detailed analysis; these texts were also to be used in the study of teachers' reading practices. Although there are associations between some of the qualities (for example those writers who were personally involved tended also to use the first person frequently, to use informal language and to have some element of narrative in their texts) these associations are not strong enough to make it possible to categorise the texts into mutually exclusive groups. Given the aim of the study to produce as full a description as possible of the characteristics of coursework texts and of the ways in which teachers might respond to the various characteristics, it was necessary to select a sample of texts displaying a wide variety of the features identified in the set as a whole. The three texts were chosen, therefore, to ensure that a variety of qualities were represented and to include both 'extreme' and 'mixed' combinations of qualities. A further criterion for the choice was that the texts should have been awarded similar marks by their teacher; this was intended to ensure that, when teacher-interviewees were asked to assess and rank the chosen texts, they would be unlikely to be able to make immediate judgements but would have to pay greater attention to detailed features of the texts, thus revealing more of their reading and assessing practices.

Richard's 'Inner Triangles' text was chosen as an 'extreme' example with a cluster of formal, impersonal qualities. The immediate impression given by this very short text is of a text book exercise. Steven's is less impersonal and is the only text that is consistently attempting to make an argument. The text suggests the writer's authority and confidence as a mathematician. The author's presence is very evident in parts of Clive's text. This was also chosen because it represents mathematical relationships as causes, a relatively unusual characteristic. All three had been awarded 13 or 14 marks out of a possible 20. Table 6.4 summarises the qualities of each of the selected texts as determined by this stage of the analysis.

Table 6.4
Qualities of 'Inner Triangles' texts chosen for case studies¹

	<u>Clive</u>	<u>Richard</u>	<u>Steven</u>
<i>participants</i>	first person inanimate	.2	first person inanimate
<i>causality</i>	mathematics	-	human
<i>authority</i>	external	-	self
<i>attitude</i>	personal evaluative	impersonal	personal evaluative
<i>reader</i>	trusted	assessor	shared community
<i>formality</i>	informal impersonal	impersonal	informal impersonal
<i>structure</i>	3	repetitive	
<i>text type</i>	narrative	list	argument
<i>algebra/ generalisation</i>	procedural	relational solution	procedural solution

The full analyses of each of these case study texts may be found in chapter 8.

6.2.3 Selection of a sample of 'Topples' texts

The application of this analysis to the set of 'Inner Triangles' texts described in the previous section revealed that, in practice, some of the variables did not provide useful information at this level of the analysis. In particular, classifying the participants merely as first person, inanimate or passive did not distinguish clearly between texts as many included examples of two or even three of the three qualities and it was not possible at this level of analysis to determine their relative importance or their function. At the stage of detailed analysis of the

¹All three selected texts were written by boys. Gender is not one of the variables taken into account as the object of study is the text itself rather than the individual student. During the selection process, the texts were identified by labels which did not indicate gender; my selection was not, therefore, influenced by this.

²Richard's text was almost entirely non-verbal and did not, therefore, contain features which could be coded in these categories.

³The lack of coding in this category indicates only that Clive's and Steven's texts did not have a repetitive structure.

chosen case study texts, examination of the participants and the processes in which they were involved play an important role, but it was decided to omit this variable from those used to analyse the complete set of 'Topples' texts. At the same time, there was considerable overlap between some of the headings; similar qualities had frequently been noted under the headings of authority, attitude, reader and formality (all related to the interpersonal). These were therefore amalgamated under the general heading of formality, a more detailed consideration of the differences being left to the case study stage of the analysis.

Table 6.5

Variables used in analysis of 'Topples' texts

<u>heading</u>	<u>qualities</u>	<u>notes on changes from previous schema</u>
causality (ideational)	human physical numbers/ patterns	Because of the practical nature of the 'Topples' task, physical causation was found in a number of texts, e.g. <i>The rod that makes the pile topple</i> . This further quality was therefore added. This is an extension of the category of 'formula as cause' used previously. (No examples of causation attributed to mathematical relationships were found. This quality was therefore omitted.)
formality (interpersonal)	formal/ impersonal informal/ personal	The 'formality' variable represents a cluster of interpersonal aspects.
text type (textual)	narrative argument school maths report	This combines the 'list' quality and the repetitive structure used previously. This is an additional quality, included to account for passages of description of patterns and procedures without a human or temporal element.
diagrams	integrated multiple examples	e.g. <i>As you can see from the diagram . . .</i>
algebra/ generalisation	procedural relational/ symbolic solution	The definition of this quality was broadened because, although symbolic generalisations normally appear to be relational (and were coded as such previously), when viewed in context they take on a more procedural meaning. (No 'Topples' texts included manipulation, so this quality was omitted.)

The schema used to analyse the set of 'Topples' texts (shown in Table 6.5) was therefore simplified, and at the same time linked more closely to the identification of Halliday's meta-functions. In addition, because of the physical nature of the subject matter of the 'Topples' task it was decided to include diagrams as an additional heading and to consider two qualities of the diagrams which had been identified in the original exploratory analysis of the texts on this task: the integration of the diagrams by references within the verba part of the text, and the use of multiple diagrams to demonstrate the practical activity.

Similar procedures to those used with the set of 'Inner Triangles' texts were followed: initial notes were made under each heading which were then coded using the qualities listed in Table 6.5; a reduced matrix containing the codings was then constructed. As was found with the 'Inner Triangles' texts, there were a number of texts which had two or more qualities under some of the headings. Because of the reorganisation of the headings for the analysis of the 'Topples' texts, however, there was only one text which could not be allocated a code under every heading (this text contained no indications of causality and no diagrams).

Again three texts were chosen for detailed analysis, the basis for the choice being to include a range of qualities and to ensure that the three had received generally similar marks. One of the texts chosen was written by Steven, whose 'Inner Triangles' text had also been chosen. This was intended to allow a comparison to be made between the same individual's writing in relation to different tasks. Table 6.6 summarises the qualities of the selected texts. Each of these case study texts is analysed in chapter 8.

Table 6.6
Qualities of 'Topples' texts chosen for case studies

	<u>Sandra</u>	<u>Ellen</u>	<u>Steven</u>
<i>causality</i>	physical human	numbers	physical
<i>formality</i>	formal/ impersonal	informal/ personal	informal/ personal
<i>text type</i>	school maths	narrative	school maths narrative
<i>diagrams</i>	multiple examples	integrated	integrated
<i>algebra/ generalisation</i>	procedural	procedural	procedural relational/ symbolic

Although the methods used to screen the two sets of texts and select the case study samples were not identical, I would argue that this does not affect the basis of the selection as the differences were not fundamental. Only those variables which did not provide useful distinguishing information were omitted from the second analysis, while those which did

distinguish between texts were retained, even if only in an amalgamated form. Additional qualities were included which were of particular relevance to the practical nature of the 'Topples' task. As the two tasks were selected on the basis that they were of different types, it seems appropriate to consider differences between texts that arise from the particular nature of the task as well as those which may be more generic.

6.2.4 Analysis of the case study texts

The analysis of each of the selected case study texts was structured around Halliday's three meta-functions (ideational, interpersonal and textual) making use of the linguistic tools described in chapter 5 to produce a detailed description of the text. Separate analyses of each text, comparisons between them and a comparative analysis of Steven's complete set of five texts are presented in chapter 8.

6.3 Detailed examination of extracts of comparable sections across texts

The analysis of complete texts provides a rich picture of the range of linguistic and visual forms used by students in writing their GCSE coursework texts and of the nature of mathematical and social world presented by them. It also, by addressing the whole text, articulates with the ways in which teachers read and assess coursework, thus informing the interpretation of the interviews with teachers reading the texts. A further concern of this study, however, is to develop knowledge of the linguistic forms that may be used in the coursework context to an extent that may allow its users the power to create effects deliberately. To this end, it is necessary to look at alternative ways of writing apparently equivalent statements and to consider the effects that the various choices may have on a reader's interpretation of the texts. Because of the varied structures and content within the sample of coursework texts, the possibility of identifying 'apparently equivalent statements' across texts is limited. The only major aspect that may be readily identified in almost all texts on a given task is the expression of a generalisation or formula derived from the empirical data. This aspect, of course, is also of particular significance in the assessment of the author's achievement on the task. It was therefore decided to make an analysis of the section containing this generalisation from each of the texts on the 'Inner Triangles' task. A sample of three contrasting extracts from this set were also selected to be used in the interviews with teachers to look in greater detail at their specific reactions to different linguistic forms. The extracts and their analyses may be found in Appendix 6; analysis of the algebraic aspects of these extracts is in Appendix 14.

7 The coursework tasks

The LEAG examination board provided seven coursework tasks for candidates entered for GCSE in Summer 1991: three identified as suitable for candidates entered at any level of the examination, two for candidates entered at Foundation or Intermediate level and two for those entered at Intermediate or Higher level¹. Each candidate entered for the syllabus with 50% coursework was expected to submit work on five of these tasks. The texts in the sample used in this study represent students' work on six of the 1991 tasks (no student attempted more than five). All the students were entered at either Intermediate or Higher level; there are, therefore, few texts related to the lower level tasks and one of these was not attempted by any of the students (see chapter 6 for details of the set of student texts). Of these tasks, the three ('Symmetry Groups', 'Topples' and 'Passola') designated as suitable for candidates entered at any level of the examination were in most cases completed during the first year of the two year course leading up to the GCSE examination, while the two ('Inner Triangles' and 'Areas Under Curves') designated for candidates entered at the Intermediate or Higher levels and one ('Pendulum') for candidates entered at Foundation or Intermediate levels were undertaken during the second year of the course. In every case, the students were provided with a question paper stating the task. The teachers were provided with this question paper and with 'performance indicators' issued by the examination board to guide their assessment. The tasks and performance indicators were subsequently reprinted, together with those for previous years, in a collected edition (LEAG, 1991); it is this edition that has been referred to in this study.

All six tasks and their associated performance indicators were analysed using the methods of text analysis described in chapter 5. On the basis of this analysis two of the tasks were chosen to be considered in detail in this study. As a distinction is made in the official discourse between 'practical problem' and 'investigation' (see Appendix 15, section 1), it seems desirable to choose examples of both types of task. In practice the distinction is not clear cut; however, it is possible to identify two tasks, 'Topples' and 'Pendulum', which may be clearly distinguished from the rest because they involve using data derived from experimental manipulation of 'real world' physical objects rather than from objects constructed according to formal rules. As the 'Pendulum' task was only attempted by a small number of the students in

¹The three levels of entry to the GCSE examination are defined by the grades that are available to candidates. At Foundation level, the maximum grade available is E; at Intermediate level it is C. The differentiated coursework tasks are designed to be suitable for two levels; the F & I (Foundation and Intermediate) level tasks allow candidates to achieve grades G to C, while the I & H (Intermediate and Higher) level tasks allow candidates to achieve grades F to A.

the sample, the 'Topples' task has been chosen as one of those to be examined further. The second task chosen is 'Inner Triangles' which varies from 'Topples' not only in its subject matter, which is formal and abstract, but also in the fact that it is designated as suitable only for those students entered at Intermediate or Higher levels, whereas 'Topples' was intended for all levels of candidate. 'Inner Triangles' was thus undertaken during the second year of the two year GCSE course, while 'Topples' was completed during the first year. In this chapter the full analyses of these two tasks and an overview of the similarities and differences between all of the tasks are presented. The tasks and performance indicators are in Appendix 2.

7.1 Task analyses

The analyses of the tasks and the performance indicators examine the nature of the mathematics in the task and the expectation of the student constructed in the text, including the consideration of different expectations for different groups of students and the values attached to various processes and products. Of particular interest because of their significance in the discourse of 'investigations' are: the extent to which a task is presented as 'practical', i.e. concerned with 'real world' objects and/or material processes; the types of reasoning expected of the student; and the degree of autonomy allowed to the student in defining and carrying out the task. In examining the nature of the mathematics in the task, the objects and processes presented will be considered while the overall structure of the text, together with paralinguistic features such as bold or italic type face will provide insight into the nature of the task that the student is expected to undertake. The student's relationship to the task and the location of authority may be interpreted through consideration of the use of personal pronouns and the modality of the text, including the use of imperatives or interrogatives to define the student's activity. A picture will thus be developed of the nature of the student constructed by the text: the types of activity, including types of reasoning, that she is seen to be capable of and the degree of autonomy that she is seen to have.

Both the text of the task itself and the performance indicators will be considered. Only the task itself was made available to the students and might thus have a direct effect on their interpretations of the task and hence on the production of their own texts. The performance indicators, produced by the same corporate author, the examination board, provide an alternative perspective on that body's construction of the nature of mathematical activity and of the students undertaking the task. While the teachers who were interviewed while assessing the students' work were not provided with these performance indicators, some of them were familiar with them from their experience of assessing these specific tasks, others were familiar with the genre through their experience of assessing other tasks set by the same examination board, and it is likely that all shared, at least to some extent, an awareness of the sorts of values incorporated within examination board documentation in general although these might vary slightly between different boards. Analysis of the performance indicators is

thus significant in so far as it contributes to a richer picture of the 'official' discourse of GCSE coursework.

7.1.1 'Topples' (Levels F, I and H) (pp. 37-38)

This task is presented with the following structure:

summary description of the practical activity

detailed description of the practical activity with illustrations

statement of the anticipated result of the activity

statement of 'Your task'

question 1: perform the practical procedure with unspecified numbers of different examples and follow a detailed list of investigative procedures: record, tabulate, make observations, generalise, explain.

question 2: 'imagine' and give the result of a further example

question 3: give the starting value that will lead to a given result and explain the 'working'

optional extension: choose any unspecified extension

The first half of this one page task focuses exclusively on the practical activity. The first sentence even suggests that this is the primary purpose of the task:

In this task you will be asked to balance some rods of different lengths on top of each other, until the pile topples.

An example of how "we" balance the rods is then described in detail as a sequence of actions. Two diagrams show two steps in the process. The piles of rods are drawn from a frontal angle which involves the reader directly in the explanation of how to construct the pile (Kress & van Leeuwen, 1993b) while at the same time reducing the naturalism of the representation by avoiding any attempt to represent the three dimensional aspect. Thus a tension between the practical and the abstract is apparent within the diagrams; this tension is further manifested as the statement of the problem proceeds. The idea that the rods should be arranged with the left hand end level is stressed by being presented in three ways: verbally in the text, pictorially in the two diagrams and verbally again through labelling the left hand side of each of the diagrams with the word *level*. The task cannot be completed successfully without paying attention to this practical instruction and using some degree of manual dexterity. Indeed, the student who achieves only the lowest available grade (G) is characterised in the performance indicators as likely to produce results which are "flimsy, few and not particularly accurate". It is only at grades G and F that the question of accuracy appears to be relevant; for those students achieving higher grades, practical activity is not an end in itself but is assumed to be successfully carried out.

When the description of the process is completed, the reader is told:

You should find that this pile of rods topples when we get to the 5 unit rod.

The shift from the first person to addressing the reader directly with the second person, as well as the modifier *should*, shifts attention from the process of the practical activity to its result. This shift of attention is continued on the next line:

So the pile that starts with the 2 unit rod at the base eventually topples when we get to the 5 unit rod.

This not only summarises the example but also moves away from the 'practical' in two ways. Firstly, it is now the pile which appears as the actor; human agency is still present but only in a subordinate clause. Secondly, the starting point (the 2 unit rod) and the end point (the 5 unit rod) are emphasised while the process that occurs in between is reduced to the indeterminate word *get* and its immediacy is lessened by moving back to use the first person *we*. The originally practical activity has been transformed into an insignificant means of achieving data for the abstract investigation described in the second half of the statement of the task. This task is now described in bold type:

Your task is to investigate the relationship between the length of the rod at the bottom of the pile and the rod which first makes the pile topple.

This description of the task contrasts with the earlier statement which suggested that the student would be primarily engaged in the practical building of piles of rods. Although this task might be described as a 'practical' task and although the collection of the data is performed practically and necessarily with a degree of experimental uncertainty, it is clear that practical activity is not what is valued most highly. What is required is the discovery of an abstract relationship between numbers. This is made explicit in the performance indicators for the task which start by stating:

The generalisation for this task is well within the syllabus at intermediate and higher level; it is a simple linear function. We would therefore expect to see an algebraic (symbolic) representation of this generalisation from candidates at grade C and above.

It is interesting to note that the practical, empirical nature of the task is mentioned just twice in the performance indicators: once for the highest attainers who are expected to offer "some explanation of why the pile topples" (which must surely lie within the domain of physics or mechanics rather than GCSE mathematics), and once for the lowest attainers whose results "are likely to be . . . not very accurate" - a clear distinction between mental and manual activity for these two groups of students.

The structure of the numbered questions moves from an open starting point, allowing the student to choose her own examples, to require specific processes and, finally, answers to closed questions:

1. Starting with rods of different lengths at the base, build up your piles until each one topples. Make sure that the rods increase by one unit of length at a time.

This is a practical task in which the student is directly instructed, using the imperative, to “build”. The instruction to increase the rods by one is ambiguous: it could be a reminder of the procedure for building piles described previously; on the other hand, it could be the starting rods of each example that increase by one unit - which would be interpreted by an examiner as working systematically. This instruction is phrased in such a way that it appears as advice to “make sure” rather than as a direct imperative. It suggests that the student has some discretion and autonomy.

(a) Record the length of the rod at the base and the length of the rod that makes the pile topple.

While the starting point is left to the student’s discretion, the processes to be followed are tightly prescribed. The relevant variables are defined.

(b) Tabulate your results.

Although (c) suggests that *any* observations would be valued, both (d) and (e) appear to assume that there is a single generalisation/result to be achieved. This is also reflected in the extract from the performance indicators given above.

(c) Make any observations that you can.

(d) GENERALISE.

The capitalisation of the word *GENERALISE* stresses its importance as a primary aim of the whole task.

(e) Explain your result. (Well argued explanations based on intuition and insight will gain at least as much credit as those based on the principles of Physics.)

The emphasis in (e) is on the argument of the explanation rather than on its validity. Again, this devalues the practical side of the task.

2. *Imagine that you start with a rod of length 100 units and build up the pile using rods of lengths 101, 102, 103, ... units.*

What will be the length of the rod that first makes the pile topple?

The instruction to “imagine” indicates to the student that this is not a practical task. The size of the rods involved also suggests that this must be a theoretical activity. By wording the request in the interrogative rather than the imperative, the student is allowed to choose their own method of answering. From the performance indicators, it is clear that the method chosen may be used to discriminate between different levels of achievement: ‘They might even, at the lower level of grade D or top grade E, answer . . . by extending their table of results.’

3. *A pile topples when we place a rod of length 50 units at the top.*

(a) What will be the length of the rod at the bottom of the pile?

(b) Explain your working.

Again the student is allowed to choose her own method. This time, part (b) makes it explicit that the method used is significant, without letting the student know what is expected.

In spite of the lengthy description of practical activity given at the beginning, it is clear from the specific tasks required of the student that carrying out the practical activity is less important than performing the standard ‘investigation’ algorithm and answering hypothetical questions in a non-practical way. The performance indicators even suggest that, for the highest attaining students, the task is really about algebraic notation and manipulation rather than about piles of rods:

From the candidates at grades B and A we would expect to see use of this algebraic form in the two specific cases given.

To summarise: this task starts with practical activity, involving experimentation with physical apparatus; its subject matter rapidly moves, however, towards abstract patterns of numbers and, for the highest attaining candidates, algebraic manipulation. Although the student’s activity appears initially to be relatively open exploration, this is directed towards gaining specific answers. The model of reasoning is essentially inductive; although ‘explanation’ is expected of those achieving higher grades, this is to be judged on its internal consistency rather than on its relationship to the physical situation which is ostensibly the subject matter of the task.

7.1.2 'Inner Triangles' (Levels I and H) (pp. 78-80)

The task for students entered at the higher levels is structured as follows:

a diagram demonstrating the drawing of a trapezium and defining the term 'unit triangle'

a description of the trapezium, introducing the term *dimensions* and introducing the idea that a trapezium "contains" unit triangles

question 1: find the number of unit triangles given the dimensions for two specified examples

question 2: find the dimensions given the number of unit triangles for two specified examples

question 3: investigate the relationship between dimensions and unit triangles

a list of processes to be included in "your report"

optional extension: do an unspecified extension

rules about what the extension might be

The first part of this task is presented in a formal way that might be characterised as similar to a traditional school text book with the structure: definitions, example, exercise. This formality is expressed through the impersonal nature of the language, e.g.

The diagram below shows a trapezium drawn on triangular lattice or isometric paper.

There is no human agency here; it is the diagram which "shows" while the creator of the diagram is hidden by the use of the passive *drawn*. The lexicon of this part of the task is tightly restricted; the mathematical objects and the attributes of these objects which are to be considered in the question (trapezium, unit triangle, dimensions, top length, bottom length, slant length) are named unambiguously and these names are used repeatedly and precisely throughout the task. One of the effects of this formality is to suggest that the subject matter of this investigation is not negotiable by any individual student but may even be a part of the conventional body of school mathematics.

- | | |
|---|---|
| <p>1. <i>How many unit triangles are there in a trapezium with dimensions</i></p> <p>(a) <i>top length 2 units,</i>
<i>bottom length 4 units,</i>
<i>slant length 2 units?</i></p> <p>(b) <i>top length 4 units,</i>
<i>bottom length 7 units,</i>
<i>slant length 3 units?</i></p> | <p>This first question is presented in a stereotypically 'school maths' form as a single stem with two questions. While the use of the interrogative allows the student to choose any method, there is clearly only a single correct answer for each part. Achieving a "lower" grade F is defined by the performance indicators as obtaining these two answers.</p> |
|---|---|

2. *Give the dimensions of a trapezium containing*
 (a) *8 unit triangles,*
 (b) *32 unit triangles.*
- The imperative *give* suggests that this is a straightforward task that can be answered either from knowledge gained from previous experience or by using a non-practical method.

In spite of this appearance of simplicity and the use of the 'school maths' format, there is not in fact a unique answer for either part of question 2. While this may be signalled by the use of the indefinite article, there is no indication given in the task that more than one answer for each part should be sought. The performance indicators, however, suggest that finding more than one answer could be used to differentiate between students at grades F and E:

*For a higher F, must obtain **one** of the answers to Part 2(a).*

while

Look specifically for . . . answers to 2(a) to define lower E.

It seems that the 'better' student is expected to read through the simple directed nature of the given question to answer a more complex question that lies behind it.

3. *Investigate the relationship between the dimensions of a trapezium and the number of unit triangles it contains.*
- In spite of the use of the word *investigate* which might suggest some degree of choice for the student, both the goal and the method of the investigation are tightly defined.

The student is to investigate "the relationship", the definite article and the singular noun suggesting that there is only one correct answer to this investigation. This is echoed by the performance indicators which specify the formula which students are expected to obtain in order to achieve a grade C or B. In practice, there are a number of alternative ways of expressing the relationship (see interviews with teachers, chapter 14).

In your report you should

show all your working,
explain your strategies,
make use of specific cases,
generalise your results,
prove or explain any generalisations.

This is the first point in the task at which the student is addressed directly in the second person. It marks a shift from the formal 'school maths' domain of the earlier part of the task, characterised by the absence of human participants, the limited specialised lexicon of the mathematical situation and the use of the imperative or interrogative to address the reader.

Here the instruction to the student is modified by *should*, suggesting the possibility of alternative approaches. The task is personalised; the report, working, strategies and results are all identified by the possessive pronoun *your* as belonging to the individual student.

The list of processes that the student should do prescribes an inductive approach to the problem through using specific examples and generalising from them. Its ordering also suggests a hierarchy within the list of processes (given the convention of school maths that questions progress from easier to more difficult). This hierarchy is confirmed to some extent by the performance indicators. For example, “explanation of methodology” is expected for a grade D while there is no expectation of proof or explanation of the mathematics until grade A where:

The quality of explanation of why the generalised result is as it is, defines the level of Grade A.

OPTIONAL EXTENSION

Extend this investigation in any way you wish. Although the student appears to be being allowed unlimited freedom to extend “in any way”, this freedom is immediately curtailed by placing constraints which, while softened by the modifier *only* are nevertheless emphasised by the repeated statements of what *must* be done and by the underlining of the whole passage.

For the extension, the only constraints placed on you are that figures must be drawn on isometric paper and that you must look at figures within figures.

To summarise: this task is located within ‘school maths’ although the specific subject matter is new and thus requires initial definitions and examples. The student’s activity at the beginning of the task is closely determined and it is not expected that the lowest attaining students will progress beyond answering specific closed questions. Those who progress further, however, are allowed more autonomy in deciding both the goal of their investigation and its methods. The type of reasoning expected is inductive although “explanation” is expected at the highest grade levels.

7.2 Similarities and variations between the tasks

The structure of all the tasks conforms to many of the conventions of school maths: in each task the ‘easier’ questions (as defined by the expectations of students at different levels expressed in the performance indicators) are found at the start, leading on to the more difficult questions, at which it is clear only the ‘most able’ are expected to succeed. Paralleling this progression from ‘easy’ to ‘difficult’ there is in the tasks intended to be attempted by the full range of students (‘Symmetry Groups’, ‘Topples’ and ‘Passola’) a movement from an initial image of mathematics as a practical activity, involving concrete objects and material actions, towards a world of more abstract and symbolic objects and mental activity. Where the tasks are differentiated by examination entry level, however, this movement is less pronounced within the individual task. In the tasks for the upper attainment range practical activity is either

entirely absent, as in 'Inner Triangles', or marginalised. In 'Areas Under Curves', for example, it is mentioned only in a note. In 'Pendulum' (for the lower attainment band), on the other hand, material objects and processes are present throughout the text and, although abstract symbolism is introduced at an early stage, the student is not expected to make use of it in any way. This distinction between material activity for the lower attaining students and mental activity for the higher attaining students is also to be found in the performance indicators for tasks set at all levels.

Two of the tasks, 'Topples' and 'Pendulum', ask students to gather data from the physical manipulation of concrete apparatus. Experimental error is clearly an issue in any such process of gathering data but the possibility of lack of accuracy in the results obtained by experiment is only admitted for the lower attaining students. The activity of those destined for higher grades rapidly becomes symbolic, their success to be measured by their arrival at the correct algebraic formula.

In spite of the physical nature of the overt subject matter of these two tasks, students are not expected to use their knowledge of the physical world or even their knowledge of school physics. The 'correct' result of the 'Pendulum' investigation could be found in a physics text book or learnt in a science lesson. Stating this result would not, however, allow the student to fulfil the assessment criteria (defined implicitly by the performance indicators), which demand that the experiment should be carried out, including the testing of implicit hypotheses that may be known to be false². Students are asked to explain the physical phenomena observed in the 'Topples' investigation but here again any pre-existing knowledge of the laws underlying the physical situation is devalued, the importance of the form of the argument being stressed rather than its validity. There is a tension here between the idea of a 'practical' task, which in these examples appears to be borrowed from the domain of school physics experiments, and that of a mathematics 'investigation', which appears to involve exploration of a self-contained system. This difficulty in dealing with references outside the immediate mathematics classroom context of the coursework activity is also apparent in some of the more 'pure' tasks, particularly those whose subject matter is part of the conventional domain of academic mathematics; 'Symmetry Groups' suggests that students might research the topic using books but gives no guidance as to how such research might be assessed, while 'Areas Under Curves' gestures towards the existence of more sophisticated techniques in higher mathematics but does not allow the student to make use of them. Although references may

²For example, the student is instructed to "Investigate what happens to the period of *the swing* . . . when you change the weight of the bob", testing the implicit hypothesis: "*The weight of the bob has an effect on the period of the pendulum*".

be made to the 'real world', to other school subject domains, or to mathematics itself, a coursework task is essentially about nothing but itself.

The form of reasoning that is overwhelmingly expected of students is inductive generalisation from empirically generated data. For all the tasks, the student is expected to examine particular examples which may be specified by the task or may be chosen by the student. Having done this, comments, observations and/or generalisations are requested. Both generalisation and explanation are accorded high status in the texts provided for the students, often stressed by paralinguistic means. It is clear from the performance indicators, however, that only the highest attaining group of students are expected to provide any sort of explanation relating their results to the structure of the original problem. Again, this differentiation between students is seen most clearly when contrasting the I & H level task 'Areas under Curves' with the F & I level task 'Pendulum'. In neither of these tasks is the student expected to provide a theoretical explanation for her results; in 'Areas under Curves', however, the desirability of proof is signalled by the statement that it is possible using a higher level of mathematics, thus suggesting that it is something towards which the student might aspire.

Much of the official and professional discourse surrounding coursework and investigations suggests that value is laid on students' freedom to determine their own questions and methods (see Appendix 15). This freedom is signalled in most of these tasks by the instruction to the student to 'investigate'³. The signal is, however, ambiguous as in each case the subject matter of the investigation is tightly defined: "investigate the relationship between . . .", "investigate what happens to . . . when you change . . .". The methods to be used are also largely determined by the use of imperatives and the explicit listing of processes to be undertaken⁴. Moreover, it is clear that, whatever variation between students might be permissible in the course of doing the tasks, they are all supposed to be aiming at the same

³Symmetry Groups is the only one of the six tasks which does not use this instruction. In Passola and Areas under Curves, it appears in the optional extension.

⁴Since 1991 some changes have taken place in the setting of coursework tasks by the London examination board. In particular, there seems to be rather less explicit guidance provided in the text of the tasks about the methods to be used. This does not, however, mean that students have more freedom to choose their own methods. Rather, the authority that in 1991 was invested in the text is, in 1994, delegated to the individual teacher who is instructed that he or she

should suggest to candidates, at the appropriate tiers of entry, that they:

- make and record any observations and comments;
- record any results or data;
- try to work in an ordered, systematic manner
- . . .

(ULEAC, 1994)

and so on.

uniquely correct 'answer' in the form of an algebraic generalisation. This uniqueness is in some cases signalled to the student explicitly by the use of the definite article: "find the generalisation"; in other cases it is only stated in the performance indicators. Thus in 'Inner Triangles', the student is instructed to "generalise your results", suggesting (by the possessive pronoun) student ownership of the results and possible variation between different students, while the performance indicators state that "The results are . . .", claiming absolute authority and no room for variation. There is a tension between, on the one hand, the principle of student exploration and autonomy expressed in the official and professional rhetoric of investigations and reflected in the use of the word *investigate* and, on the other hand, the absolutist nature of mathematics expressed in these texts. This tension may be a consequence of the assessment context in which the texts are located. Although a note inside the front cover of the collection of tasks and performance indicators (LEAG, 1991) states that the performance indicators

are provided as a *guide* to assessing the level of performance demonstrated by the student. They are *not* hurdles and must be treated flexibly. Alternative approaches to solving the problem or tackling the task must be assessed using different measures of performance (original italics)

the forms of language used within the tasks and within the performance indicators associated with individual tasks do not reflect such flexibility.

7.3 Conclusion

There are tensions between some of the values expressed in the official and professional rhetoric of coursework and investigations and their manifestation in this set of coursework tasks. 'Practical' activity is one of the six ways of working demanded by the Cockcroft report (Cockcroft, 1982: para. 243) and also appears in the general GCSE criteria (DES, 1985) as a compulsory component of any scheme of assessment by coursework. The intrusion of 'real world' or practical aspects into the coursework tasks, however, produces uncertainty and problems with the definition of task and solution. This tension has been resolved by ensuring that each task includes material objects and actions while simultaneously circumscribing these material aspects to such an extent that their 'real world' existence is marginalised. In this way, each task takes place within an imaginary abstract world that may share some characteristics (or at least vocabulary) with other discourses but which is entirely self-contained. These imaginary worlds are also assumed to be uniform systems; inductive generalisations from empirically generated data are presented as the primary valid means of reasoning.

Although autonomous student activity is valued and called for through instructions to 'investigate', this is in tension both with the absolutist nature of mathematics portrayed in these tasks and with the need to validate assessment decisions by reference to common criteria for all students. Little variation is accepted in the methods that may be used by students and it is clear in most cases that only one 'correct' result is possible.

One of the most striking features arising from this analysis of coursework tasks and performance indicators is the way in which the tasks construct students of different attainment levels both through the differentiated tasks that are set and through different expectations of performance within the same tasks. While all students are expected to undertake 'practical' activities, these constitute the whole task for the low attaining students but are marginalised for the high attainers. Any attempt to reason theoretically is preserved for the very highest attaining students.

8 Case study analyses of GCSE coursework texts

The student texts selected for further analysis were chosen on the basis that they appear to contain a range of characteristics. The preliminary analysis outlined in chapter 6, while providing a general description of the texts, did not provide detailed knowledge of the linguistic and non-linguistic forms used within the texts. Such detailed knowledge is necessary in order to address the aims of this study: to investigate the ways in which teacher-readers respond to specific aspects of coursework texts and to form a basis for providing concrete support for students to develop their awareness of the genre of investigation reports and hence to improve their ability to produce texts that will be likely to be judged to be successful. Each of the three texts selected from each of the sets on the two tasks 'Inner Triangles' and 'Topples' has been analysed in greater detail using the method described in chapter 5. In each case, the ideational, interpersonal and textual aspects of the text are considered in order to describe the nature of mathematics and mathematical activity, the relationship of the student-author to the task and to the reader and the type of text constructed by the writer. The texts and detailed analyses may be found in Appendix 5; in this chapter, the main features of each text are summarised and comparisons are made between the three texts on each of the two tasks. These give an indication of the range of characteristics found in the texts as well as specific similarities and differences that may be significant in affecting teachers' responses to the different texts. Finally, Steven's texts are examined across the tasks. Both his 'Inner Triangles' and his 'Topples' text were included in the selection for detailed analysis. These analyses are compared and further evidence from his other three coursework tasks is considered in order to construct a picture of an individual student-author's repertoire.

8.1 The 'Inner Triangles' texts

The qualities of each of the three selected 'Inner Triangles' texts and the textual features indicating these qualities are summarised in Tables 8.1.1 to 8.1.3. In each case, the ideational aspects are presented first, followed by the interpersonal and finally the textual aspects. The texts themselves and the full analyses from which these summaries are derived may be found in Appendix 5.

Table 8.1.1

Clive: Summary analysis of 'Inner Triangles' text

qua ty	indicated by
<p>IDEATIONAL</p> <p>Mathematics is about patterns of numbers (although these may reflect patterns in shapes .</p> <p>Tables are autonomous mathematical objects.</p> <p>The role of human mathematicians is to use tables and diagrams to find answers (which may be specific numerical results or formulae).</p> <p>The overall aim of mathematical activity is to find formulae which express algorithms.</p>	<ul style="list-style-type: none"> - many relational objects - tables presented as actors - human actors mediating between tables and diagrams and results - human agents find formulae - first person to make ownership claims
<p>INTERPERSONAL</p> <p>The original task was imposed upon the author but the extension is his own.</p> <p>The author has qualified confidence in the quality of his work.</p> <p>It is not important to use conventional mathematical terminology and symbolism</p> <p>The reader is expected to assess the author's individual contribution to the mathematical work.</p> <p>The reader is expected to be interested in products (including general methods) rather than processes.</p> <p>Reader and author share a common understanding of the problem situation and the reader is expected to interpret the text sympathetically.</p>	<ul style="list-style-type: none"> - contrast between 'copying' the original question and the passive "we were given" and the use of the first person in the extension - explicit but modified statements of confidence - variation in the terminology used for the basic objects of the task - distinction between 'the group' and 'I' as actors - use of the second person in relation to tables and answers - lack of specialist language - some deixis and lack of specificity
<p>TEXTUAL</p> <p>The text is a display of a number of relatively discrete products: tables, patterns and formulae.</p> <p>It is a 'school mathematics' text but has both 'question-answer' and 'project' features.</p>	<ul style="list-style-type: none"> - a repeated structure of existential declaration of tables etc. - copied questions - question numbers and headings determined by task <p>but</p> <ul style="list-style-type: none"> - title page and conclusion

Table 8.1.2
Steven: Summary analysis of 'Inner Triangles' text

<u>quality</u>	<u>indicated by</u>
<p>IDEATIONAL</p> <p>Mathematics is about patterns and relationships between variables.</p> <p>Fulfilling the requirements of mathematics coursework involves finding and testing a formula.</p> <p>Practical activity such as drawing and counting takes place in the background but is not important.</p> <p>The purpose of presenting data in tables is to illustrate patterns that occur in it.</p> <p>The role of human beings is to manipulate parameters and to observe the results.</p>	<ul style="list-style-type: none"> - many relational objects - nominalisation of changes in variables - discontinuity between the general focus on description of patterns and the display of the formula and its test - small number of practical examples displayed through diagrams without verbal elaboration - tables as actors in verbal processes - human actors 'change', 'see' and 'find'
<p>INTERPERSONAL</p> <p>The reader is concerned with the mathematical content, not the author's processes.</p> <p>The author is an enthusiastic authority on this task.</p> <p>The author is confident of the validity of most of the work, although he has 'hedged' the claim with the highest level of generality.</p> <p>The reader is invited, as a colleague, to learn from the author.</p> <p>It is not necessary to use conventional mathematical terminology.</p>	<ul style="list-style-type: none"> - few first person references - only first person process is to 'find' - explicit expressions of interest - fluent variation of vocabulary - generally unmodified statements - modal adverbs expressing certainty but - modification of most general statement by 'may' - second person 'can see' - variation in terminology used for basic objects of the task
<p>TEXTUAL</p> <p>The text incorporates 'school mathematics' question and answer features within a 'project' framework. There are discontinuities in genre between the various sections of the text.</p> <p>It is a report describing patterns.</p> <p>Evidence for the description is presented in a logical sequence.</p> <p>A formula is included because it is a required part of coursework.</p>	<ul style="list-style-type: none"> - copied questions - answers signalled by equals signs but - title, contents page and conclusion - colour, elaborate headings - high proportion of topical themes where the topic is a relational object - a section with themes indicating sequence and reasoning - discontinuity of this section from the rest of the text

Table 8.1.3
Richard: Summary analysis of 'Inner Triangles' text

<u>quality</u>	<u>indicated by</u>
<p>IDEATIONAL</p> <p>The subject matter of the task is shapes and their measurements.</p> <p>Mathematical activity consists of stating relationships between mathematical objects.</p> <p>No human activity is reported.</p> <p>The focus of the text is on product rather than process.</p>	<ul style="list-style-type: none"> - only basic and derived objects as actors - only relational processes - no human actors - nominalisation of problem solving processes - highlighting of formula by positioning and heading
<p>INTERPERSONAL</p> <p>The communication between author and reader is formal.</p> <p>The reader is an assessor.</p> <p>The reader is an 'expert', familiar with mathematics and with this task.</p>	<ul style="list-style-type: none"> - no reference to author or reader - unmodified statements - labelling of answers - specialist vocabulary - lack of verbal elaboration
<p>TEXTUAL</p> <p>The text is a product being presented for assessment.</p> <p>It is a 'school maths' text, containing both question-answer and stereotypical 'investigation' components.</p>	<ul style="list-style-type: none"> - title pages - careful presentation - question numbers - repetitive structure - data-pattern-generalisation structure

As may be seen from the ideational sections of the tables above, the subject matter with which each of the texts is concerned varies. While the subject matter of Richard's text is shapes and their measurements, both Clive and Steven appear more concerned with patterns; Steven, in particular, focuses on patterns in numbers and relationships between them rather than on the concrete objects which formed the original data for the task. Similarly, the nature of mathematical activity and, in particular, the role of human actors varies. For Richard, the only explicit mathematical activity is the stating of relationships between mathematical objects; there is a complete lack of human presence in his text. On the other hand, Clive's text constructs a role for humans in using tables and diagrams to find required answers, while Steven shows human beings to have a more active role, manipulating parameters and observing the consequences.

The analyses reveal substantial differences in the interpersonal aspects of the three texts. Richard's impersonal style, referred to above, contributes to the extreme formality of his text, while both Clive and Steven have produced relatively informal texts, including the use of unconventional vocabulary. Their constructed readers, although all apparently interested in the products rather than the processes of mathematical activity, are thus very different. Both Richard and Clive appear to be addressing an assessor, although this is indicated in different ways; in Richard's case, however, this assessor is a distant expert, whereas Clive's is more personally interested in the author, assessing his individual contribution and sharing his attitudes and concerns. Steven, in contrast, confidently addresses a colleague who is expected to share his interest and enthusiasm about his results.

It may be seen that all three texts have some 'school maths' type features; in all cases the questions posed provide at least some of the structure for the students' texts through the use of question numbers or copied headings. In the case of both Clive and Richard, the texts thus appear as a set of relatively discrete products, although Clive provides some coherence to his text by drawing the reader's attention to each component. Steven's text is more of a coherent whole, although even here there are some discontinuities between different parts of the text. He includes sequences of descriptive report and of logical reasoning as well as displaying products such as his formula.

A theme that emerges from the analyses of all three 'Inner Triangles' texts is the focus on product rather than process. This is indicated in the stress laid on the formula, in the ways in which the reader's interests are directed, and in the structure and presentation of the texts.

8.2 The 'Topples' texts

The qualities of each of the three selected 'Topples' texts and the textual features indicating these qualities are summarised in Tables 8.2.1 to 8.2.3. In each case, the ideational aspects are presented first, followed by the interpersonal and finally the textual aspects. The texts themselves and the full analyses from which these summaries are derived may be found in Appendix 5.

Table 8.2 1

Steven: Summary analysis of 'Topples' text

<u>quality</u>	<u>indicated by</u>
<p>IDEATIONAL</p> <p>Although data may arise originally from practical activity, the subject matter of mathematics is number patterns and calculations.</p> <p>Formulae and numerical answers are important outcomes of a coursework task.</p> <p>A formula is a way of presenting a procedure for finding an answer.</p> <p>The role of human beings is to observe and discover patterns and formulae and to perform calculations.</p>	<ul style="list-style-type: none"> - preponderance of numerical and relational actors - highlighting through presentation - lack of integration of practical context - explicit statement that formula is to be 'used' - emphasis on exemplification of procedure - humans as actors in mental processes and in calculations
<p>INTERPERSONAL</p> <p>The author is confident and authoritative.</p> <p>The reader is concerned with checking that all requirements are fulfilled.</p> <p>The requirements of the task include not only answering all the questions but also demonstrating the use of 'investigation' processes.</p> <p>The reader is also interested and involved with the mathematics.</p>	<ul style="list-style-type: none"> - fluent variation of vocabulary - instructions given to the reader - confident modality - use of the present tense, claiming generality - translation of the given problem and example into a first^{person} narrative - use of given questions as headings - ticking off requirements - use of the first person as actor in investigative processes (e.g. predict, estimate, etc.) - use of the second person as actor in material processes
<p>TEXTUAL</p> <p>Different parts of the problem demand different styles of response.</p> <p>The text is both 'school maths' and 'project'.</p> <p>The author has attempted to present a coherent whole text.</p>	<ul style="list-style-type: none"> - discontinuities in the types of themes used in different sections - copied questions followed immediately by answers but - coloured title - framing with title page and "the end" - some explicit cohesive links made, including references to earlier parts of the text

Table 8.2.2

Ellen: Summary analysis of 'Topples' text

<u>quality</u>	<u>indicated by</u>
<p>IDEATIONAL</p> <p>The subject matter of this task is material objects and their behaviour.</p> <p>The role of human beings is to observe this behaviour and to perform calculations which produce parallel results.</p> <p>Tables need to be accompanied by verbal text containing parallel information.</p> <p>The purpose of formulae is to describe methods of calculation.</p>	<ul style="list-style-type: none"> - preponderance of basic and derived objects as actors in material processes - capitalisation and overlexicalisation of physical objects - human actors primarily in calculations and mental processes - numerical results paralleled by references to the material world - tables and 'equivalent' verbal text juxtaposed - imperatives used to express generalisations
<p>INTERPERSONAL</p> <p>The text is divided into two sections which address two different types of reader:</p> <p>The reader of the first section is an examiner</p> <p>The second reader is a trusted and interested colleague.</p>	<ul style="list-style-type: none"> - general obscuring of human agency - 'false trail' presented and rectified - increased use of first person - admission of personal failure
<p>TEXTUAL</p> <p>The text is largely descriptive of observations and procedures.</p> <p>It has many features of a 'project' but also has 'school maths' aspects.</p>	<ul style="list-style-type: none"> - a sequence of topical themes followed by - a sequence of imperatives - elaborate presentation with colour, headings, etc. - statement of the problem heavily paraphrased - deviations from the given order of the task - headings related to investigation stages <p>but</p> <ul style="list-style-type: none"> - structured by question numbers

Table 8.2.3
Sandra: Summary analysis of 'Topples' text

<u>quality</u>	<u>indicated by</u>
<p>IDEATIONAL</p> <p>The subject matter of the task is material objects and their behaviour.</p> <p>Mathematical activity consists largely of performing repetitive but largely unconnected procedures.</p> <p>Mathematics is a human activity and the human role is to manipulate material objects and observe the results as well as to pose problems.</p> <p>The task is to describe general procedures rather than to provide a specific narrative of the author's activity.</p> <p>The purpose of an 'extension' is to pose further problems rather than to solve them.</p>	<ul style="list-style-type: none"> - preponderance of basic or derived objects as actors - naturalistic diagrams - repetitive structure, listing individual results - human actors involved in material and mental processes - use of a general second person as the main human actor - use of the present tense - there is an explicit statement of this
<p>INTERPERSONAL</p> <p>The relationship between author and reader is distant and formal.</p>	<ul style="list-style-type: none"> - the author's presence is largely obscured - copying and echoing of the words given in the task - lack of expressions of attitude - meticulous presentation
<p>TEXTUAL</p> <p>The text is in the 'school maths' question-and-answer genre.</p> <p>The extension is a display of the author's activity.</p>	<ul style="list-style-type: none"> - question numbers as headings - copied questions - repetitive structure - interpersonal and existential themes in the extension

Considering the ideational aspects first, the summaries show that all three present a procedural view of mathematical activity, yet both the nature of the subject matter and of human involvement in mathematics vary. In Steven's case, the subject matter of the task is number patterns, formulae and calculations; human beings are involved in observing and discovering the patterns and performing the calculations. While the material objects and practical activity were necessary in order to generate the numbers which form the patterns, they play a minor role. For both Sandra and Ellen, on the other hand, material objects and their behaviour are central to the subject matter of the task. Even here, however, there are differences between the two in relation to the role of human activity: while Sandra presents this as the manipulation of the material objects, Ellen's text suggests that the material objects'

behaviour is independent of human intervention and that the human role is to observe and describe such behaviour.

Again the analyses show substantial differences between the interpersonal aspects of the three texts. Sandra's text is very impersonal and formal, giving the impression of a distant relationship with a public audience and displaying little personal involvement with the subject matter. Both Steven and Ellen also demonstrate to an examiner that they have fulfilled the necessary requirements of the task (all three display the numbers of the questions posed by the examination board). In addition, however, parts of these two texts construct a relationship with a reader who is interested in what is being read, although, as may be seen in Tables 8.2.1 and 8.2.2, this interest is indicated in different ways. While Steven presents himself as confident and authoritative and involves the reader in the mathematical activity, the latter part of Ellen's text show her as tentative but willing to take the risk of exposing her uncertainty to a trusted reader.

As was also seen in the summaries of the 'Inner Triangles' texts, the use of question numbers by all three authors gives each of the 'Topples' texts a 'school maths' aspect; the strength of this aspect, however, varies between the texts. In the case of Sandra, in spite of her decorative presentation, it is reinforced by her extensive use of the wording from the question paper and by the 'question-answer' structure of much of her text. The other two texts both have project-like features, being structured around the components of investigation (formula, extension, etc.). Although neither is consistent in style throughout the text, Steven has attempted to create a coherent whole through the use of explicit cohesive links between different parts of the text.

8.3 Steven: a case study of an author across texts

Steven's 'Inner Triangles' (Table 8.1.2) and 'Topples' (Table 8.2.1) texts were both selected as part of the samples to be analysed in detail and to be used in the interviews with teachers. In this section the analyses of these texts will be compared in order to construct a picture of those features that are common to both, and may thus be seen as a constant part of Steven's repertoire within the genre, and those features which differ, and may thus be specific to a given task or type of task. At the same time, the extent to which the other three texts produced by Steven in response to the 'Passola', 'Symmetry Groups' and 'Pendulum' tasks¹ also exhibit such features will be considered. Where features used commonly across a number of tasks are identified, it will be particularly important to consider teachers' responses

¹All the tasks set by the examination board are in Appendix 2 and Steven's texts on the additional three tasks, 'Passola', 'Symmetry Groups' and 'Pendulum', are in Appendix 7.

to them; if a feature that is likely to be assessed negatively is used consistently by a student across his or her whole coursework submission, this could have a significant impact on the student's ultimate performance in the GCSE and means of intervention would need to be considered.

8.3.1 Ideational aspects

It may be seen from the summaries in Tables 8.1.2 and 8.2.1 that representational and relational objects (tables, patterns, formulae) play an important role in Steven's 'Inner Triangles' and 'Topples' texts. These are stressed by the use of headings and colour and announced to the reader by declarations such as "I have found a formula". The major human involvement in both texts involves finding patterns and formulae or reading information from tables. A similar emphasis on tables and formulae is found in the other three texts as well. Even in 'Pendulum', in which the question paper explicitly instructed the student to include tables and graphs, these are further highlighted through the use of headings and colourful presentation. In Passola, a table containing all the results and colour coded to demonstrate patterns within them plays an important role: the reader is referred to it at several points in the text and it is even presented as an actor in its own right:

The grid demonstrates my theories.

(Steven: Passola)

In 'Symmetry Groups', as in both 'Topples' and 'Inner Triangles', a formula is presented on a separate page under a large heading announcing its status.

The major differences in the pictures of mathematics and mathematical activity constructed in Steven's texts appear to be related to the subject matter of the tasks he was set. Thus the 'practical' text ('Topples') does involve some manipulation of the concrete objects involved in the task, although this is only apparent in the early part of the text in which the problem is described. The more 'pure' 'Inner Triangles' text focuses on manipulation of the variables and on changes in the patterns of numbers. Interestingly, the 'Pendulum' text, a 'practical' task completed during the second year of the course, contains no human involvement in the manipulation of physical objects. Although the first part of the task instructed the student to make a pendulum, Steven's report obscures this practical activity by using a labelled diagram of the apparatus, the nominalisations *this design* and *positioning*, and the passive *the experiments were started*; the only human involvement is to observe and make decisions:

This design started out to be very promising, but when the experiments were started I found the bob got caught on the wall, and this could have affected my results. I decided against the positioning of pendulum but the design was very good.

(Steven: Pendulum)

Later in this text, as in the 'Inner Triangles' text, human involvement is restricted to changing the values of variables.

The explanation in the 'Topples' text makes use of the vocabulary of physics, introducing the terms *gravity* and *weight*. This is also found in Steven's explanations of the behaviour of his

pendulum in 'Pendulum'. Elsewhere, Steven restricts himself to the domain of objects defined by the set task. This reflects the difference in the nature of explanation in practical and pure tasks; in the practical situation it is necessary to refer to the real world context beyond the confines of the task, whereas, in the pure situation, explanation is provided by considering relationships between the variables within the restricted context of the task itself.

8.3.2 Interpersonal aspects

The major interpersonal characteristic identified in both Steven's 'Inner Triangles' and 'Topples' texts is the expression of confidence and authority. This is to be found both in his use of statements of his achievement in the form "I have found a formula" and in the ways in which he addresses his reader directly. He uses the second person *you* not only to describe general processes but also to address his reader, who is constructed as an active participant, observing patterns in the tables (in 'Inner Triangles') and performing calculations (in 'Topples'). The reader is expected to be interested in Steven's discoveries and Steven himself is presented as the 'expert' displaying his knowledge. Similarly in Passola the reader is addressed with authority, for example:

You can see these patterns demonstrated on the grid on page 8. Just match the colours.

(Steven: Passola)

and is expected to take an active part in making sense of Steven's solutions. Steven does not address the reader directly in his other two texts although in 'Pendulum' a sense of his authority is conveyed through statements of confidence in his decisions and the results of his work:

When I did my timing I decided that you could not trust one result, so I took three results for each time and then found the average time. This worked very well and all the results fell in place.

(Steven: Pendulum)

As was mentioned above, Steven's response to the 'Symmetry Groups' task follows the structure of the question paper very tightly with predominately one sentence answers to each of the questions. The picture of the relationship between student-author and teacher-reader thus constructed is one in which the reader is assessing simple factual responses to closed questions without concern for the author's individual processes or explanations. It seems likely that this is a response to the structure of the task itself which consists almost entirely of a set of short closed questions and hence allows little scope for the student to claim 'ownership' of any part of the task.

It was noted in the analyses of both texts that Steven has overlexicalised the basic objects of the two tasks, using a variety of words and phrases to refer to the dimensions of trapezia in 'Inner Triangles' and to the components of piles of rods in 'Topples'. As well as emphasising the significance of these objects this too contributes to his expression of confidence. Although in some other contexts, such flexibility of vocabulary might be considered a sign of a fluent writer, it is more conventional in mathematics to maintain consistency in the terms used.

This variation is not found in the other texts but the more consistent usages in these texts may have been influenced by the ways in which the tasks were set. For both 'Pendulum' and 'Passola' the names of the variables to be used in the problems were emphasised in the question paper through the use of italics or capitals, thus making it obvious to the student that it was important to use these names. For the 'Symmetry Groups' task, Steven uses the term *element* consistently; again this term was defined in the question paper and, in this case, Steven's text is structured very tightly by the given questions, echoing the wording of each question in his response.

8.3.3 Textual aspects

All Steven's texts largely follow the structure of the given question papers, including question numbers and copies or paraphrases of the questions. They also, however, have 'project-like' features such as a title page, coloured or underlined headings and, in some cases, additional headings such as "Formula" which indicate the presence of sections which play an important role in the conventions of investigation. Because of this structure, the texts appear disjointed. A minimal level of cohesion (beyond that provided by reference to the question paper itself) is achieved through the consistent use of coloured and underlined headings and other 'signposts'. In addition, in 'Topples' and 'Passola' there are a small number of explicit links between different sections, each referring the reader to a formula, pattern or table to be found in another section of the text.

The analyses of Steven's texts for both 'Inner Triangles' and 'Topples' reveal some discontinuities in genre. Both texts contain passages of descriptive report and passages in which the thematic structure suggests the construction of a train of reasoning. In neither text, however, is there any narrative recounting Steven's actions or processes apart from simple announcements that "I have found a formula". This lack of narrative is also apparent in the other three texts, although Steven refers to his decisions in 'Pendulum'.

One of the features which led to the choice of Steven's 'Inner Triangles' text as one of the sample to be analysed in detail was the presence of passages of argument. This characteristic was relatively uncommon among the whole sample of 'Inner Triangle' texts but is found at a number of points in Steven's work. In all cases in which Steven is providing explanations of phenomena he has observed, he signals this clearly by the use of terms such as *this is because* or *the reason is*. This serves to draw attention to the causal relationships and to the fact that he is providing an explanation.

As was mentioned in the section above, 'Symmetry Groups' consisted almost entirely of single sentence responses to the given questions. 'Passola', however, contains both passages of descriptive report:

When the person number is even and the step number is odd then the moves taken will be the same as the person number. There are a few exceptions.
(Steven: Passola)

and passages of explanation:

This happens, because the step number is odd it will not go into the person number which is even, so it has to go around a few times before it gets back except for a step number of one but this still has the same move number.
(Steven: Passola)

These two passages are given as responses to different questions, appearing on different pages; there is no explicit cohesive link in Steven's text that would indicate that *This* in the second passage refers to the phenomenon described in the first. It is only through reading the question paper alongside Steven's text that coherence can be achieved. In 'Pendulum', however, the descriptive and explanatory passages are juxtaposed:

*When the angle is changed from 10° up to 40° the period of swing changes, it is a very small change but it is there.
This could be because when the bob is dropped from a higher height there is a faster speed and therefore it takes nearly as long.*
(Steven: Pendulum)

Here Steven is responding to a more open instruction in the question paper to:

Comment on your results, offering any observations or generalisations.

It would seem that Steven is capable of writing complex passages of exposition but that he is constrained by the format of the task given to him.

8.3.4 Summary of Steven's language across tasks

There is a high degree of consistency in the language used in all five of Steven's texts. In particular, Steven focuses consistently on patterns and formulae as the subject matter of the tasks and constructs himself as a confident authority in relation to an interested and actively involved reader. He appears more concerned to describe and explain the results of his mathematical activity than to present the processes he has gone through. When providing explanations, he draws attention to causal relationships by announcing them explicitly. Although his texts contain several passages of coherent exposition presenting chains of reasoning and the form of presentation suggests that Steven is seeking to construct each text as a coherent 'project', the structure of each of the texts follows the structure provided in the corresponding question paper, leading to a disjointed collection of styles in some cases.

In general, the differences between the texts appear to parallel differences between the tasks. The disjointedness mentioned above is greatest where the questions posed by the task are closed or relatively narrowly focused and least where they ask for more general comment. Where the task defines technical terms explicitly or emphasises them, Steven uses these terms consistently; where the given statement of the problem is less formal in defining its terms, he writes less formally, using a variety of expressions to describe the same variable.

The practical nature of the 'Topples' and 'Pendulum' tasks is reflected in the presence of some manipulation of physical objects, although even in these tasks this is restricted to the introductory part of the problem and human involvement is kept to a minimum. Probably the most significantly different characteristic of these tasks is the introduction of vocabulary relating to the physical world outside the task in order to form explanations.

The picture of Steven that emerges from this comparative analysis of his five texts suggests that he has a fairly consistent view of the nature of mathematics and of his relationship both to the tasks he is set and to his reader. Although he presents himself as confident and authoritative in relation to his subject matter and his reader, the form of his text is constrained by the form of the questions set in the given task. He has not, however, transferred the formal characteristics of writing encouraged by the structured tasks into his repertoire to be used in the less structured tasks.

It must, of course, be remembered that the five texts were written over the period of a two year course leading up to the GCSE examination. 'Passola', 'Topples' and 'Symmetry Groups' were completed during the first year of the course, while 'Inner Triangle' and 'Pendulum' were completed during the second year. During this time it might have been expected that Steven's skills as a writer would have developed and other changes might have taken place affecting his orientation to the tasks. The remarkable consistency actually found suggests either that he received very little feedback relating to his writing, or that whatever feedback he received was entirely positive, or that it was largely unsuccessful in effecting any change.

9 Investigating teachers reading coursework: methodology

One of the aspects of the discourse of GCSE coursework that is most significant in terms of the relationships between the participants and in terms of its importance for both students and teachers is the fact that coursework forms part of a summative, externally validated examination whose results may have significant consequences for the students' future educational and employment opportunities. The texts produced by students cannot be looked at in isolation but must be viewed within this examination context. An important part of this context is the teacher who reads the students' texts and has the primary responsibility for evaluating them. An investigation of teachers' readings thus complements the investigation of students' writing. This investigation seeks to explore, through interviews with teachers experienced in working with students undertaking GCSE coursework, the discourse of the assessment of coursework in general as well as teachers' specific practices in reading selected coursework texts. In particular, it is intended to identify those features of students' written texts which are significant in the assessment process.

It is important to recognise that answers provided by teachers in an interview situation cannot be taken as an absolute, objective indication of the 'truth' about the way they read and assess their students' texts. The activity of taking part in an interview is different from that of assessment and may therefore draw upon different practices and resources. Not only may the teacher, in being removed from her normal setting, be deprived of "resources - shared knowledge with which to approach the task, shared values, familiar procedures for analyzing data, widely agreed on criteria - which may be essential for succeeding with a given task" (Doheny-Farina & Odell, 1986: p.507), but in asking a teacher to verbalise her practice and to explain or justify it she may be prompted to reflect on it in new ways. The interaction between interviewer and interviewee constructs a new text (Paget, 1983; Mishler, 1986; Mellin-Olsen, 1993) which must be interpreted within its own context. Analysis of the texts produced during the interviews must, therefore, be informed by consideration of the whole context within which the interviews took place and the way in which the participants are positioned within that context.

9.1 Design of the teacher interviews

In order to investigate the ways in which teachers read students' written work and respond to its various features it is not possible to rely solely on the teachers' self-reporting of their practices. Not only are such reports unlikely to reflect practice fully or accurately but they are also unlikely to address the specific issues of concern to the current study. In particular, the analysis of students' writing on which this study is based has identified features of the work and differences between texts which may not be describable in a theoretical way within the

discourse familiar to the teachers yet which may have a concrete effect upon the teachers' reading practices. For example, both a narrative of mental processes and a narrative of material actions are likely to be described merely as 'telling what was done'. At a preliminary stage in the study, semi-structured interviews (without student texts) were carried out with a number of teachers, asking the question "Can you give some examples of 'good' or 'poor' communication in coursework?". Responses to this question tended to list structural features, such as introduction, explanation and conclusion, and highly visible non-verbal features, such as tables and diagrams. The following extract is typical in this respect although it also includes one of the few attempts to describe the form or content of any of these features.

A well presented piece of coursework probably would have the . . . headings, clearly sequenced, tables, charts, calculations, explanations, you know, findings and then probably posing a list therefore "this question was posed so I tried it, this was what I found, and I'm going to test this theory" and then it would show examples and whether that therefore proved they were right or wrong. So there would be (. . .) bits "Now I'm going to try this to prove it" that sort of thing. And obviously, end results very clearly presented, maybe in a couple of different forms, you know. And if the extension was done, clearly what they set out to do in the extension and then following the same sort of things. A poor piece of coursework would be bits and bobs, probably not a table, probably a working out and then something written and very little written work, you know.
(Mandy)

Such responses are, of course, significant in that they form part of what appears to be 'common sense' teacher talk and are likely to reflect the sort of advice and feedback that is provided to students. The hypothetical examples of student narrative that Mandy provides are particularly interesting, suggesting both her approval of such a personal narrative style and the importance of portraying mental processes. Terms such as 'well presented' and 'clear', however, are unanalysed 'transcendental signifieds' which, by assuming (without necessary justification) that all participants share a common understanding of their meaning, serve to maintain teachers' authority within the discourse (Cherryholmes, 1988); we do not gain a sense of what this meaning might be. One of the objectives of this study must be to attempt to analyse such common sense terms in relation to students' texts and teachers' practice in reading these texts. Addressing this objective requires that the data collected should include teachers' responses to concrete examples of students' writing as well as their self-reports of their assessment practices.

It was therefore decided that the design of the interviews should be task-based, centred around the assessment of a number of students' written texts. As one of the issues to be addressed through teacher interviews is the possible influence of different types of features of these texts on the teachers' readings and evaluations, texts which contain a variety of such features need to be used. In an exploratory study (reported in Morgan, 1991; 1992b) a discussion was conducted with a group of four experienced secondary mathematics teachers on an MSc course in Mathematical Education who were asked to rank and comment on three

short 'coursework' texts which had been constructed by the researcher to differ in respect to: the ways in which diagrams were used; the relationship between numerical results, algebraic generalisations and verbal text and their relative quantities; the presence of a narrative of processes; the degree of formality, including the use of personal pronouns. The variables used arose from the initial stage of the development of a method of analysis of student texts described in chapter 6. The use of such artificially constructed texts, while possibly allowing analysis of the teachers' assessments to isolate reactions to some of the variable features deliberately incorporated into the design, nevertheless loses some of the potential richness of data gathered from teacher readings of texts actually written by students. Moreover, even when using these constructed texts, unanticipated features of the texts appeared to be significant to the teachers; for example, differences in the handwriting of the three texts were used by at least one teacher to draw conclusions about both the gender and the age of the 'authors'. The complexity of the resources used by teachers when reading such texts means that the goal of isolating the influence of specific textual features is likely to be a forlorn hope. While the variables identified and used in this exploratory work are certainly of interest, in seeking an answer to the question 'What features of students' writing are significant to teachers?', the use of constructed texts presupposes that those variables incorporated into the construction are those which are most significant.

This use of texts as a focus for interviews to research writing in its social context is similar to the 'discourse-based' interview introduced by Odell & Goswami (1982) in a non-academic setting and adapted by Herrington (1985) to investigate the judgements about academic writing made by undergraduate engineering students and their tutors. In both these cases, authentic texts and constructed variations on these texts were used to investigate writers' and readers' reasons for preferring "specific stylistic and substantive features" (Herrington 1985: p.337). By explicitly drawing their interviewees' attention to the differences between the texts used and asking for explanations of the reasons for making particular choices between the variations, both studies succeeded in eliciting responses which provided insight into the interviewees' perceptions of the audience for the texts and into the ways in which knowledge of the social context in which the writing takes place influences both writing and reading. Although Odell & Goswami and Herrington made use of authentic texts, the use of constructed variations on these and of explicit questions again restricted the features of the writing which might be considered to be significant to those identified by the researchers in advance.

As has been argued, the complexity of teachers' reading resources and of students' texts makes it difficult to isolate the influence of specific features without oversimplification. Moreover, given the lack of previously existing knowledge of mathematics teachers' reading practices and the hypothetical nature of the interpretation of features of students' texts in the

present study, the construction of texts would be likely to overlook features which might prove to be significant. It was therefore decided to use authentic student texts because these, although lessening the researcher's control over the situation variables, would allow the exploration of teachers' reading and assessment practices to be less constrained by preconceptions. The selection of the student texts to be used in interviews with teachers is described in chapter 6.

The field of interest of the present study is not restricted to the students' texts in isolation but seeks to place them within a wider discourse of school mathematics practice. The design for the interviews thus seeks to provide a setting in which teachers' responses to features of students' texts (identified through consideration of the literature and during the analysis of the texts themselves) may be investigated while simultaneously maintaining the possibility of exploring other aspects of the teachers' reading that arise during the interviews. The setting in which the teachers are operating is not, however, 'natural' in the sense claimed by Rapaille (1986) for his study of teachers' assessment practice; this must be taken into account when interpreting the results.

Each interview is based on one of two coursework tasks: 'Inner Triangles' or 'Topples'. The first of these may be considered to be a 'pure' investigation, while the second involves use of practical equipment and is designated a 'practical task' by the examination board. An analysis of each of these tasks may be found in chapter 7. The use of these contrasting tasks provides the opportunity to identify which aspects of the teachers' reading practices vary between the two tasks, building up a picture of a complex genre, and which are common to both and may thus be part of a general coursework practice.

The interview schedule (see Appendix 10) is in three parts, each of which addresses the reading and assessment of students' texts in a slightly different way. In the first part, the teacher is asked in general terms about the assessment task that forms the focus for the interview:

What are you going to look for when assessing responses to this task?

The teacher has been provided with the examination board's specification of the task several days before the interview and has been asked to read it and to prepare to answer this question. The primary objective of the question is to elicit the teacher's 'common sense' notions of the nature of coursework writing and assessment and to identify those features of students' work that the teacher herself appears to identify as significant. It is not necessarily the case that the responses given during this part of the interview will correspond with those given during subsequent parts in the presence of student texts. Such differences between theoretical and practical aspects of the discourse are themselves of interest.

During the second part of the interview, the teacher is provided with three complete student texts in response to the same task. The method of selection of these texts is described in chapter 6 and a summary of the analysis of each of them is provided in chapter 8. While the texts were selected in order to provide a number of contrasting features, the teacher's attention is not drawn to these features. The explicit focus of the interview is still on assessment:

I'd like you to look at and assess these three pieces of pupils' coursework. Please talk aloud while you are doing it so that I will know how you are making your judgements.

Further questions are used where necessary in order to clarify the teacher's responses, to encourage an expanded response, or to remind the teacher to talk about what they are doing. In addition, after all three texts have been read, the teacher is asked to rank them in order of merit if this has not already been done without prompting. This part of the interview is intended to gain insight into the ways in which the teacher makes sense of her reading of the student texts and into the nature of her assessment practice. By providing student texts with specific contrasting features, the influence of these features on the teacher's reading and assessment may become apparent. However, by constructing the purpose of the reading as assessment rather than as discussion of textual features, the identification of other, unanticipated features is not inhibited. Moreover, there is more likelihood that the teacher will adopt a position within the discourse of coursework assessment rather than being clearly situated within the interview discourse. The question of teachers' positioning within the coursework assessment discourse or within the interview discourse will be addressed in section 9.2 below and throughout the analysis of the interviews.

In the final part of the interview, the teacher is presented with three extracts from further students' texts, representing the students' presentation of a generalised solution to the 'Inner Triangles' task. Again, these extracts have been chosen to display a number of contrasting features (see Appendix 6). Moreover, the brevity of the texts allows them to be examined simultaneously, enabling direct detailed comparisons to be made between them. Each extract is less than a page in length and has been typed, implicitly emphasising the idea that they are not whole texts belonging to individual students. The teacher is thus encouraged to focus on the form of the text rather than attempting to assess its author. This time, the teacher's attention is explicitly drawn to the writing of these extracts:

These are some extracts from some other pupils' work .

Which of them do you think has expressed themselves best?

Why?

What advice would you give to each of the pupils to improve their work?

This part of the interview is closer to Odell & Goswami's (1982) discourse-based format in that its explicit focus is on the way in which the work is 'expressed'. Whereas the previous

part of the interview should provide a rich picture of the teacher's reading practices and is likely to identify the features found to be significant within each text, this part is intended to elicit comparisons between texts that are based on specific textual features rather than on general overall impressions. Although the brevity of the extracts means that the features that may be discussed are strictly limited, the fact that they are related to the formation of a generalised solution means that they are of particular significance within the mathematics coursework discourse.

9.2 The teacher as interviewee

In interpreting responses to questions concerning the reading and assessment of texts, it must be remembered that the questioning, the reading of the text and the judgements made by the reader are always situated within a context which affects, among other things, the understanding by the participants of the nature of the text being read and of their reasons for reading it. What sort of text is it? What might be the circumstances of its production and consumption? What are the generic criteria by which it ought to be judged? Is the aim of the task to assess the student's work or to evaluate the teacher's assessment? However carefully the interview may be designed to create conditions in which 'authentic' readings may be made by the interviewee, it is impossible to control the positionings that s/he may adopt. This raises questions about the interpretation of discourse-based interview data in general. Odell & Goswami (1982) and Herrington (1985), while paying attention to the design of the interview itself in order to maximise the likelihood of an 'authentic' response, do not examine the responses achieved in a critical manner that questions the position from which the response is given or the social context, both immediate and wider, within which the response has been constructed. Their analyses, by reducing their data to mere counts of types of reasons given for judgements, assume that each interview provides a static picture of the homogeneous practice of an individual. This type of analysis is in a positivist tradition of the type criticised by, among others, Mishler (1986) and Jensen (1989) as inappropriate for data produced through social interaction.

The idea of 'positioning' as a factor in the analysis of interview data is discussed by Evans (1994). He suggests that subjects are positioned by the practices at play in the setting of the interview (in the case of his own study of adult students' numeracy he identifies these as Academic Mathematics and Research Interviewing). Then, depending on this positioning of the individual, further practices may be called up which "provide the context for that subject's thinking and affect in that setting" (p.323). Evans' study suggests that the positioning of interviewees, by making certain practices more or less likely to be called up, affected not only expressions of beliefs or feelings but also performance on the mathematical tasks which formed the focus of the interviews. It is thus relevant in the present study to consider the positioning of the teacher-interviewees and its possible influence on their judgements about

students' texts. However, whereas Evans identified a single dominant positioning for each of his interviewees, this would not take account of the complexity of the practice of assessment and the context within which it takes place.

As well as the teachers' positioning as interviewees, the practice of GCSE coursework assessment involves a number of different, and sometimes contradictory positions that teachers may at various times adopt. As teachers of individual students, they are clearly in a pedagogic relationship: concerned to further the students' mathematical learning. They are also concerned to ensure that each student achieves as highly as possible on external examinations, not only because of personal loyalty to individual students but also in order to secure the teacher's own professional standing in the eyes of the school management and others who might have influence on their promotion or employment prospects. At the same time, however, the teacher assessing coursework is acting as an agent of the external examination board and as such is guided by the rules of that body. Failure to abide by those rules, causing major changes in students' grades following the moderation process, would be likely to result in loss of standing in relation to students, parents and the school. The external assessment context also ensures that concepts such as standardisation, rigour and evidence play a part in the teachers' discourse.

The interplay and movement between these various reading positions is illustrated within the following passage from the interview with Fiona, discussing Richard's Inner Triangles coursework text.

- 5 *F This is a major problem because he's got these results but unless one is there in the class and you're a teacher you don't know whether this is his results or somebody else's. He hasn't shown any diagrams of where these results have come from. He hasn't done any drawings as far as I can see. He's come up with a formula which is Z equals Z . Z must be the slant height. Is equal to X plus Y equals T . I assume that's right, I don't know.*
- 10 *I Yes, I think that's right*
- 15 *F That is right is it? Ok . . . but again even that's not, I mean he's given . . . one thing that I think they have to do is when they give a formula they should explain it using quite a few examples and show how it works. The thing that I always look for and I say to the kids is: you write it up as if you're writing it for somebody who's never seen this problem, who's never done that but would be able to understand it if they were to read it.*
- 20 *And from this, somebody . . . I don't think it's clear enough for somebody to use it and then work out, I mean he hasn't done even one example of how it works. So I think he's got a major problem here, he hasn't shown how he's got it. If you were in the class, obviously we can award marks when they've done something in class but it's not written down. If you're the teacher in the class and you knew that he's just omitted it by mistake um but also he hasn't included his rough work and that's, when this happens, often you find what you need in the rough work and that can, you know. So none of his results are justified there at all. I think he could have problems with that.*
- (Fiona: 57-80, Richard)*

Several times Fiona identifies “problems” with Richard’s work. She is reading here in the role of an examiner looking for evidence that the results ‘belong’ to Richard (lines 1-3), for evidence of “how he’s got it” (lines 17-18), and for justification of the results (line 23). Her discomfort in this role, however, is suggested by the use of the word *problem*. There is a tension between the rigour of the examiner, for whom use of the assessment criteria determines unproblematically a decision about the value of a piece of work, and the wish of a teacher that a student should get as high a grade as possible. In this case, Fiona reading as teacher, acting as advocate on behalf of her student (although she did not have personal knowledge of Richard), feels that the missing evidence might have been available in the classroom but when she reads as examiner she cannot take account of this possibility. The problems are Richard’s problems but they are also Fiona’s problems in resolving her two roles.

This tension is expressed again (line 10) when Fiona describes “one thing that I think they have to do”; there is an ambiguity in this phrase which is also present in the intonation used: is this her own opinion of what is appropriate or is it her belief about what is expected by the examination board? I would suggest that this ambiguity serves the function of enabling Fiona to reconcile her different roles: when she (as teacher concerned to support a student) is uncomfortable with the severity of her judgement of the piece of work, she is able to shift the blame to an anonymous authority that lays down what “they have to do”.

At line 12, Fiona again expresses a dual role - this time a more comfortable one - as examiner and as teacher/adviser, looking for the specified characteristic of the writing as an assessment criterion and simultaneously advising the students of the criterion that is being used. The nature of the criterion she describes, however, forces her to adopt yet another reader position, as an imaginary naive reader “who’s never seen this problem, who’s never done that but would be able to understand it if they were to read it” (lines 12-14).

In this short passage extracted from Fiona’s interview, she thus adopts a number of reading positions, some of which are potentially contradictory:

- examiner, using externally determined criteria;
- examiner, setting and using her own criteria;
- teacher/advocate, looking for opportunities to give credit to a student;
- teacher/adviser, suggesting ways of meeting the criteria;
- imaginary naive reader.

The tension between taking on an examiner role and acting as teacher/advocate is a familiar one for teachers involved in any summative assessment; it is to be expected that one way in which it may be resolved is by appeal to the anonymous authority of the examination board. When the assessment criterion is expressed in terms of suitability for an imaginary audience,

however, there is an assumption, not only that the student will understand the nature of this hypothetical audience and actually address it, but also that the teacher will successfully adopt the position of the specified audience when reading and judging the student's work. Neither of these assumptions is justified (see, for example, Redd-Boyd & Slater, 1989; Gilbert, 1989). Where, as in this case, the teacher is reading as an 'expert' examiner, judging the mathematical quality, and simultaneously attempting to read as "somebody who's never seen this problem", the tension is less easily resolved. This is particularly the case where the characteristics of the specified imaginary audience are unclear; in Fiona's case it appears that "somebody" has some degree of mathematical understanding as they "would be able to understand it if they were to read it" and yet they are constructed as needing an example in order to be able to use a simple formula.

A further reading position occasionally taken by the teachers, not exemplified in Fiona's passage above, is that of a reader interested in the mathematical content of the text being read. This occurs infrequently, probably because of the routine nature of much of the coursework produced. Where it does occur, however, there is a tension between this position and that of examiner. For example, Dan expresses interest in Richard's investigation of the area of stars:

Perimeter times slant height . . . mm! . . . Blimey, I would definitely have liked a bit more explanation of that, cos that's quite interesting isn't it. The perimeter multiplied by the slant height gives you the area inside, that's quite an interesting find which I doubt if many other people did so it's quite innovative and something a bit different but given us absolutely no . . . not even 'oh gosh look at this' would have been . . .
(Dan: 164-169, Richard)¹

On the one hand, Dan would like more explanation of this result because he finds it interesting and possibly wishes to understand the mathematics of it himself. The modality of the exclamations, the repetition of "interesting", and the statements that it is "innovative" and "different" suggest a personal involvement with the subject matter. At the same time he is evaluating Richard's work as an examiner, comparing it to other students' work and then criticising it because no evidence or comment has been given to "us". Use of the first person plural here marks Dan's shift from reading as an interested individual to reading as an examiner acting as part of a group with common expectations. The suggestion that "oh gosh look at this" would be an appropriate comment is an evaluative suggestion also from a position as examiner; such a comment would not help an interested reader to understand the mathematics.

Although the format of the interviews was designed to construct the teachers as expert informants demonstrating their normal practice, there were occasions when it appeared that

¹This passage from Dan's interview is also discussed in chapter 14.

this role was not fully adopted. Joan, for example, expressed insecurity about her assessment of Clive's work:

. . . so probably a level 7 again. But you're going to tell me now that somebody else has done this and come up with something completely different - but don't tell me that. (Joan: 186-187, Clive)

She appears to be seeing herself in the interview situation as an examinee trying to come up with the 'right' answer. Similarly, Charles, on finding that the author of the third Topples text he read had arrived at a different result from that given in the first two texts, reacted to this difficulty not by treating it as a problem that might arise during the assessment of a set of coursework scripts but by accusing the interviewer of giving him a trick question:

You horror! Could I have looked at these in any order? (Charles: 172, Sandra)

It appears that, at least at these points in the interviews, Joan and Charles are positioned as interviewees rather than within the practice of coursework assessment. Such explicit examples are rare but their existence suggests that there may be other points at which the teachers are attempting to justify themselves and their judgements to the interviewer rather than merely 'thinking aloud' while undertaking their normal assessment practice. This may be indicated where general descriptions of practice are given to back up comments on specific texts. This possibility, however, does not invalidate the interview data in relation to the aims of the study. The justifications used by the teachers must be drawn from their repertoire of 'members' resources' related to coursework and its assessment. As such, they form a part of this discourse and need to be reconciled with the rest of their practice.

In their analysis of the discourse of scientists, Gilbert & Mulkay (1984) explained apparent contradictions (in the ways in which errors were accounted for) as arising from the scientists' simultaneous participation in two practices with different sets of beliefs about the nature of scientific knowledge and activity. Similarly, the teachers in this study are participants in the practices both of teaching and of examination as well as that of the interview. As Galbraith (1993) points out, the 'constructivist' paradigm associated with the current discourse of teaching is not compatible with the 'conventional' paradigm of external examinations. Contradictions within the teachers' discourse should not, therefore, be interpreted as indications of irrationality or incompetence but as signs of movement between the various practices in which they are situated in the process of making sense of the text being read and of their own relationship to the text and to its author. The reading positions identified above will be used in the analysis that follows to describe and account for some of the characteristics of and variations within teachers' readings of students' texts. Where shifts between these positions occur, as in the passage from Fiona's interview analysed above, this appears to mark an area of difficulty for the teacher concerned and is thus an indicator of the significance of the features of the text being read at this point.

9.3 Analysing interview texts

Mellin-Olsen identifies two ways of approaching the interpretation of interview texts: from within the researcher's theoretical perspective or making "every possible attempt to step back during conversation and interpretation in order to reduce the effects of his presuppositions" (1993: p.151). Writing from a constructivist point of view, he advocates the second approach as one which sees the interviewee as subject rather than as object, participating actively in the joint construction of meaning in the interview. As it stands, however, this proposal to make "every possible attempt" appeals only to well-meaning liberalism as it provides no practical means of reducing the effects of presuppositions. An analytic method is needed to anchor the interpretation - not to make a claim of objectivity but to make explicit the way in which the researcher's presuppositions are effective.

Given the theoretical standpoint on the motivated nature, lack of transparency and contextuality of language use taken in the analysis of students' written texts on which this study is based, it would be contradictory to adopt a more naive representational view of the data arising from interviews, which are also essentially linguistic events. As the language of interviews is not itself the subject matter of the present study, it is not intended to conduct a full textual analysis of entire interviews. However, many of the tools of textual analysis described in chapter 5 for use in the analysis of written mathematics are also useful to help answer Jensen's (1989) criticism of much interview-based research:

It is the exception rather than the rule that qualitative researchers *analyze* the language of their materials. Even if the importance of language is acknowledged in an abstract sense, very often the interviews only appear as quotations illuminating the researcher's own narrative, so that the reader is left wondering how the discourse of the interview was transformed into the discourse of the report. (p.100; original emphasis)

These tools assist the 'transformation' from the discourse of the interview at two points in the analysis: firstly at the stage of identifying parts of the interview text which may be particularly significant and subsequently in a more detailed analysis of the nature of this significance.

The first approach to an interview transcript involves the identification of passages which are of interest either because they address certain themes which have been identified a priori:

- the linguistic or other symbolic forms used in students' writing (e.g. diagrams, tables, 'algebra');
- a student's use of personal narrative and logical argument;
- the explicit statement and use of criteria either provided by the examination board or particular to the school or individual teacher;
- creativity, originality or error;

or because they contain what Jensen (1989) refers to as “linguistic danger signals”. Such signals² include:

- the hiding of agency (through passives or nominalisations);
- changes in personal pronoun use or tense, which may indicate distancing, differing degrees of generalisation, or a change between different practices³;
- positive or negative modality;
- emphasis through repetition of related semantic terms - a sign of the ideational significance and possibly contested nature of the theme.

Having thus selected passages of interest, further themes are identified within these passages which appear significant either to an individual teacher or to groups of teachers. The process is recursive as the identification of additional themes through the use of linguistic signals necessitates a return to the original data to discover other occurrences of these themes.

Teachers’ explanations and justifications of their judgements are also identified at this stage. Describing the ways in which teachers explain their own actions and those of others is an important aspect of building up a picture of what events and ideas are significant within the discourse. The expression of causal relationships is therefore of interest to the analysis of the interview texts. However, as Polkinghorne points out, ‘logico-mathematical reasoning’ is not the only form of explanation used:

People ordinarily explain their own actions and the actions of others by means of a plot. In the narrative schema for organizing information, an event is understood to have been explained when its role and significance in relation to a human project is identified.

(1988: p.21)

The narratives⁴ that teachers relate may reveal the interpretive resources they are using to make sense of the practice of assessment or to justify their judgements.

teachers’ narratives do not necessarily reflect classroom reality, though they may refract teachers’ perspectives. This is an advantage for the researcher wishing to investigate such perspectives. Teachers’ narratives of personal experience can be thought of as impression-management, as the presentation of their professional selves.

(Cortazzi, 1993: p.42)

²The general interpretation of such linguistic features is discussed more fully in chapter 5.

³An example of the use of similar linguistic signals in the study of teachers’ assessment practice is provided by Rapaille who suggests that the obscuring of agency achieved by use of the indefinite pronoun ‘one’ instead of ‘I’ “can be interpreted as an attempt of the teacher to protect himself from blame from others” (1986; p.138), where “others” may include the researcher listening to the recording of the assessment event.

⁴A narrative may be defined as involving temporality, causation and human interest, which “determines whether the events and causes fit together in a plot” (Cortazzi, 1993: p.86)

Cortazzi, in a study of over a hundred primary teachers in which narrative analysis was the main analytical tool, suggests, following Polanyi, that such narratives provide insight into “core cultural concepts” of the occupational group. While in the present study there is no attempt to seek substantial generalisability on the basis of a similarly large sample, it is nevertheless the case that consideration of teachers’ narratives can provide insight into the objects, events and causal relationships that are significant for them.

Having identified themes and passages of the interviews related to them, the analysis proceeds to consider the nature of the teachers’ reading practices and interpretive resources used in relation to each theme, including their orientation towards the students and the texts and their positioning as pedagogue, examiner and interviewee. This is achieved through close attention to the interview texts, making use of the tools of critical text analysis described above and in chapter 5. Linguistic features of particular significance in this analysis include the modality of the text and uses of personal pronouns as these affect the expression of relationships between the teacher, the student text, the student-author and other participants in the assessment context.

Just as every text fulfils ideational, interpersonal and textual functions, a single passage of interview text may contain a number of different aspects of interest to the analysis. For example, the same passage may provide an example of a teacher shifting between different reading positions while simultaneously containing evidence of the way in which the teacher is using algebra as an indicator of a student’s ability. It is not possible, therefore, to pigeonhole extracts of interview text into exclusive categories. Consequently, during the analysis that follows, it has been necessary to include some extracts more than once in order to discuss them from different perspectives. Where this has been done, cross-references are provided in a footnote.

9.4 The sample of teachers

In all, eleven teachers have been interviewed: six teachers from three schools read the selection of complete ‘Inner Triangles’ texts, while five teachers from another three schools read the selection of ‘Topples’ texts. All of the teachers also read the ‘Inner Triangles’ extracts. The selection of this sample was made on the basis of contacts with heads of mathematics departments made through PGCE or INSET work. In each case a request was made to interview all members of the mathematics department but in practice this only occurred in one small school (school J); within the other schools, the teachers interviewed were volunteers who were available at the time of my visit. School R provided the students’ coursework texts used in the study; two of the teachers from this school, Andy and Dan, had taught or knew in other ways some of the students whose texts they read. Their personal

knowledge of these students, although some time removed, provides an additional dimension to their readings of the texts.

Prior to the interview, each teacher was asked to complete a brief questionnaire to provide some background information (see Appendix 8). An overview of this information is provided in Appendix 9. It is clear that this group of teachers may in no sense be considered a representative sample. They are, as a group, relatively highly experienced both in terms of their years of teaching and their levels of responsibility within both school and department. With the exception of the two teachers from school J, they are also more than usually experienced with the examination of coursework, having been involved with this before it became compulsory. Their willingness to be interviewed suggests a degree of interest and commitment to coursework although several also expressed doubts about their own expertise. Having said this, the study of experienced teachers is likely to identify more stable practices while their professional status may be influential in shaping the practices of their less experienced colleagues. This study does not claim to make any universal generalisations on the basis of the data collected from these interviews. Rather it attempts to describe a range of practices and to point to areas of tension and conflict within the discourse of coursework, both between different teachers and within individuals engaged in reading and assessing coursework texts.

9.5 Summary

The aim of interviewing teachers is to explore their reading practices, identifying the features of these texts to which they respond, and the interpretive resources used to make sense of the students' coursework texts and of their own assessment practice. This complements the analysis of the coursework texts themselves by examining the context of their consumption. As observed earlier, the interview setting is not a naturalistic one and the texts produced within it are secondary texts and must be analysed as such. They contribute to a picture of the discourse of coursework and its assessment but do not provide a representation of the authentic practice. Research in this area might benefit from consideration of further alternative perspectives on the discourse through, for example, observation of moderation meetings in which teachers assess each others' students' coursework and negotiate meanings among themselves, although, even in such 'natural' settings, the very act of observation changes the context and hence the discourse. For example, Rapaille (1986) identifies points during his naturalistic recordings of teachers assessing students' work when the teachers appeared to be orienting their talk towards the (absent) researcher.

The task-based design of the interviews, making use of original student texts, attempts to position the teacher-interviewees as far as possible within the discourse of coursework assessment. A minimum structure is provided and it is intended that interventions by the

interviewer should not introduce new themes but should only be used to facilitate communication. The incidence of a theme in an interview text may thus be taken to indicate its significance to the teacher's reading. The interviewer is, however, a participant in the construction of the interview text and this must be considered in the analysis. The overtly artificial use of short extracts of texts in the final section of the interview is intended to elicit responses more explicitly related to the textual form of the extracts.

The non-transparent nature of language must be taken into account in the analysis of the texts produced in the interviews just as it is in the analysis of the students' written texts. Meanings are constructed within the social context in which the participants are positioned and it is, therefore, necessary to attempt to identify these positionings and, in particular, the shifts and possible conflicts between them. The discussion of the analysis will be structured according to themes within the interview texts; some of these themes are derived from consideration of features identified in the analysis of the students' texts or from the official and professional public discourse concerning coursework and investigations (see Appendix 15); other themes are identified through the occurrence of linguistic signals indicating their significance or contested nature for the interviewee. Linguistic tools are also used to anchor interpretations of the meanings produced in the interview texts.

10 Comparisons between teachers reading the same texts

As has been seen earlier (chapter 9), teachers appear to experience tensions between their various roles in reading and evaluating students' coursework texts. Because of the demands of the examination system, they have to find some way of resolving these tensions in order to arrive at a single final assessment of each piece of work. Different individual teachers do not necessarily do this in the same way; indeed, of the six teachers asked to rank the three 'Inner Triangles' texts, three placed Richard's work first while the other three placed it last. In order to examine variations in the ways in which teachers read students' texts and resolve such tensions, two teachers' readings of the same text will be examined in detail for each of the two tasks. Because of the variations in assessing Richard's work, this has been chosen as the 'Inner Triangles' text to be considered. The rankings produced by the teachers reading the 'Topples' texts were far more consistent; for this task, Steven's text has been chosen.

10.1 A comparison of two teachers reading Richard's 'Inner Triangles' text

The teachers chosen for this detailed examination are Joan and Fiona, both experienced at GCSE coursework and neither of whom knew the author of the text. Joan ranked the three 'Inner Triangles' texts in the order: Richard (1), Steven, Clive. Fiona ranked them: Steven (1), Clive, Richard.

During the interview, each teacher first read and talked about their assessment of each student's work individually. Later in the interview, they were asked to rank the three pieces of work¹ and to explain their assessments. At this stage the teacher returned to re-read at least part of each of the texts. These different parts of the interview may be seen as providing different contexts for the teacher's reading of the texts. In particular, the first reading may be closer to the teacher's 'authentic' assessment practice while the later readings may reflect their need to justify this practice to an outsider. Joan's and Fiona's first readings of Richard's text will be compared initially and this will be followed by consideration of their later comments. In order to relate the initial readings to the student's text, the teachers' comments have been divided according to the page(s) being read at the time. The pages have been grouped together on the basis of the section of the set task that the student is responding to; the teachers' readings were also made in this way, sometimes pausing to read the page or group of pages in silence before proceeding to comment. Transcripts of the relevant sections

¹Ranking a class set of coursework texts is a part of the normal procedure of GCSE assessment, prior to assigning grades. This was referred to spontaneously by several of the teachers before they were asked to do it.

of Joan and Fiona's interviews are in Appendix 11. Richard's text may be found in Appendix 5.

10.1.1 First reading of Richard's text

pages 1-3: Title: answers to questions 1 and 2: 'Working out' in the form of d'agrams related to questions 1 and 2 drawn on isometric paper.

Both Joan and Fiona treat these pages, the answers to the closed questions 1 and 2, as a single unit. While Joan passes over them quickly with little comment, Fiona elaborates the functions that they are fulfilling in her reading of the text. Firstly, by answering the questions correctly, Richard is indicating his "understanding" not only of the specific questions asked but also of "the problem" and of the convention of this particular coursework genre which expects the student to answer the given questions rather than to investigate "on his own tangent". 'Understanding the problem' is one of the general assessment criteria for all such tasks². These answers also serve a formal function as indicators of a particular grade. Fiona is clearly reading as an examiner at this point, checking that Richard has fulfilled the requirements of the task and building up a picture of the general criteria that he has met. Joan's reading here appears to consider this passage less significant for assessment purposes; she merely notes the features of Richard's text without making any explicit evaluation. This noting suggests that, while it is necessary to complete this section of the task it does not play an important part in the overall evaluation of the student.

Diagrams are picked out as significant by both these teachers, being identified by both as demonstrating his "working out". In addition, Fiona, again reading as examiner, uses Richard's diagrams as evidence not only of the method of counting that he has used but also of the fact that he did not copy the answers from someone else. The features of these diagrams that might lead to this reading are their 'rough' appearance (probably drawn without a ruler) and dots which appear in some of the triangles, suggesting a record keeping process to assist Richard's counting of the triangles.

page 4: Results table, diagram and formula

The major issue in this section for both Joan and Fiona is the lack of any verbal explanation or elaboration of the formula. As examiners, both of them identify this as a problem but they resolve it in different ways. Joan takes on the role of teacher/advocate, suggesting that the necessary explanation probably took place in the classroom (although there is no suggestion of this in the text). Fiona recognises the possibility of taking this position but rejects it in

²The London examination board non-task-specific grade descriptions include: at grade F "Shows an understanding of the task"; grade C "Shows good understanding of the task"; grade A "Shows excellent, clear understanding of the task". (LEAG, 1989; see Appendix 1)

favour of an examiner role; the teacher role is apparently only acceptable to her "if you're the teacher in the class".³

While reading this page of Richard's text, the practices of the two teachers are very different. Joan starts by making sense of the formula, interpreting the variables and then using the data in the table to check whether the formula works. The table is used as a tool for the reader to make sense of the mathematics. The diagram is also seen as a way of helping the reader to understand. Joan's initial position while reading this section of text is as an interested reader; it is only once she has made sense of the mathematics that she takes on an assessor role.

Fiona, by contrast, does not attempt to make mathematical sense of Richard's formula until after she has considered and evaluated the other features of his work. While Joan used the values in the table to check the formula, Fiona demands that examples should be provided for her, taking on the role of a naive (and uncooperative) reader. For her, the table is not a means of communicating mathematics but an indicator of systematic working, i.e. fulfilment of an assessment criterion. Her reading is a search for evidence which leads her to read in a non-linear way, searching through the whole text for the "working out" that she claims is missing. The diagram which Joan interpreted as an explanation is completely ignored by Fiona, who even states that "he hasn't done any drawings". As the drawing on this page is a generic diagram it cannot fulfil the "working out" identity and is thus not significant for Fiona. After this search for evidence, she returns to the formula but even here does not appear to wish to make sense of it, apparently willing to assume it is right or at least to accept the authority of the interviewer.

pages 5-6: Title (Extension): Triangles - diagrams, table, formula

On proceeding to read Richard's extension, each of the teachers starts by acknowledging the existence of this section of the work. This is significant in the examination context because doing an extension can enhance the grade given to the whole piece of work. Although not asked to do so at this point in the interview, Joan compares Richard's work on triangles to that of another student, remarking on the absence of a geometrical justification that she had appreciated in Clive's work. Fiona again looks for (and this time finds) drawings as evidence of working out. She also notes the formula that Richard has found. The comments "that's quite good" and "right" might indicate appreciation of Richard's mathematical achievement in finding a correct formula. The speed with which they were made and the intonations used, however, suggest that it may be more appropriate to interpret them as acknowledgements of the presence of the formula rather than confirmation that it is correct. This interpretation

³Fiona's discomfort with this situation, displayed through her shifting between several different positions within this brief passage, is analysed in more detail in chapter 9.

would be consistent with Fiona's general practice of searching for signs which meet the assessment criteria she is using rather than reading in order to make mathematical sense of the text.

pages 7-10: Four similar pages each with diagrams, table, formula for Hexagons, Stars, Squares and Hexagons (the last two on squared rather than isometric paper)

The contrast between the reading practices of the two teachers is again apparent in this section. Joan seems to be engaging with the mathematical problem, considering the connections between trapezia, triangles, hexagons and stars. She takes Richard's diagram of a hexagon as evidence that he has thought in a particular way and presents her own way of thinking about the star as a collection of triangles. During this passage, Joan shifts between a position of interested reader and a more assessment oriented position. Although she has inferred Richard's thinking from the diagram, a written explanation would have been "a good idea". Similarly, a summarising table and overall conclusion would have been good. As well as identifying (as an examiner) what is missing from the text, the way in which Joan hypothesises about what Richard could have done suggests that she is reading partly as a teacher/adviser, suggesting ways in which the student might improve his work.

Fiona, on the other hand, has nothing to say about the pages on hexagons, stars and squares, each of which contained diagrams, a table of results and a formula. She passes quickly over these but stops at the page of hexagons drawn on squared paper, commenting that there is formula produced without any evidence in the form of "answers" in the table. Her reading is only evaluative: criticising what has not been done, making a comparison between the standard of the extension and of the earlier work and summing up the whole of Richard's work as lacking "detail". At no point does Fiona demonstrate any attempt to make sense of the mathematics or even to check the accuracy of Richard's work; her assessment of all parts of this text appears to be based on the presence or absence of formal features: diagrams, tables, examples, formulae, writing.

Summing up immediately after completing the first reading

It appears that Joan and Fiona validate their assessments by using different reference points; Joan uses the National Curriculum (she had recently been on a course run by the Southern Examining Group about relating coursework to Attainment Target 1, Using and Applying Mathematics) while Fiona refers back to the wording of the Inner Triangles task itself⁴. In

⁴The task included the statement: "In your report you should: show all your working, explain your strategies, make use of specific cases, generalise your results, prove or explain any generalisations."

spite of Joan's use of criterion referencing she nevertheless compares Richard's work to Clive's in attempting to decide/justify which level to award.

Both teachers pick out Richard's use of symbolic algebraic notation and his lack of "justification" as significant to their overall evaluation of his work, praising the algebra as "quite advanced" and condemning the lack of justification. The ways in which they resolve the tension between these two aspects again reflect different reading positions. While Joan identifies aspects that are missing from Richard's text, she does so from the position of teacher/adviser suggesting what he might do to improve the level of his work and, in spite of her lack of personal knowledge of the student, states as teacher/advocate that she is "sure he'd have . . . got a level 8 if . . ." An unwillingness to take on an examiner role is suggested by the way in which she hedges her final evaluation; although she claims Richard could have achieved level 8, she nevertheless says that level 7 may be generous and that this assessment is in any case only "a gut reaction".

Fiona, on the other hand, is unambiguously positioned as examiner in this passage. Although she does say that "it would have been nice" if Richard had shown his working, this is not a suggestion about how he might have improved his grade but is a request for evidence that he has not cheated. Having listed all the things that Richard has not done (or has not shown he has done) she dismisses what he has achieved "by whatever method I don't know". Unlike Joan, she is not prepared to hypothesise about Richard's potential or hidden attainment but will only consider what is written on the paper, measuring it against her 'ideal text' and finding it wanting.

10.1.2 Subsequent comments

After reading all three students' texts, the teachers were asked to rank them and to justify their ranking. In undertaking this process, the key features used in order to arrive at an assessment are picked out by each teacher, often contrasting strengths and weaknesses both within and between students' texts.

The features of Richard's work that Joan identifies as significant to her evaluation are:

- use of symbols
- extent and variety (or creativity?) of the extension
- justification
- explanation of method

As during her initial reading, Joan's dominant position is as teacher/advocate; she draws conclusions from the "finished product" about work that Richard might have done but has not displayed. Thus the lack of explanation of the steps towards the solution is excused to some extent by her assumption that he must be "hiding loads and loads of rough work". Her final

evaluation of Richard's work is made on the basis of the amount of "different things" he achieved in his extension of the original problem.

Joan's reluctance to take on the role of examiner is apparent in a number of ways. Firstly she complains that it is difficult to make a decision. When pressed, she hedges her decision:

I think you would have to say that's the best one

not only expressing the tentative nature of the decision but also passing the responsibility to an anonymous authority and claiming that this is not her own personal decision but one that must be shared by the general "you", presumably of the teaching community. The suggestion that the three texts could be combined to make "an excellent piece of work" diverts attention from the need to evaluate the individual student in order to allow Joan to take on a more comfortable teacher/adviser position. Finally, she again attempts to avoid the responsibility for making a final decision by suggesting that any assessment is "a personal interpretation" and that the problem of reconciling the positive and negative aspects of the student's work would be better resolved through a communal discussion.

The significant features identified by Fiona, like those used by Joan, include:

- use of symbols ("the way he has written it out")
- the extent of the extension
- showing "working out"

In addition, she mentions:

- the correctness of the formula
- the amount of work, or "effort", on the "investigation" (i.e. question 3)
- answers to the specific questions 1 and 2
- a table of results

Throughout this section of the interview, Fiona is firmly positioned as an examiner. Even when, at the end of passage A, she suggests that Richard's problem "would have to be addressed", this is only a hypothetical acknowledgement of a teacher/adviser position which she herself does not attempt to fill. Whereas Joan was willing to assume that rough work existed, Fiona will not accept this without "evidence" and apparently measures the amount of "effort" put into the work by the number of pages of text presented to her. Her evaluation is very firmly tied to the task as defined by the question paper provided by the examination board and, although she acknowledges the quality of Richard's extension, this is dismissed as not counting for more than "a few extra marks". The feature of Richard's work which is critical to Fiona's decision to rank him third is the lack of evidence of "effort" in the part of the task that she identifies as "the actual investigation".

10.1.3 Summary of 'Inner Triangles' comparison

To a large extent these two teachers identify the same features of Richard's text as significant both during their initial reading and when asked to compare it with the other students' texts.

The main exception to this is the generic diagram on page 4 which is read as “explanation” by Joan while being ignored by Fiona. This exception appears to be symptomatic of the different ways in which the teachers read diagrams generally. Although elsewhere both comment on the presence of diagrams, Joan interprets them as indicators of Richard’s thoughts while Fiona reads them as evidence of “working out”.

However, the ways in which they interpret and resolve the tensions between positive and negative features vary considerably, leading to opposite rankings of Richard’s text. One of the sources of this variation, particularly apparent during the re-reading while comparing the three texts, seems to be a difference in the definition of the “investigation” task used by each of the two teachers. This leads them to place very different values on the extension section of the text. I would suggest that this difference is closely related to differences in their reading practices. As has been shown, Fiona’s dominant reading position throughout this part of the interview is as an examiner. As well as searching for “evidence” that the work belongs to Richard himself, she is concerned to determine whether or not he has fulfilled the assessment criteria. Her reading is thus a search for indicators that allow her to compare his work to an imaginary ideal answer. This imaginary ideal is determined by the structure of the task set by the examination board which not only includes a number of closed questions but also specifies processes to be demonstrated while “investigating”. The extension is officially described as “optional”, which suggests that it is peripheral to the main task. It is hardly surprising, therefore, that Fiona attaches little weight to Richard’s extension when evaluating the whole piece of work.

Joan, on the other hand, appears uncomfortable with the examiner role when forced into it by the demands of the interview and generally adopts reading positions which allow her a less formal relationship both with the student-author of the text and with the task itself. Rather than seeking for performance indicators she appears much of the time to be reading in order to understand what has been written; she thus accepts the definition of the task given by the student and does not refer explicitly to the questions set by the examination board. Her frequent adoption of a teacher role (adviser or advocate) leads her to treat Richard as an individual with an existence outside the text. Although the text is the only evidence available to her, Joan’s reading leads her to make statements about what might have happened in class or what Richard might have done if he had been advised differently. The extent of the extension section of his work, therefore, causes her to make assumptions about Richard’s understanding and about the existence of work that was done but not recorded. Hence she evaluates the text as a whole highly, largely on the basis of the extension.

10.2 A comparison of two teachers reading Steven's 'Topples' text

There was substantial agreement among the teachers who assessed the 'Topples' texts on the ranking of the three pieces of work. In all cases Sandra's text was ranked lowest. The two teachers whose readings of Steven's text are compared here, Jenny and Charles, both ranked Steven's text highest, as did all but one of the teachers who read it. In both cases, Steven's was the first text that they read of the three, hence there are a number of points at which they check or make assumptions about the validity of the data. Like the teachers whose readings of Richard's 'Inner Triangle' text are compared above, Jenny and Charles taught in different schools and had several years experience of coursework assessment; neither of them knew the author of the text.

During her interview, Jenny frequently distanced herself from the assessment process, positioning herself as an interviewee commenting on the process rather than committing herself to the role of examiner. While her positioning as interviewee must raise questions about the extent to which her practice during the interview corresponds to her genuine assessment practice, there is no reason to suppose that she was not taking the task she was given seriously. Although not unwilling to be interviewed, Jenny was clearly not as committed to the practice of investigations and coursework as were the other interviewees. This in itself makes it useful to consider her readings and to compare them with those of a teacher who appears to have adopted the discourse of investigations more whole-heartedly. Charles had been involved several years earlier in the trialling of GAIM (Graded Assessment in Mathematics) materials which included a number of investigative tasks. He had thus participated in the development of assessment criteria for such tasks.

The procedures followed both in the interviews and in this comparative analysis are the same as those described in section 10.1 above for 'Inner Triangles'. Transcripts of the relevant sections of Jenny's and Charles' interviews are in Appendix 11 and Steven's 'Topples' text may be found in Appendix 5.

10.2.1 First reading of Steven's text

pages 1-2: Title: statement of the problem

Steven's statement of the problem is a close paraphrase of the statement provided in the question paper. Neither Jenny nor Charles paid a great deal of attention to this or to the title page; Charles, in fact, turned the pages quickly without apparently trying to read them. Jenny, on the other hand, spent enough time reading the statement of the problem to identify it as a copy of the question paper and to express a negative reaction to it, although she did not read it thoroughly enough to recognise that Steven had in fact included a statement of when the pile used as an example topples.

It is not clear on what basis Jenny made her judgement of the author's gender as she did not appear to have noticed the name before making the comment. Knowing the gender of the student seems to have been important for her as she made a number of comments while reading the other texts and when summing up her assessments which suggest that it influences her reading of the text. In particular, her comments on Sandra's presentation included a number of generalisations about the ways in which girls work.

As at a number of other points throughout her interview, Jenny refers to the interviewer for confirmation of the accuracy of her reading. Although this might be interpreted as a sign of a lack of confidence in her own judgement, I would suggest that, when seen in the context of her generally cynical tone of voice and a number of other comments about the validity of the process of assessment and of the National Curriculum, it may serve the function of distancing herself from the activity. She appears to be deliberately positioning herself as an interviewee.

page 3 (top): table of results

As the data for the Topples task had to be generated practically, there was no evidence within the text that the results displayed in the table were accurate. Although the teachers had been given the problem in advance and asked to try it out before the interview (Charles' opening remark provides evidence that he did engage in the practical task prior to the interview), neither of them appear to be familiar enough with the task or confident enough of the accuracy of their own results to make an absolute judgement of the validity of Steven's data. Both, however, state a willingness to accept his data, at least provisionally.

Both teachers comment on the difference pattern which Steven had included next to his table. They appear to have found the pattern itself unexpected; thus both read it aloud and Charles qualifies it as "nice" while Jenny questions its validity and rather grudgingly accepts it. These different reactions are found in other teachers perceiving unusual or unexpected aspects of the texts (see chapter 14).

While Jenny's concern appears to be related only to the accuracy of Steven's results, Charles brings in two further aspects of his reading of the table: it displays "a reasonable set of results" and it is "systematically laid out". Given his subsequent questioning of their accuracy, it would seem likely that Charles is using the term "reasonable" to refer to the form of the results or the number of them rather than their content. The association of the table with 'system' was commonly made by the teachers interviewed (see chapter 11) and relates to the examination assessment criteria.

page 3 (bottom): description of pattern and comment on generalisation

In this section, Steven's response to the request to 'generalise' was to make the comment:

<p><i>With such a definite pattern a formula should be easy to find</i></p>

This response clearly strikes both teachers as unexpected, although they mark their surprise and deal with it in different ways. Charles' sequence of incomplete sentences signals his difficulty in making sense of this section in the light of his expectations. As his next action was to search forward in the text for a formula, it seems clear that he was expecting to see the formula itself at this point in the text. Jenny, on the other hand, merely greets the comment with amusement. Although she appears to find the remark incongruous, she reads it for its own sake rather than seeing it as a place holder for something more acceptable. It is Steven's "enthusiasm" which appears out of place rather than the lack of a formula.

page 4: explanation in words and diagram of physical reasons for the results

Both teachers passed quickly over the page containing Steven's response to the request to explain his result. In Charles' case the position of this section clearly did not match the order in which he expected the text to be arranged as, having just read the table and description of the difference pattern, he was engaged in searching for a formula. Having found the formula on the next page, however, he did not return to this section until he was revisiting the whole text after completing his first reading. While Jenny did comment at this point, she merely remarks on the general nature of the explanation, perhaps influenced by Steven's introduction of vocabulary such as *weight* and *gravity* to identify it as in the domain of physics. Her response to this section is to add to her picture of Steven himself (as a student of physics) rather than to attempt to make sense of or evaluate the explanation.

page 5: formula and examples

As a first step to making sense of the formula, both teachers refer back to Steven's table of results to see whether his formula fits the data. For Jenny, seeing that the formula works appears to be the only criterion she applies and, having established that it does, she approves the page. At a number of points during the interview it was apparent that Jenny was reading the texts only partially, making her judgements of each section on the basis of her reading of the first part only. In this case, she did not pay attention to the examples that Steven had provided on the second half of the page, forming her opinion that the formula worked only for even numbers on the basis of her own use of the formula with the earlier table of results. Only after an intervention did she attend to Steven's statement that it was necessary to round up some of the answers achieved by applying the formula. Although at another point during the interview Jenny herself commented on her reading style:

I'm a great not reader of tasks. I look at it and hope.

(Jenny: 79)

this cannot be taken as an unambiguous demonstration of her normal assessment practice. As mentioned earlier, Jenny appeared to be more strongly and frequently positioned within the interview setting than most of the other teachers. Her meta-comment "one ought to check" and the interjection "it's not me being stupid is it?" are evidence of such positioning during this passage.

Following his earlier difficulty in coping with the mismatch between Steven's 'generalisation' and his own generic expectations, Charles marks the fulfilment of his expectation of finding a formula; the emphasis provided by the modified "we *do* have a formula" indicates that he is not merely cataloguing the existence of the formula but is simultaneously commenting on its previous absence. As at a number of other points during his interview, Charles spent a long time reading in silence and had to be prompted to provide a commentary; his subsequent description of his reading suggests that he is looking for more than just the correct formula that satisfied Jenny. Not only does the formula have to fit the data but the student must also demonstrate that he has understood the formula and is expected to show how it was derived. It is not, however, clear how 'understanding the formula' is being distinguished from the mere fact that the formula fits the data. There is some tension between the requirement to show "how they arrived at the formula", the value placed on 'understanding', and Charles' image of how the student might have been working. This tension is manifested in the repeated hedging and modification of his demand to see where the formula came from:

I'm not sure where they've got it from. So I'd have liked to have seen something to show how they arrived at this formula. So it may have been something that they got from someone else and then fair enough they may be able to understand the formula and use it but did they know how to arrive at the formula is something I would be looking for.

His use of the first person here in describing the assessment process, although suggesting that he is claiming personal responsibility for the criteria, simultaneously makes the process appear hesitant and possibly idiosyncratic rather than authoritative. On being prompted to be more explicit about the forms of text which might be interpreted as showing how the formula was arrived at, Charles is tentative about committing himself. He suggests that tables, calculations and a narrative indicating a trial and error approach might be appropriate, but again hedges his demands, suggesting that he might be wrong to make them but nevertheless claiming to need 'evidence'. The demand for narrative in this passage is discussed further in section 11.5.

In contrast to Jenny's cursory reading of the examples provided at the bottom of the page, Charles read both of them carefully, having some initial difficulty in making sense of Steven's unconventional lay out:

$$(1+1)=2 \quad \left(\frac{1}{2}\right)=0.5$$

$$2+0.5=2.5. \text{ (you then round this up)}$$

$$\text{ans. [3]}$$

but eventually succeeding in making sense of it. His use of personal pronouns is once again of interest here. Having used *they* to refer to the author of the text when discussing his actions and his understanding, Charles shifts to a general *we* and *you* when making sense of the procedure of applying the formula. This mirrors Steven's own shift from using *I* when

describing his actions to the general *you* in his statement of the procedure. It suggests that Charles is concerned at this point with the mathematical content rather than with Steven's own processes. At the end of this section, however, he returns to his discomfort with the unconventional way of presenting the formula and examples, juxtaposing this with his uncertainty about Steven's thinking:

It's very difficult to judge what's going on in somebody's head. Um .. so perhaps this . . yeah .. It's a funny way to express it, um . .

At this point he appears to be taking the "funny way of expressing it" as evidence towards judging "what's going on" in Steven's head.

page 6: answer to question 2 showing three alternative ways of calculating the result for 100 units

The first reaction of both teachers is to attempt to make sense of the alternative methods presented by Steven for finding when a pile starting with a hundred unit rod will topple. They focus in particular on Steven's second method, which involves scaling up his result for a pile starting with a ten unit rod by multiplying the result by ten. This method could be seen either as a relatively sophisticated appreciation of the way in which linear relationships behave (although in this case it could not be completely generalised because of the difference in the relationship for odd and even starting points) or as an unfounded simplistic assumption. Both Jenny and Charles took some time to make sense of the method, suggesting that they found it unusual, but neither expressed any doubt about the basis for the method. Jenny focuses only on her own understanding of the procedure, moving on once she is satisfied that she knows Steven's intentions; she does not comment on the validity of the procedure but, given her concern with 'correctness' elsewhere in her interview, it seems likely that the lack of comment indicates a willingness to accept it. Charles reads much more into the text, suggesting that the method indicates a "good understanding of the problem".

The value that Charles places on this scaling procedure contrasts with his demand when reading the previous section to see evidence of the processes leading up to the formulation of the original procedure. This difference may arise from the different positions of the two procedures within the text and the different functions that they thus fulfil. The original procedure was presented under the heading "Formula" and hence had to conform to Charles' generic expectation for narrative, trial and error, etc. leading up to it. The second procedure, in contrast, appears as 'working out' for a specific problem rather than as a generalised method (although Steven actually makes use of this method twice in this section and once in the next, suggesting that he himself sees it as generalisable). There is apparently no demand that the method for 'working out' should be justified; the fact that it gives an acceptable result is enough to validate it.

page 7: answer to question 3 showing two alternative methods, including a second formula

The lack of attention to the details and to the lack of justification of Steven's scaling method when it was originally presented in the previous section is paralleled by a complete absence of comment on its use in this section. Although this time the scaling method is presented first, both teachers immediately focus on the algebraic formula which Steven presents second as an "alternative" method. I would like to suggest two possible explanations for this: firstly, Steven presents his scaling procedure in a paragraph, starting with the words "I estimate that it will be 20 units on the bottom". Although the rest of the paragraph describes a precise procedure rather than an approximate one, identifying the result as an estimate may immediately make it appear of lower value than the algebraic formula which follows. The teachers, scanning the page initially, may therefore focus on the more highly valued algebra and, having decided on its validity and hence on Steven's ultimate level of attainment, do not feel the need to return to make sense of a section demonstrating a lower level. Alternatively, they may be making sense of the whole page by fitting it to their expectations of the genre. Given that there is a formula at the bottom of the page, a description of the formula in words might be expected to precede it (see chapter 12). The teachers may therefore be taking the first paragraph to be a verbal description of the formula that follows; again, the formula itself is of higher status and thus demands more attention. Whatever the reason for their neglect of Steven's first method, it seems likely that parts of a student's work on an investigation that do not fit easily into the standard expectations of the genre may not be read with the sort of attention that is probably necessary to make sense of them unless they are very clearly signalled in a way that is likely to be highly valued by a teacher-reader, perhaps as 'extension' work.

Interestingly, both Jenny and Charles make the assumption that Steven derived his second formula from his first one. The two formulae are shown in figure 10.1 below. Although the second does express the inverse relationship, its form makes it appear more likely to me that the two were derived independently, particularly as Steven introduces the second with the words "I have found another formula" rather than providing any indication of its derivation. There is no evidence here or elsewhere within the text that Steven has performed any manipulation of either algebraic expression. It seems that the teachers are imposing their own understanding of the inverse relationship between the formulae onto their interpretation of Steven's text.

Figure 10.1

Steven's first formula

and

Steven's 'inverse' formula

$$(A + A) + \left(\frac{A}{2}\right) = b$$

$$\left(\frac{a}{2}\right) - \left(\frac{a}{10}\right) = b$$

The two teachers differ in the amount of effort they expend in reading the 'inverse formula'. Charles appears to assume that the formula is correct and takes the mere presentation of the inverse procedure as a sign of Steven's good level of understanding. Jenny, on the other hand, goes to great lengths to validate the formula by manipulating it with pencil and paper. This is consistent with her focus throughout the interview on making sense of the mathematical 'content' presented in the text rather than on identifying the student's 'understanding' or the processes gone through. Her stated reason for performing the manipulation is to see if Steven "knows what he's doing". The emphasis on 'correctness' and on testing the student's knowledge suggest that Jenny is working within a more traditional framework of assessment that values content over process.

Summing up immediately after completing the first reading

Apart from the fact that Jenny was making use of the National Curriculum statements that she had brought with her to the interview while Charles was giving a less formal evaluation, the two teachers appear to be using different frameworks to form their overall impression of the piece of work. Although Jenny looks at each of the National Curriculum statements and goes through the action of checking Steven's work against them, she is not concerned to find specific evidence for each statement but is willing to apply them based on her initial assessment that Steven ought to be given a grade C. Her use of *I'm sure . . . , I suppose . . .* acts to express her attitude towards the National Curriculum while going through the form of complying with it. The essential criterion that she appears to be using is her belief that Steven has manipulated algebra. Again we see her focus on content together with an explicit dismissal of the processes contained in the National Curriculum Attainment Target 1.

Charles' initial summing up, however, is concerned with the coherence of the text as a "nice piece of work". Algebra is picked out for particular comment, suggesting that it plays an important part in his assessment too, but his focus is on Steven's understanding of the formula and ability to apply it rather than on his alleged demonstration of further skills. Although he criticises the form of Steven's algebraic expression, he hedges his criticism and eventually plays down the importance of this point.⁵ During his review of the text at this point he returns to Steven's 'explanation', which he had passed over without reading earlier in the interview. His interpretation of this section is vague and he merely marks its status as "an explanation" and hence its fulfilment of a generic requirement. The nature of the explanation does not appear to be important to him. On the other hand, he returns again to his wish to see some description of Steven's supposed actions leading up to the 'discovery' of the formula.

⁵This passage and Charles' way of dealing with his perception of error is discussed further in chapter 13.

10.2.2 Subsequent comments

Both teachers referred back to Steven's work and appeared to use it as a reference point while reading the other two texts. Jenny used Steven's table of data to check Sandra's work:

Go back to his table. I liked his table, I could read his table [referring to Steven's work]
(Jenny: 130-131, while reading Sandra's text)

When comparing the form of Steven's table with those used in the other two Topples texts, there are no obvious reasons why it should be considered easier to read. It is possible that Jenny's use and approval of his table at this point is a consequence of her more general evaluation of his work.

Charles spontaneously compared Ellen's generalisation with Steven's:

she's understood the problem as much as the other person who's .. the other person's managed to use a little bit of algebra, but as I say it wasn't particularly well written, but I'd put both those pieces of work on a similar sort of footing really
(Charles: 144-146, while reading Ellen's text)

We see the tension he displays between the value he places on 'understanding' and the importance of 'algebra'. In this case he resolves it by devaluing the form of Steven's algebraic formula in order to justify his general evaluation that the two students have the same level of understanding.

Immediately after reading each of the three texts, Jenny's practice was to allocate a National Curriculum level to the student. As she assessed each of the three texts at a different level, an explicit ranking had already emerged by the end of the first readings. It was thus not possible to ask her to compare the texts in order to form a ranking as was done in all but one of the other teacher interviews. (The other exception was Carol, a teacher at the same school as Jenny, who used the National Curriculum statements in a similar way.) She was, however, asked to sum up by saying what was good and what was bad about each text. Unlike ranking, such a summing up is not a part of the practice of assessment. Jenny's response at this stage of the interview cannot be compared directly with Charles' comments while ranking the three texts because she is engaged in a different sort of activity. It does, however, contribute to the picture of her reading of Steven's text.

The features of Steven's work that Jenny identifies as significant are:

- the lack of extension;
- the explanation in terms of 'physics';
- the use of algebraic symbolism, in particular the perceived relationship between the two formulae.

The importance of Jenny's general evaluation of Steven's 'ability' is strongly demonstrated here. As was seen earlier, Jenny has constructed an image of Steven as the sort of student who should achieve a C grade. Weaknesses in his work are thus blamed on his laziness rather than on any intellectual lack. In revisiting Steven's 'inverse' formula, Jenny notices that

the variable names used have been reversed. At this stage in the interview, this is not enough to make her reconsider either her assumption that the second formula was derived from the first or her overall evaluation of Steven's level of attainment.

The key to Charles' final assessment and ranking of the texts is clearly 'algebra' and, although he still criticises the form of Steven's algebra and even suggests that it might have been "picked up" from another student, its very existence is enough to outweigh his doubts about Steven's understanding. The tension between 'understanding' and algebra is still present and Charles appears to feel the need to justify his final decision to rank Steven higher than Ellen. In extract A he uses the general *you* to distance himself from the responsibility for the criterion and, when finally committing himself in extract B, he hedges his decision heavily: suggesting he does not have enough evidence, using the conditional "if I had to do it" to indicate that this is not a real high-stakes assessment, and modifying his action with repeated use of *probably*. His awareness of the discomfort caused by this tension is demonstrated by his nervous laugh at the end of extract A and by his expression of embarrassment about the final ranking.

10.2.3 Summary of 'Topples' comparison

In general, both Jenny and Charles paid attention to similar features of Steven's text and gave similar interpretations of their mathematical meanings, both, for example, assuming that Steven saw his second formula as an inverse of the first. Their interpretations of how the features contributed to the assessment process differed, however, suggesting that they have different understandings of the object of coursework assessment. While Charles wished to see evidence of the processes that Steven had gone through, in particular before arriving at his formula, Jenny's focus was strongly on the 'content' and on Steven's display of his knowledge. Indeed, she was explicitly dismissive of the focus on process that she perceived in the National Curriculum and elsewhere in the discourse of coursework assessment. For example, when asked at the beginning of her interview to describe what she would look for in students' work, she replied:

I wouldn't look and see if they were working systematically and all that sort of nonsense
(Jenny: 3-4)

Steven's use of algebra played an important role in both teachers' readings, apparently being the determining factor in their final assessments, although Charles experienced tension between this and his valuing of 'understanding'. He resolved this tension by taking on a position of subservience to external assessment criteria in an attempt to distance himself from his decision. While Jenny also referred explicitly to the external criteria provided by the National Curriculum, this was certainly not in order to defer to them. Her confidence in her own judgement allowed her to make the criteria fit her assessment rather than the other way round. Assessment did not appear to be a problematic activity for Jenny. The lack of tensions between different criteria displayed in her interview may be a consequence of her

clear focus on content knowledge and her conscious rejection of the officially sanctioned 'process' criteria. Her way of dealing with the tensions between valuing content and valuing process that are inherent in the discourse of investigations and coursework assessment (see chapter 4, Appendix 15) is to position herself outside the discourse. During the interview she thus adopted two roles, both of which distanced her from the coursework discourse: that of interviewee, free to comment on the practice of coursework assessment from the outside, and that of autonomous examiner, setting her own criteria by which to judge the students' work.

In spite of the similarities between the features identified and the final assessments of the work, the ways in which the two teachers arrived at their assessments were very different. Jenny appeared to build up an overall picture of Steven, identifying his personal characteristics from evidence in the text: this picture eventually included the facts that he was male, 'able' (in particular able to manipulate algebra), lazy and had knowledge of physics. From this picture she derived a further characteristic, that of 'C-ishness', which enabled her to arrive at a final assessment. Charles, on the other hand, while reading the text, seemed to be searching for components of the text that were compatible with his preconception of an 'ideal' piece of coursework. Features which did not fit into this preconception were passed over quickly. Although he expressed concern about 'understanding', his final assessment was of the text itself and the extent to which it conformed to his ideal rather than of the hypothetical characteristics of the student who had produced it.

10.3 Conclusions

It is not possible to make direct comparisons between the teachers' readings of Richard's text and those of Steven's 'Topples' text. Nevertheless, it is possible to conclude that, even where teachers identify similar features within the text, the uses that they make of these features may differ. Jenny's focus on content rather than process and her oppositional relationship to the coursework discourse were unique among the teachers interviewed; this cannot, however, be taken as a sign that similar opinions and practices may not be found among other mathematics teachers.

One of the major variations between teachers' practices, found in both comparative analyses, is the degree to which a teacher focuses on the student or on the text itself. Although they seemed to do so in rather different ways, both Joan and Jenny based their assessment on assumptions or deductions about the nature of the student and about the nature of the type of work that he might have done but did not appear in the text. The coursework text itself appears to serve as a source of evidence towards the construction of a picture of the student. In Joan's case, this method of reading appears to be associated with her frequent adoption of a positioning as teacher/adviser or teacher/advocate which places her in a more personal relationship to the student-author of the text than would a positioning as examiner. It is

interesting that this focus on the student is found in teachers who have no personal knowledge of the student himself; the present study does not address the interaction between a teacher's previous background knowledge of an individual student and her ways of reading that student's coursework text.

In contrast, both Fiona and Charles based their assessment on the extent to which their reading of the text corresponded to their idea of what should be in an 'ideal' text. At some points during their readings, they appeared to be searching for features that they would expect to find in their ideal. Features of the texts that did not match the teachers' expected structure were passed over with little attention or had little value attached to them. The characteristics of teachers' 'ideal texts' are investigated in chapter 11.

It is not clear to what extent these various approaches to reading and assessing coursework texts may affect the final assessment of a student's work. Whereas Richard's text was evaluated substantially differently by the two teachers, both readers of Steven's work eventually relied on his use of algebra as the final determining factor in their assessment and hence ranked his text highest of the three read. It seems likely that there would be interaction between the nature of a text, the nature of the resources brought to the text by a teacher-reader, and the amount of variation in teachers' assessments; in Richard's case, his text differed substantially from Fiona's ideal text, whereas Steven's text contained most of the features that Charles expected to find.

11 Teachers' identification of and response to features of students' texts

In order to build a picture of teachers' 'ideal' coursework text, the teachers' identification of and comments on specific features of the students' texts have been analysed. The process of selection of the students' texts to be used in the teacher interviews (described in chapter 6) ensured that each teacher read and responded to a variety of styles of text, thus encouraging not only a range of responses but also comparisons between different texts. During the main part of the interview in which the complete coursework texts were read, the teachers were not explicitly asked to comment on the ways in which the texts were written as the intention of the interviews was to elicit reading practices as close as possible to those used during the normal assessment process; questions related to the form of the writing would have substantially changed the nature of the teachers' relationship to the text. Nevertheless, there were frequent spontaneous evaluative comments on the form of a text, particularly in response to perceived weaknesses or absences of 'writing'; when such comments had been made by teachers it was sometimes possible to follow them up with further questions to encourage elaboration of the reasons for the judgements. The features that were commented on included the use of specific forms of representation or communication (e.g. words, tables, diagrams, algebraic symbolism) and more substantial structural elements of the text (e.g. the statement of the problem, passages relating what was done or explaining the results). In this chapter, the teachers' expressions of attitude towards such features are examined, together with the meanings that they appear to ascribe to them. For example, some features appear to serve as signals of particular types of achievement by the author, e.g. 'understanding the problem' or 'working systematically'. The small number of explicit comments and observable reactions related to the 'style' of the texts will then be considered. Teachers' responses to the form of generalisations and other algebraic aspects of student texts are addressed in chapter 12.

11.1 A generic demand for 'writing'

There were numerous instances during the interviews when teachers expressed a wish to see more 'writing'. In most of these cases, however, they gave no appearance of difficulty in understanding the student's text in the absence of such 'writing'; the demand seems, rather, to be an expression of a generic expectation. Grant stated his expectation explicitly in response to a general request to say what he is looking for when assessing students' work:

A written introduction, sentences. What I usually tell the class to do is to describe in a few sentences what the problem is about before they start. So I'd expect to see almost a repeat of my explanation that I've just given to the class about the project, written down in a paragraph or perhaps a couple of sentences if it's a low ability group. Further on from that then I'd look for how

they start the problem, describing again what they do along the way, so sentences going into the project along the way (Grant: 3-9)

He repeated his desire to see 'sentences' at various points during the course of the interview. At one point, when reading a specific text, he justified his criticism of the quality of the writing by reference to the nature of the genre rather than to any difficulty in communication:

I suppose it's pernickety isn't it to look at short sentences like that or the way they said it but after all I think that investigations are a report on some work so they should be writing about what they done and not just going for an answer (Grant: 201-203, Ellen)

This attempt to justify his "pernickety" judgements suggests that Grant experiences a tension between the normal value placed by school mathematics on correct answers, his evident ability to make coherent sense of Ellen's text, and his understanding of the genre of investigations. This tension is seen perhaps most strongly in the teachers' reactions to Richard's Inner Triangles text which was almost exclusively non-verbal. Although Richard's conclusions were recognised to be correct, and some of the teachers suggested that they considered Richard more mathematically able than the other two students whose work they had read, his work was nevertheless universally criticised for its lack of 'writing'; two teachers' ways of dealing with this were discussed in chapter 10. While all the teachers interviewed made similar demands that the students' texts should contain 'writing', some appear less convinced of the justification for this demand. Dan, for example, repeatedly hedged his condemnation of the lack of 'writing' in the texts he read by appealing to the external authority of the examination board's requirements, relating an anecdote about a boy whose work was "marked down" by the moderator because it was very "blank". When reading Richard's text he referred to the LEAG performance indicators for the Inner Triangles task to explain his judgement:

he got a formula for it, I think he's into B country isn't he immediately. But you see again it [the performance indicators] says 'This generalisation with a very good write up is a lower A' So he's cut himself out from there immediately by not doing any writing, and it's still a bit difficult because all the way up they've asked for explanation so my guess would be he could only get a B and probably a lower B. (Dan: 195-199, Richard)

Dan's account suggests that, for him, the requirement for writing is an arbitrary assessment criterion rather than an essential part of the practice of doing investigations.

The reasons given by teachers for making this demand to include 'writing' thus include the perceptions that:

- it is appropriate to the genre of report writing (as opposed to more usual school mathematics which may be entirely symbolic);
- it is an expectation of the examination board.

There is, however, a tension for teachers between their perception that 'writing' is essential and their ability to make sense of what has been done in the absence of such writing. The

requirement for 'writing' is also in tension with the more traditional assessment criterion that a student who has given correct answers should not be penalised.

11.2 Non-verbal forms vs. words

Although the teachers interviewed generally approved explanations expressed in words, there were a few cases in which individual teachers remarked that the verbal explanations were unclear and expressed a preference for other forms of text. In this section, I discuss a number of incidents in which teachers expressed such a preference in specific cases. More widespread approval of tables (section 11.3) and diagrams (section 11.4) is discussed separately.

Fiona, in particular, stated a preference for non-verbal forms on several occasions, suggesting, for example, that both diagrams and examples are easier to read than Clive's verbal explanation:

I think his explanation is fine, I can, I understand what he's saying um because he's also shown it in the diagram 'the numbers can be added together to get the next row of numbers. It can also tell you that the answer from the two top two slant [...]' It'd have been nice if he had actually done that to show us what he meant. You know, again, example. And it's always easier to understand from examples rather than from text, kind of. But it's not too bad, I mean you know, I've seen a lot worse when it comes to explanations.
(Fiona: 187-193, Clive)

and that Steven's pattern could have been better explained by adding arrows to his table "rather than just having a sentence about it" (Fiona: 228, Steven). Similarly Harry considers Steven's ('Topples') diagram to be easier to understand than his verbal explanation:

he's tried to explain here that the weight is too much on the top block, but he doesn't actually say - there isn't any point about what he means by too much. But I think the illustration shows that we know what he's trying to say cos he's actually got this centre of B line which I think is maybe the halfway point um I think that's what he's trying to say.
(Harry: 127-130, Steven)

In each of these cases the verbal explanation provided in the text is complex and the teachers' comments may be in response to their difficulties in understanding what has been written. They do, however, also appear to represent a belief that, at least in some circumstances, non-verbal forms may be more appropriate than verbal explanations. The extract from Fiona's interview above contains a general claim about the usefulness of examples which provides evidence for such a belief.

In general, the absence of words was condemned even when the teacher was clearly able to make adequate sense of the non-verbal text (i.e. provided an interpretation of the meaning of the text and expressed no difficulty in understanding it). However, there were a small number of cases in which it was admitted that a non-verbal form used without verbal accompaniment was more effective in communicating meaning. This was rather grudgingly admitted by Harry in the case of Sandra's illustration of her extension problem:

she's thrown some illustration of what she actually means I think something like that to explain in words would be quite difficult so that's a good use of an alternative method of presenting it.
(Harry: 241-244, Sandra)

and applauded by Carol:

the explanation I find acceptable here, it's done just as a mathematical calculation, I think sometimes that word explain causes problems. Some of our girls start to write you reams whenever it says explain. So the fact that this student has used a simple calculation and left it at that actually at this stage makes me into an even more positive frame towards them because they see that a mathematical calculation can be sufficient explanation.
(Carol:135-140, Sandra)

Both these extracts, however, show the teachers struggling with the normal requirement to put things in words. There is a tension between the general rule that 'you must explain in words' and its application within a specific context in which words might be difficult to understand and other forms of text may be completely effective in communicating the required meaning.

In general, the use of conventional algebraic symbolism was approved (although see chapter 12 for a discussion of the need to use words as well as symbols). One exception to this was Grant's reading of extract No. 1. The student had defined her variables by listing the, but Grant objected:

G . . . They haven't written this particularly well I don't think. This could have been more in a sentence
I That's the . .
G The "a=slant line, b=base line, c=top line". That would have read better to me. You get the meaning of it from this but it would have read better as part of an investigation perhaps to me.
(Grant: 284-291, No.1)

Although he admits that the given form is meaningful, his strong understanding of the generic requirement for 'sentences' in an investigation report (see section 11.1 above) overrides other considerations.

It is clear that some of the teachers interviewed have different views about the place of verbal and non-verbal forms within the investigation report. While Grant's position appears extreme, his statement of the generic requirement for 'sentences' does appear to be commonly agreed. The extent to which non-verbal forms may be allowed to substitute for sentences varies, however, as does the degree of teachers' approval of non-verbal forms supplementing the sentences. The cases discussed in this section suggest that approval is given to non-verbal forms in specific cases where it is perceived that they are easier to make sense of than an equivalent verbal form. There is, however, a tension between the importance laid by teachers on their ability to understand the text and the generic requirements for verbal explanations.

11.3 Tables

Not only did all the students' texts contain tables at some point but all of the teachers also commented on the presence of tables. The table is clearly an important, if not essential,

component of the investigation genre, frequently mentioned as one of the things that would be looked for when assessing and in several cases included in teachers' accounts of the advice they provided for their own students. The teachers' responses to the tables varied, indicating a number of different ways of reading and evaluating them. Some appeared to value the table for its own sake, without indicating any specific interpretation of its function within the text:

- merely marking the presence of the table

He's tabulated the results

(Harry: 114, Steven)

- non-specific approval of the presence of the table

Then he came up with a table, I like that.

(Amy: 35, Richard)

- non-specific approval of the form of the table

He's tabulated his results nicely.

(Andy: 86, Steven)

In some of these cases, it appeared that the presence of the table was being used simply as a mark of fulfilment of an assessment criterion. Other responses gave some indication of the role that the table played in the teacher's reading of the text:

- helping to communicate (in a non-specific way)

I liked his table, I could read his table.

(Jenny: 130, Steven)

- helping to communicate because it is an organised form

A reasonable set of results. They're systematically laid out.

(Charles: 50-51, Steven)

- signalling that the student has worked systematically

Tabulated results . . . she's done this systematically.

(Grant: 56, Sandra)

The identification of the table with a systematic way of working was made explicitly by four of the eleven teachers. As none were asked specifically to give their reasons for approving tables, these spontaneous comments suggest a degree of agreement that makes it appear likely that other teachers are also making this identification. The use of a table as a sign of systematic working raises some issues about the assumptions that teachers make concerning the relationship between the form of the text and the student's problem solving activity that may be represented by the text. This is illustrated by the contrasting readings by two teachers of a table used by Clive in his Inner Triangles text. This table was unusual in that it was organised as a two dimensional array rather than as a one dimensional list. (Figure 11.1 illustrates the difference between these forms.)

Figure 11.1

		two dimensional table				vs.		one dimensional table		
		slant length						slant length	top length	number of triangles
top length	1	1	2	3	4			1	1	
	2							1	2	
	3							1	3	
	4							1	4	
								2	1	
								2	2	
								

The two dimensional table might be considered to be appropriate to the task, which involved a situation with two independent variables (top length and slant length); the relationship of the structure of the table to the structure of the situation was recognised by Joan:

... his tables are probably the best because he's put, you know, he's actually correlated two different things (. . .) I actually like these tables. (. . .) So he has actually looked for a relationship that way, you know, joining things together rather than just that it builds up in 2's or it builds up in 3's.

(Joan: 243-252, Clive)

Joan has taken the form of the table as an indication of the way in which Clive was thinking about the problem; the more complex form of table is taken as evidence of more sophisticated forms of thought. Fiona, on the other hand, also uses the form of this table as evidence of Clive's problem solving behaviour but interprets it in an entirely different way:

at least he's tabulated his results and made some effort to, so that . . so there's some organisation there. Um, he doesn't seem, he's gone one two three four five on the slant, one two three four five on the top. He hasn't kept anything constant, you know, at any point, and certainly there's no evidence of it here.

(Fiona: 182-186, Clive)

'Keeping something constant' is one of the signs of 'working systematically' and hence contributes to the assessment of the student's problem solving processes. A one dimensional table, by suggesting that the entries in the table are arranged in the order in which they were derived by the student, appears to act as a sign that the student's chronological actions have taken place in a logical order. Clive's two dimensional table is, however, organised by a different logical principle that does not include any chronological aspect. It cannot, therefore, be read as a record of the order in which the work was done and, as Fiona says, provides no evidence that Clive "kept anything constant". Joan's acceptance of Clive's table suggests that she is taking it as a sign of his way of thinking about relationships between the variables rather than a sign of his way of organising his data gathering. As forming relationships

between variables may be taken to be a more highly valued aspect of his work, she is not concerned with the lack of evidence of systematic variation of the variables.

This variation in the ways in which these two teachers interpret the same table raises a dilemma for students. The conventional way to provide evidence to fulfil the assessment criterion related to 'working systematically' appears to be to include a linear table in which the value of the independent variable is increased in some consistent way. Where there is more than one independent variable, a non-linear representation may be more effective in arriving at a solution. Although this may be interpreted by a teacher-assessor as a sign of a higher level of thinking about the problem, it is also possible that the consequent loss of evidence of a 'systematic' linear approach will be penalised.

The identification between a one-dimensional table and a systematic way of gathering data assumes that the table contains a chronological record of the work carried out. While in some cases this may be true, there is no necessary connection. Even data gathered on an entirely random basis can be organised into a table of the conventional form. The 'systematic' nature of the student's work would thus reside in the organisation of the data rather than the original collection. If this were recognised, the organisation of the data into a two dimensional table could also be taken as a sign of a systematic way of working. Differences in teachers' readings of such a table may arise from different beliefs about the nature of mathematical problem solving and from different assumptions about the relationship between the nature of the processes that the student has gone through and the nature of the text that she produces as a consequence of those processes.

Apart from a general approval of tables for their own sake, the roles that tables appear to play for teachers include:

- to fulfil the assessment requirements;
- to help communication;
- to provide evidence of the student's thinking and problem solving processes, and, in particular,
- to signal that data has been gathered systematically.

While the teachers did not appear to experience any problems in interpreting and placing values on tables within the students' texts, the different possible readings of tables described above present a challenge to the student wishing to demonstrate her organisational processes.

11.4 Diagrams

The specifications of both the Inner Triangles task and the Topples task include diagrams and may thus be seen at least to sanction and possibly to encourage the use of diagrams in

students' responses to the tasks. In fact, all the students' texts on these two tasks in the sample used in this study contained at least some diagrams, even where these were merely copies of those given in the task. Like other non-verbal features of the students' texts, diagrams appear to be valued in their own right. Thus Joan included drawing diagrams in the list of guidelines given to her students for doing any investigation:

These things are sort of "remember to if possible" because obviously they don't always work: "draw a diagram, make a table, plot a graph, predict and check". Graphs don't always get them anywhere of course, so you wouldn't necessarily put it in, and I've still got quite a few people who'll put bar charts when a line graph would obviously have been much more useful.

(Joan: 446-451. The quotation marks indicate that Joan was reading from the printed list of guidelines given to students in her department.)

While she recognises that this advice may not always be appropriate, it is presented to the students in a form that values the drawing of a diagram for its own sake without any indication of when or why it might be possible and appropriate.

Similarly, Charles, when asked what he would look for when assessing students' work on the Topples task, listed diagrams among his other performance indicators:

Well, shall I say we would do the thing like this, ok. Have a look for children being systematic in their approach, I've got a couple of things written down here . . . putting down, tabulating results, perhaps drawing diagrams, putting down ideas, putting forward hypotheses, testing out ideas, um what else . . . perhaps coming up again eventually at the other end with algebraic formulas, things like that, . . .
(Charles: 4-8)

By listing the indicators in this way, "drawing diagrams" is presented as of equivalent status to "putting forward hypotheses" and other items in the list (although the ordering of the list may suggest that items near the beginning are more fundamental in some sense and the ones near the end more advanced). Charles is not distinguishing here between items related to the solution of the problem (ideas, hypotheses) and those concerned with the form in which the work is presented. Again there is no indication of possible reasons for drawing diagrams; all potential uses of diagrams are apparently equally valid. It is interesting to note that none of his indicators are specific to the particular task; tables and diagrams, like ideas and hypotheses, do not need any context in order to justify their existence.

Where teachers commented on particular diagrams, their comments were almost always positive even when suggestions were made about how the diagram might be improved. The existence of a diagram thus appears to be being used as a positive performance indicator, irrespective of the context in which the diagram is found. There is, however, considerable variation in the ways teachers read and interpret particular diagrams. These interpretations include:

- data gathered by the student
- working out
- the student's attempt to explain his or her activity or thinking

- evidence of the way the student was thinking
- evidence of student ownership of the results

They may also be seen to help the teacher's own reading process, either by supporting the verbal text or by providing a check on the accuracy of the results provided. A more thorough discussion of the interaction between the form of a diagram and the ways in which it may be read may be found in Morgan (1994).

In spite of the value generally ascribed to diagrams, there were also a number of indications given in the interviews that in some cases diagrams might not be appropriate or might even be a sign of work done by a 'less able' student. Too many diagrams or diagrams which are not arranged according to an easily recognisable system may be taken as evidence that the student was working at a more concrete, practical level. Thus Dan, having described a boy he remembered who had very quickly 'seen' a formula as a solution to the Inner Triangles task, went on to describe in general terms the characteristics of work by other groups of students:

But if they don't see that [the formula] then I think I'd be looking for somebody who would be quite carefully and systematically looking at some diagrams that they could make that had 8 units and then sort of transferring those ideas into 32 triangles. I can't remember . . . as this was offered as an Intermediate/Higher level piece of work I don't remember anybody having enormous difficulties. If it was offered lower down, maybe children might get a little bit, might find it a bit difficult, might try varying kinds of diagrams before they actually come up with some answers to that. (Dan: 20-27)

He has identified a hierarchy of types of children in terms of the way in which they make use of diagrams in attempting to solve the problem. The highest level of abstraction is characterised by an absence of the practical activity which is embodied in the drawing of diagrams. Thus Andy states as one of the things he would look for when assessing that the student should be able to

calculate the area of a triangle [sic] just by looking at the dimensions given for any particular trapezium without drawing them and counting. (Andy: 27-29)

There comes a point in the assessment of a solution to the problem at which the absence of a diagram becomes a positive performance indicator.

Moreover, some types of diagrams may be read as indicators of 'low level' activity. In particular, Sandra's 'Topples' text contains naturalistic diagrams of piles of rods showing the use of several rods to make up lengths over twelve units. This seems to have contributed to the way in which Harry made sense of her mathematical activity, causing him to question the validity of Sandra's results based on the story he constructed about her practical activity. While the constraints of the materials used for this task mean that the other students and Harry himself must have combined rods in order to build their piles, the effects of this on the results generated by the practical activity are only considered in Sandra's case and are used to explain why her results are different from those produced by the others. (The full text of

Harry's discussion of this point is in Appendix 12.) Although he suggests alternative sources for Sandra's errors - general inaccuracy in building the piles even when they are not split or blindly continuing a pattern of numbers - the remedy he prescribes for her difficulties is still based on the persistent idea that the split rods are causing the errors. The naturalistic form of Sandra's diagrams so strongly presents her work as being of a very concrete nature that this overrides all the other possible interpretations that Harry had identified.

Similarly, while 'good presentation' is valued (although usually only "by the way" (Joan: 140)), it is also possible that a teacher may interpret some forms of presentation as detracting from the mathematical content of the work. Thus Harry, reading Sandra's Topples text, begins by praising her presentation:

Three dimension illustrations to start with, very nice, nicely presented.
(Harry: 190, Sandra)

but quickly moved on to criticise the form of the presentation:

Oh, she's colour coded anyway. She's used some sort of key. Um, not really necessary. She could have just put the numbers on, . . .
(Harry: 195-196, Sandra)

There is a suggestion here that the use of colour is an unnecessary elaboration. Using numbers to label her diagrams would not only have saved her time but might also be interpreted as a more abstract, and hence more highly valued, way of thinking about the problem. The presentation of Sandra's work was also commented upon by Charles in summing up his evaluations of all three Topples texts:

C Then you've got Sandra, well, . . . got some nice diagrams [. . .]
I You said that she had some nice diagrams.
C Yeah, I mean, well, artistic ones [laughs] I think perhaps she probably went a little bit over the top but I mean . . . there's nothing wrong with that. Er nicely presented work. In fact they've all presented their work pretty well.
(Charles: 235-247, Sandra)

In both cases, the presentation was the first feature of Sandra's text to be commented upon; in both cases the initial praise is then qualified. Being "artistic" or using colour seem not to be considered appropriate to the genre.

These qualifications of teachers' reactions to the presence of diagrams and elaborate presentation in students' texts imply that the context free approval of diagrams described above does not reflect the way in which the teachers actually read. Their advice to their students to "draw diagrams" may not be helpful to students as it does not provide any means of distinguishing between those uses of diagrams which will be approved and those which will be taken as signs of a low level of mathematical achievement or will be judged to have been a waste of time.

11.5 Statement of the problem: copying vs. 'own words'

Apart from Richard's Inner Triangles text, each of the students' texts on both tasks started with a statement of the task itself, either copied from the question paper or restated in the author's own words. Although two of the teachers passed over all these sections, all the rest commented on at least some. Amy and George both required such an introduction as a communicative device, commenting that it helped them to "follow" the text but making no apparent distinction between an introduction that was copied and one which was written in the author's own words. The general attitude, however, appeared to be that merely copying the task from the question sheet was a waste of time and might even be taken as a sign of laziness or lack of thought, while a statement in the student's own words was approved. Joan, beginning to read Clive's Inner Triangles text, exemplifies this general attitude:

Right, he's explained the problem in his own words so that's good. Um . . . possibly . . . a bit of a waste of time there although . . . no I'll take that back cos he has actually done a bit more work on the questions so he hasn't just rewritten them so that's okay.
(Joan: 83-87, Clive)

While in this and some other cases the 'own words' introduction is merely stated to be a 'good thing' in itself, there are some indications that other functions are ascribed to it. One of these functions is to demonstrate that the student understands the nature of the task:

Okay, so her introduction is a good start actually because she's laid down some of the rules of the investigation which is something that I can see that makes me aware that she understands what the task is.
(Harry: 32-34, Ellen)

It may also be taken as evidence that she has worked through the examples provided:

And the pupil has carried out the initial task that's given . . . done the initial task as was given on the sheet and checked that the information as given on the sheet was correct as they were supposed to do.
(Carol: 76-78, Sandra)

In the last example, it is interesting to note that the section of Sandra's text that Carol is commenting on was actually copied verbatim from the question paper. This is not, however, recognised by the teacher-reader who takes the copied words as a sign of material activity and of engagement with the task.

Carol was not alone in mis-identifying introductory statements of the problem as copied or written in the student's own words. Table 11.1 below summarises the judgements made of this section of each of the texts. Although in some cases, for example Steven's Topples text, the differences between the question paper and the student's version are subtle and may not be noticed or considered significant by the reader, it appears that even substantial differences between the versions may be neglected while, conversely, verbatim copies may be read as original. In every case, the question paper was in front of the teacher throughout the interview but it is not usually clear from the transcripts whether or not the teacher was making use of this.

Table 11.1

Identification of problem statements as copied from the question paper or composed in the student's own words

Student text	identified by teachers as:		remarked - no identification	no comment
	copied	own words		
Steven (Inner Triangles): own words	Fiona	<i>Dan</i>	Amy, George	Joan, Andy
Clive: copied apart from very minor errors	<i>Dan, Fiona</i>	Joan		Amy, George, Andy
Richard: none			Amy	George, Joan, Andy, Dan, Fiona
Steven (Topples): own words (a very close paraphrase, see b.5.1)	Jenny	<i>Harry, Grant, Carol</i>		Charles
Ellen: own words		<i>Harry, Grant</i>		Charles, Jenny, Carol
Sandra: copied apart from the first sentence.		Harry, Grant, Carol, Jenny		Charles

(Correct identifications are in italics.)

Although there are hints that different teachers have consistently different reading practices in respect to the problem statements, the number of texts read is too small to allow such generalisations to be made. However, it does appear either that there are different standards of what counts as 'copied' or that some of the teachers, in making their judgements about these sections, are using other aspects of the text as well as or instead of the actual words.

The relatively consistent readings of Sandra's introduction as being in her own words is particularly interesting. Sandra omitted the first part of the first sentence of the question paper "In this task you will be asked to", starting her introduction with the next words "Balance some rods of different length . . ." and continuing subsequently to copy both the words and the diagrams of the rest of the statement of the problem exactly. It is possible that some of the teachers judged this to be in her own words merely on the basis that the first sentence started in a different way. However, Grant actually looked more carefully at the whole introduction, suspecting that it might be copied, yet still concluded eventually that it was not:

Now this is a good introduction. This is the type of thing that would put me in a good mood before I started to mark it I think in terms of whoever it is just described what they got to do with pictures which always help. It's in fact a copy of that isn't it? Almost? No it's not. They've rewritten this in their own words which is fairly good I think.
(Grant: 48-51, Sandra)

The positive impression made by Sandra's immaculate presentation, using colour, elegant handwriting and a well-balanced layout of the page, may be a stronger influence on the reader than the actual words she has used. One explanation for this might be that the interpersonal effect of the presentation is such as to suggest a great deal of effort and personal engagement in the task, which is then interpreted by the teacher-readers as personal 'ownership' of the task.

In addition to a general wish by teachers to see an introduction for its own sake, the roles played by an introduction written in the student's 'own words' appear to include:

- to demonstrate that the student has understood the problem;
- to show that the introductory examples have been worked through;
- to display 'ownership' of the problem.

On the other hand, if the introduction is not perceived to have been rewritten in the student's 'own words', the student may run the risk of being condemned for wasting time.

The inconsistencies between the teachers' readings of these sections of text raise a problem for the student who wishes to demonstrate that she has 'understood the problem' or performed the required material actions. While paraphrasing the words given on the question paper may be the best strategy, this does not in itself guarantee that the teacher-assessor will recognise that it has been done. Where an introduction was missing, its absence was generally not remarked upon; it may, therefore, be possible to persuade a teacher-assessor that one has 'understood the problem' by other means.

11.6 Narrative

The value placed on processes in the discourse of investigation leads to a requirement that students should demonstrate those processes. One of the ways in which they appear to be expected to do so is through writing a narrative of their actions and/or thought processes. A failure to do so may be considered "suspicious":

I think to arrive at a formula like that you're not going to arrive at it straight off the top of your head first time. I think there'd be some attempts perhaps to look at other sets of numbers that might fit in with these. Perhaps looking at, I mean he's got square numbers coming hasn't he? He's got two a . . . Oh, right, I see why it works now [laughs]. . . Perhaps I'd have liked to have seen perhaps some more tables of values with some calculations along the way. "So I tried this and it didn't quite work, when I tried that I noticed this." Often what I get from kids is "oh well I just plugged away on my calculator and suddenly it was there." And I'm highly suspicious of it, you know. But then there are some kids who will set down and show how they got to it and I'm much more happy about that, that I've got something in front of me. Perhaps I'm wrong, I mean, because often kids can do these things in their head or they can suss it out without having to write it all down. But I like to see some evidence on the paper [laughs] That's something I can judge isn't it. It's very difficult to judge what's going on in somebody's head.

(Charles: 67-79, Steven)

The need for the teachers to assess “what’s going on in somebody’s head” is converted into a requirement for the students to externalise their thoughts in writing. Charles also has quite clear ideas of what sort of actions and thoughts Steven ought to have had; these include getting things wrong as well as performing calculations. Although he hedges the statement of his expectations by admitting that he may be wrong and that a student may in fact compose a formula without going through such processes, this nevertheless does not negate his demand to see ‘evidence’ in writing.

This demand for a personal narrative of thoughts and actions was made by most of the teachers. Some also gave more specific statements of what they considered to be appropriate and inappropriate types of narrative. For example, extract No. 1 (see Appendix 6) contained a personal narrative aspect that was strongly criticised. The temporal themes in this extract (*Once . . . , First . . . , In a very short time . . .*) and the use of the past tense structure it as a story of what the author and her collaborators did. This does not, however, satisfy its teacher-readers’ expectations. Andy, for example, found the lack of specificity alarming:

. . . it’s this “In a very short time” I must have read that a thousand times and “We suddenly discovered that” and that always rings alarm bells. I’d like in their own words to say what ab plus ac actually means. What they’re actually doing to, I’d love to know geometrically what they were doing to produce that - as evidence.
(Andy: 335-340, No 1)

Although he claims to want to know “what they’re actually doing”, Andy appears to be seeking insight into the author’s thought processes rather than her actions. In his advice to another student, Andy provides examples of the sort of narrative he would prefer to see:

Once he’s found, he’s got the data say “I’ve noticed this pattern and I predict that this is going to happen or this is the pattern which I can see”
(Andy: 281-283, Richard)

The author’s processes that Andy wishes to see in the text are all mental processes: *I’ve noticed, I predict, and I can see.*

In a similar vein, Joan suggested that the type of narrative provided by No. 1 is a sign of an immature writer:

J It’s a rather anecdotal description of what they did, but er again that’s something to do with maturity. Um . . .
I Don’t you like it being anecdotal?
J That’s one of my pet hates. It’s the sort of, you know, “Miss put this task on the board and we copied it down” but not quite that bad. . . “In a very short time we had discovered a relationship . . . we were able to put this into a formula . . . and we checked it to see if it worked”
(Joan: 297-303, No. 1)

Although Joan’s own written advice to her students included an admonition to “Write down what you did in the order that it happened” (Joan: 444), this student’s attempt to do so is not acceptable. Later in her interview, Joan recognised this tension and provided some examples of the sort of narrative that might be more valued:

it's difficult though isn't it to get the balance between what I'm saying about being too anecdotal, and giving every single detail of what you did, and not leaving out anything important. it's very very difficult to get the balance and I'm quite sure that even if I was doing an investigation I'd never get it quite right. Um, what I encourage my pupils to do is as they're going through their work to put little think bubbles at the side so that whenever a thought occurs to them "I've noticed a pattern" or "I wonder if this will work for squares" or whatever, that they will actually put that in at the side as they go along because it saves having to write it out in a sort of essay form and yet the evidence of their thinking or their thought processes rather is there as you're going along.
(Joan: 367-381)

Again it is the students' thought processes (*I've noticed, I wonder . . .*) that are required rather than the discovering, putting into a formula and checking that were described by No. 1. Several of the teachers provided examples of the sorts of statements that they approved or did not approve. These are summarised in Table 11.2. Some statements were quoted wholly or partially from the student's text that was being read; others were constructed as examples of good or bad types. Where 'bad' examples were constructed rather than quoted they often appeared to be intended facetiously.

Most of the examples that are condemned by the teachers are simple, single clause statements of single actions or strings of such statements connected only by 'and'. Many of them also merely declare the discovery or the existence of a result. On the other hand, the examples that are recommended tend to be more complex, expressing causal relationships between the actions.

Although the teachers tended to express their demand for such narrative writing in terms of wanting to see what the student did, a closer examination of the types of writing that were approved and of the more elaborated reasons that were given in response to probing during the interview suggest that there are actually several different functions that such writing is expected to fulfil:

- it should provide not only description of the student's actions but also the reasons for taking them;
- it should provide a basis for conclusions made in the text;
- it should give enough detail to be taken as evidence that the results are the student's own work;
- its existence fulfils an examination requirement.

Table 11.2
Teachers' examples of statements of processes

teacher	approved	disapproved
Joan	I've noticed a pattern. I wonder if this wil work for squares.	Miss put this task on the board and we copied it down. In a very short time we had discovered a relationship . . . we were able to put this into a formula . . . and we checked it to see if it worked.
Andy	I have noticed that, if you take the, if you add the parallel lengths together and you multiply by the slant height, then it gives us the area I've noticed this pattern and I predict that this is going to happen or this is the pattern which I can see. Do the top times the side and then the bottom times the side and the two numbers added together gave us . . .	In a very short time . . . We suddenly discovered that . . .
Dan	I then looked at different shapes and I kept the slant length the same, the same while the top length was varied and I got down the number of triangles that were in that shape and when I'd done this table I then looked carefully at the results to see if there was a pattern, a pattern I recognised and this was . . . and so I eventually came to this formula.	In a short time we discovered it. I have found this formula
Harry		I've found another formula.
Charles	So I tried this and it didn't quite work, when I tried that I noticed this.	Oh well I just plugged away on my calculator and suddenly it was there.
Grant	I'm going to use my base as one and then increase that by one each time. I found a formula earlier so I'm going to use it on a hundred and see what my answer comes up to.	This is the formula
Carol	My rule appears to be, I am going to predict it.	

A further function that the narrative appeared to serve in some cases was to help the text to be read as a coherent whole. For Charles this seems to be related to the desire to see the student's thought processes. He compares Ellen's text with Steven's:

There seems to be a little, well I don't know, even the word she's given seems to be a little bit more build in it. Seems to be more of a thought

process going on. I mean she's come up with an idea early on about, she's got one idea about something that's working for rods of length two and then later on she's found out something else so there's a little bit of progression in that one but this seems to have a gap and then suddenly suddenly you're diving in and something's happened and you don't know where the . .

(Charles: 195-200, Ellen vs. Steven)

Whereas Steven's formula was merely introduced with the words "I have found a formula" (and was criticised by some of the teachers for this), Ellen's text included an early attempt at a formula that only applied to even numbered rods. She had constructed cohesive links to connect this section, introduced with the words:

I thought I had a formula but it only worked for even numbers

to the next section which contained a formula which worked in all cases:

THE REAL FORMULA - works for odd & even numbers.

In examining Charles's reading of these two texts it is difficult to separate his judgement of the content of the texts from the influence of the level of coherence. However, he does appear to be talking about both the "thought process" and the words used to express it which provide "a little bit more build". Similarly, Andy wishes to see words in Richard's text:

*I would like to see some words between the results and the formulae. (. . .)
This way you get a nice systematic progression through the piece of work.*

(Andy: 280-285, Richard)

Again the narrative is required to provide coherence in the text.

Although there was general agreement that narrative was required and a certain amount of similarity in the forms which were approved by the various teachers, some expressed concern about the relevance of the requirement. While expressing his unease with the examination board requirement for 'writing', Dan identified the source of his discomfort as a perception that students who are good at mathematics may not have matching competence in writing:

'Explain your strategies' is one they find quite difficult, cos they know what they're doing and they do it quite efficiently. To actually put down on paper . . the strategies they're using. I think sometimes they find it too - especially with an intelligent child - they find it too mundane to actually say well this is what I did and so on. And some children have very good mathematical skills and can see through things but actually their English skills don't match it in terms of explaining what they do to go on.

(Dan: 37-43)

As well as suggesting that the requirement to include a narrative of the process of solution is perhaps not relevant for the "intelligent child", he is making a clear separation between mathematical competence and 'explaining', suggesting that he places more value on the 'product' of the investigation than on the processes. Carol also appeared to experience difficulty in resolving her judgement of the mathematical value of a piece of text with her judgement of the language used in it:

I mean the immediate thing that I notice when I read up is that this appears, this is actually telling me what the person's actually been doing, it's not just results as was in Sandra's more. It immediately communicates more. As I

say we're not supposed to be assessing their English skills but it does help with marking it if they tell you what they've been doing, even if their spelling is awful and their English is awful, but the trouble is the one's with English are awful are reluctant because they've got hang-up about that

(Carol: 331-335, Steven)

She, however, acknowledges the role that the narrative of “what they’ve been doing” plays in influencing her own understanding and assessment of the text and recognises an interaction between achievement in the two fields.

The narrative in the students’ texts appears to play the following roles for teachers:

- to demonstrate what the student has done;
- to demonstrate what the student was thinking;
- to provide reasons for the student’s actions;
- to provide a basis for the student’s conclusions;
- to provide evidence of the student’s ownership of the results;
- to fulfil examination criteria;
- to provide coherence in the text.

There is a tension between the common assumption that the narrative provides a window onto the student’s thoughts and actions and the equally common perception that the writing actually produced by students is inadequate for this purpose. Those teachers who question the requirement to provide a narrative of the student’s processes appear to be placing greater value on the mathematical ‘content’ that may be displayed in lists of results and formulae than on the ‘processes’ that are displayed through narrative. This is perhaps in tension with the more general value placed on processes within the discourse of ‘investigation’.

11.7 Explanation

Both the Inner Triangles and the Topples tasks asked students to ‘explain’ their results and ‘explanation’ was one of the features of texts whose presence or absence was frequently remarked upon by the teachers. One of the difficulties in looking at teachers’ readings of ‘explanations’ in students’ writing, however, is the multiplicity of ways in which words such as *justify*, *explain* and *prove* are used. They are used in different ways by different teachers and, in some cases, an individual teacher may use one of the terms in a variety of ways. In particular, they are frequently used to refer to a narrative of the processes gone through in achieving a result or to a set of examples demonstrating how a formula may be applied or demonstrating that it ‘works’. ‘Explanation’ in the form of a narrative of processes explaining how a result was achieved has already been discussed in the previous section. In this section, only those references to explanations which address the question of why a result is as it is will be considered.

The performance indicators provided by the examination board make it clear that, although all students are asked to explain their results, it is only expected that the highest achievers will

actually succeed in doing so (see chapter 7). This identification between explanation and high achievement is also made by the teachers. In some cases it is made by explicit reference to assessment criteria. Joan, for example, used the statements of attainment in Attainment Target 1 of the National Curriculum:

Did he try and justify it? . . . Yes because he said 'a triangle is like a trapezium but without a top' so there is some element of justification there. So it's definitely a level 6.
(Joan: 120-122, Clive)

while Fiona identified explanation with an A grade:

For the really, the more able, if they found a formula can they explain why it works, and then I would have given probably an A, certainly an A grade.
(Fiona: 19-22)

The 'Inner Triangles' texts read in the interviews contained very little that could be considered explanation in this sense. Its absence was remarked upon in only a few cases, suggesting that it is not a general expectation of all students. Dan's comment on the lack of explanation in Clive's text:

I would have thought he [Clive's teacher] could have clued into a kid who was working at this level to say something about why.
(Dan: 243-244, Clive)

appears to imply that, because Clive is assessed to be at a high level on the basis of other evidence within his text, he ought also to be producing explanation. The relationship of high achievement to explanation is thus a two-way implication.

One of the questions posed in the 'Topples' task specifically asks that the student should explain the results they have achieved through their practical work.

e) Explain your result. (Well argued explanations based on intuition and insight will gain at least as much credit as those based on the principles of Physics.)

The mention of the "principles of Physics" makes it clear that this demand for explanation is expecting the student to make connections between the numerical results and inductive generalisations achieved through practical experimentation and the nature of the physical phenomena involved. There is a tension expressed between the value that is being laid on the form of the argument and the physical validity of the content of the argument. The capitalisation of "Physics" confers a status that seems at odds with the apparent valuing of "intuition and insight". This tension is also apparent in the teachers' interviews, both in their readings of students' attempts to provide such explanations and in their general comments about their expectations.

Two of the three texts read during the interviews included responses to this question. Of the two attempts at explanation, Ellen's:

The smaller the unit at the bottom the more likely the load will fall quicker. ie a one unit rod can't even balance a 2 unit rod, yet a 3 unit rod can balance 8 units.

was largely ignored or dismissed by the teacher-readers. Although the fact that she had made an attempt to provide an explanation was appreciated, those teachers who commented on it at all suggested that they would not lay much importance on it. For example, Carol dismissed it as "intuitive"

there is an intuitive feel for um an explanation of the original situation um and they've got a got a she's got a generalisation and there's an intuitive kind of justification and the National Curriculum says giving some degree of justification, a lovely phrase 'some degree'. Um I would say enough to show that the pupil's reflecting on what's happening and trying to get a feel for the situation.
(Carol: 307-311, Ellen)

While Grant's judgement of the explanation was again related to the 'ability' of the student:

a low ability pupil I'd give some credit for that, a high ability pupil I would expect to mention it towards the beginning of the project but not necessarily give credit for it.
(Grant: 227-229, Ellen)

Ellen is credited with effort but not with any degree of understanding. Steven, on the other hand was judged more highly. His explanation not only uses a vocabulary that identifies it as 'Physics' (i.e. weight, gravity), but also makes it explicit that he is answering the question 'why', starting with a statement of a causal relationship:

The reason that the pile topples could be because the weight over the starting pile becomes too much and gets pulled down by gravity. . .

Those teachers who spent time reading his explanation appeared to find it difficult to make sense of it. Thus Grant, for example, struggled to quantify the explanation in order to make it fit in with the numerical results achieved through experiment and in an attempt to understand the physics of the situation himself:

'There's too much weight on the right hand side so the pile topples over'. Um, he could have gone a bit further into it. He could have counted the units of weight and perhaps given an example for this one being perhaps, what would that be - two - so that would be eight on one side and er seven on that side, so is that going to topple? Two three four five. It will won't it, so it doesn't work in that one but it should do shouldn't it? Am I right? No, cos it would depend on how much was above the base, it wouldn't depend on the base one so much, yeah. So he could perhaps have gone into that a little bit more. He's obviously quite a good pupil in terms of his thought processes about it, but could have explained more.
(Grant: 137-150, Steven)

Although Grant judges the explanation itself to be inadequate, he nevertheless credits Steven with being a "good pupil". It appears that the form of the explanation as well as the validity of its content contribute to the evaluation of the argument; this distinction between form and content is made explicitly by Charles:

I think any argument would be good so long as they got a good explanation and a good argument as to why it might be the case. I'm not too worried about it necessarily being correct but I would be more interested in them whether they could actually argue the point for themselves. So I would be looking more in the assessment on the strength of the argument than actually what they're trying to put across.
(Charles: 29-34)

The explanations provided by Ellen and Steven differ in both form and content. While Steven's is clearly in the form of an argument, stating a causal relationship, Ellen's statement

describes a general association between two states without making any explicit claims about causality. At the same time, Steven makes use of the vocabulary of physics, suggesting that he is searching for explanations outside the confines of the task itself, while Ellen has confined herself to terms used within the question paper itself. The data available is too limited for it to be possible to determine which of these factors plays the greater role in influencing the teachers' judgements of the value of the argument. It does, however, appear that students would be well advised to use linguistic forms that clearly signal that they are making claims about causality, whatever the validity of the explanation they are attempting to provide.

The roles of 'explanation' of the reasons for the results are two-fold:

- it is a sign of high mathematical achievement (on the way to 'proof');
- it shows ability to construct a coherent argument.

There is a tension between these two roles that is related to the tension between content and process in the 'investigation' discourse as a whole. In many of the coursework tasks with which students engage, a mathematical proof of the result is likely to be beyond the capability of the vast majority of the students. This is signalled to both teachers and students (see chapter 7). On the other hand, a substantially larger group of students is expected to be able to produce some form of argument in an attempt to 'explain'. For most students (and their teachers), therefore, the production of an argument in an acceptable form is of greater importance than producing a 'correct' explanation.

11.8 Use of a conventional mathematics register

In addition to the comments on various forms and sections of text discussed above, there were a number of specific words or phrases in the students' texts which were identified by teachers. Usually this identification indicated that the usage appeared to be considered incongruous; this was signalled by an explicit comment, by the tone of voice used while reading aloud from the student text, or by laughter or facial expressions accompanying the reading. The types of items identified in this way suggest that the incongruity may have arisen from conflict with the conventions of vocabulary and tenor of formal writing in mathematics. There were also a small number of general references to a student's style of the writing.

There is clearly an expectation that students should use conventional forms of mathematical vocabulary although this expectation is not always strongly enforced. Dan, for example, was the only reader who identified Clive's use of the term "conversion table", commenting that it didn't convert anything. Similarly, Jenny was the only teacher who objected to the way in which the word 'unit' was used in the Topples texts, complaining that it was not used "in a very mathematical way" (Jenny: 112, Ellen). Jenny also objected to Sandra's use of the

equals sign as an operator (Sandra had several statements of the form $50 - 2 = 48 + 2 = 24$). The use of succinct descriptors for variables was desired by several teachers; for example, Steven's variable name was read disparagingly as "the distance along the bottom, as he describes it" (Andy: 94, Steven). Similarly, Grant indicates that he sees a hierarchy of forms of variable names:

Yeah. I think that, well with higher ability pupils it [using letters] is [better]. We are looking for more of the good algebraic work from them. If this had come from a low ability pupil, writing this, which is algebraically correct and well written even down to using top length instead of the top of the trapezium, it cuts it down what they've got to write down for the formula and shows some knowledge . . . so if I was to get that from a low ability pupil I would be well pleased
(Grant: 298-302, No. 2)

While a single letter name is clearly preferred by all the teachers, labels such as 'top length' are also acceptable (see chapter 12). One feature, apart from its shortness, that might distinguish the acceptable 'top length' from the less acceptable 'top of the trapezium' or 'distance along the bottom' is the objectification of the measurement. This distances the quantity from the physical object that gave rise to it, thus placing it at a slightly higher level of abstraction.

Teachers reacted with what appeared to be affectionate humour to some of the more informal and personal aspects of students' texts. Ellen's heading "THE REAL FORMULA" was marked by laughter from Carol, while Jenny laughed at Steven's expression of confidence:

Well I like his enthusiasm that he thinks 'with such a definite pattern a formula should be easy to find.'
(Jenny: 22-23, Steven)

Such expressions of attitude are not a common part of most other forms of mathematical writing. Similarly, statements about the way in which the student worked (in a group or individually) were greeted with laughter by Fiona and with rather condescending approval by Dan:

It's quite nice when they say things like that as well 'a formula that our group worked out'
(Dan: 208-209, Clive)

Unlike the expected narrative of the student's processes, this mention of the group provides the reader with access to the context within which the student's activity took place. Although the reaction of laughter suggests that the teachers see such inclusion of the personal and contextual as incongruous (or at least unusual), the data available is not sufficient to make it clear how this may affect their evaluations of the text.

Aspects of students' texts which some teachers appeared to find incongruous thus include:

- the unconventional use of specialist mathematical vocabulary;
- the use of the equals sign as an operator (in situations where the expressions on either side of the sign are not strictly equal);
- lengthy variable names, maintaining a close relationship to the concrete referent;
- explicit expressions of confidence or other attitudes;

- reference to the context within which the task was done.

The relatively small number of teacher comments on the phrases identified in this section does not necessarily indicate that other teachers were not influenced by these aspects of the students' texts. The teachers' attention was not deliberately drawn to specific features of students' texts during the interview, nor were they asked to comment on style of writing. Where they have identified specific features, this suggests that they are conscious of the effect these have on their evaluation of the student's work.

More general comments on 'style' suggest that, even where the teacher cannot articulate the nature of the stylistic incongruity they experience, this is likely to affect their assessment. Dan, for example, commented repeatedly on his discomfort with Steven's text, stating in a non-specific way that "the writing's not all that marvellous" (Dan: 271, Steven) and eventually linking his evaluation of the standard of the writing to a more general evaluation of Steven's ability:

He writes perhaps not at the same level as the others . . . on intellectual terms
(Dan: 281, Steven)

and concluding that Steven, although he had produced "a good sound piece of work" was a "not particularly able mathematician" (Dan: 314, Steven). It is possible that the use of variable names such as "the distance along the bottom", discussed above, may have been one of the features which contributed to Dan's discomfort with Steven's writing. Similarly, Dan compared two of the extracts from Inner Triangles texts (see Appendix 6):

Number 2 gives me the impression they obviously know what they're talking about whereas this one, although it says almost exactly the same thing in different words, er, it doesn't give me the same impression.
(Dan: 361-364, No. 2 & No. 3)

The comparable parts of the two texts, as Dan said, say almost the same thing in slightly different words. Obvious differences include the fact that No. 2 had included two examples and had used verbal variable names, while No. 3 had used algebraic symbols for her formula. Dan had commented on these differences earlier, claiming that they did not greatly affect his assessment of the students although he would wish to advise each of them to include the features found in the other's text. His "impression" appears to be based rather on the verbal descriptions of the procedure which, as he says, appear very similar in form. There are, however, a number of aspects of the two texts which may have affected Dan's reading:

- The use by No. 3 of *times* rather than *multiply* is less formally 'mathematical' and may be read as a remnant of the early years of mathematics schooling and hence a sign of immaturity.
- No. 3's procedure is more 'wordy' using *add together* rather than simply *add*, and *times it by* rather than *multiply by*. The number of unit triangles is also qualified as being *in that trapezium*. A common characteristic of these additional words is that they include

reference to the concrete lengths, numbers or shapes. The procedure may thus be read as being at a lower level of abstraction.

- No.3's use of *you will end up with* rather than *you get*, by using the future tense, also suggests a more concrete procedure, located in time.
- The presentation of the final formula by *You can write this as . . .* presents the symbolic formula merely as an alternative to the verbal procedure. No. 2's announcement *This therefore is the formula*, on the other hand, displays the formula as a product in its own right which follows logically from the procedure rather than merely being equivalent to it. This may be read as an indication that No. 2 has a better understanding of the importance of the relational formula in mathematics, even though she has not used algebraic symbols to express it. The contrasting modality of these two statements also suggests that the two students differ in their levels of confidence.

Any of these features might have contributed to an impression that No. 3 is less competent mathematically. While it is not possible to say precisely which aspects of the writing contribute to Dan's assessments of Steven and of extract No. 3, there is clearly a mismatch between the students' texts and Dan's expectations which appears to be affecting the teacher's evaluation of the whole of the students' performance¹.

This analysis of possible sources for Dan's different evaluations of extracts No. 2 and No. 3 points to the subtle nature of the relationship between the linguistic form of the text produced by the student and the teacher's evaluation of her general intellectual 'ability'. As Dan himself was unable to identify the features of the text which gave rise to his impressions it seems unlikely that he would be able to provide advice to a student on how to produce an acceptable text. Moreover, his identification of the style of writing with 'intellectual level' and with 'knowing what they're taking about' suggests a view of language as the transparent representation of thought. It is a logical consequence of such a view that 'improvements' to the text can only follow (and will necessarily follow) developments in the students' thinking about the mathematics. A teacher holding this view of language is unlikely to consider it necessary or useful to address the form of writing with his students.

11.9 Coherence: a case study

In the discussion above, the teachers' readings of features of students' texts have been largely isolated from the rest of the text. This is justified to some extent in that it reflects the way in which many of the features were read; they were remarked upon as data or 'evidence' or as performance indicators in their own right rather than being seen as part of a coherent

¹It is, of course, possible that, in the context of a whole text rather than these brief extracts, such features might be read differently or carry less weight in Dan's overall evaluation.

argument. However, teachers read and assess whole texts. In this section, therefore, the ways in which meaning may be made of more extended passages of text are considered through an analysis of the readings by one teacher, Andy, of extracts from the Inner Triangles coursework by two students whom he had taught himself, Clive and Richard. These extracts (see Appendix 13) differ substantially in their degree of explicit cohesion.

11.9.1 Andy reading Clive

In this section (which is identified as starting with the first diagram and finishing at the bottom of the page), Clive has integrated diagrams and verbal text using several cohesive devices. It is linked explicitly to the previous section of the text with the words "Also my formula is the one above but mine is below", indicating that it should be read as a "formula" and that it forms an alternative or an addition to the previous section. In the verbal text between the two diagrams, references to "the numbers" and to "the next row" make explicit links between diagrams and verbal text; the words do not have any reference without the diagrams to provide context. Similarly, the use of the pronoun "It" must refer to one or both of the diagrams as there is no previous verbal subject for it to refer to. The final verbal statement "but this didn't carry on" also appears to refer to some aspect of the pattern of numbers displayed in the diagram next to it. This statement, moreover, brings the section to a close; the echoing of the beginning statement "but mine is below" by "but this didn't carry on" forms a frame around the two diagrams and the intervening verbal text which further marks this as a self contained coherent section of the whole text.

The second diagram is a copy of the first one with the addition of further annotations in the form of lines separating out pairs of rows (thus forming trapezia of slant length 2) and brackets with numbers showing the sums of the numbers of triangles in each pair of rows. The addition of numbers is referred to in the intervening verbal text, as is the trapezium of "2 top, 2 slant and 4 bottom" which is formed by the top two rows of the diagram. It thus appears that Clive is presenting both his solution (or "formula"), in the form of a diagram that "can also tell you the answer", and an explanation of how this solution is constructed, deriving the second diagram from the first.

This analysis of the cohesion of this section of text was achieved by paying explicit attention to cohesive devices used within the verbal part of the text and to similarities and differences between the two diagrams. In doing this, not only is the formal linguistic cohesion of the text described but also the logical coherence and textual function of the section is clarified. It is not, however, an easy section of text to read and make sense of. Factors contributing to this difficulty include: the apparent inaccuracy of the first statement that adding the numbers gives you "the next row of numbers"; the discontinuity between this sentence which describes a pattern within the diagrams and the next one which makes connections between the diagrams and the original trapezium problem; some technical errors in punctuation and lack

of capital letters; the elliptic “but this didn’t carry on”, which suggests a weakness in Clive’s mathematical solution without making it obvious what “this” might be². Rather than merely reporting what he has done, Clive has attempted to describe both a pattern and a general procedure. The task that he has undertaken is perhaps a difficult³ one and this may account for a greater difficulty for the reader in making sense of it. The lack of control of the written language displayed in Clive’s verbal text may contribute to the way in which his teacher, Andy, reads and assesses it.

He’s illustrated the difference in the area - a nice attempt to show that as you increase the slant height by one so the area increases by two each time. He’s tried to do that all on one diagram. It’s not easy but it’s er, I can understand what he’s getting at there. And he’s tried that then for a . . . a second occasion.
(Andy: 146-150, Clive)

Andy is not reading this section as an integrated text. While the first diagram is analysed and identified as a form of communication, providing an explanation of an observation about the trapeziums, he makes no reference to the verbal text and the second diagram is interpreted as a repetition or variation rather than a further step in the argument. The brevity of this extract from Andy’s interview indicates the speed with which he passed over the whole section. Given the difficulties that might be involved in reading the verbal part, he is unlikely to have followed the logic of Clive’s text in this time. This fragmented reading is paralleled by the ticks with which he had originally marked Clive’s work. The page is structured visually as four parts of almost equal size: writing, diagram, writing, diagram. Each of these parts has a tick next to it, suggesting that Andy was validating the form of each part rather than the content.

While he expresses approval of the first diagram, this approval is qualified by the modality of the whole passage. Clive has made a “nice attempt” and he has “tried”. By repeating this theme of “trying”, Andy suggests that Clive has not been very successful. He is getting credit for effort rather than for the quality of his work.

²While such deixis is one of the devices which creates cohesion in a text by forming links between consecutive statements, in this case its reference is outside the text. This is typical of spoken rather than written discourse (Halliday, 1989); it makes an assumption that the audience shares a complete knowledge of the context. Where this assumption is not justified it contributes to the difficulty that a reader may have in making sense of the writing. Hoey (1986) identifies the major problem with secondary school children’s write-ups of science experiments to be an assumption that any reader will share a complete knowledge of the context.

³There is some evidence that children find structuring their writing more difficult when it is not time related (Harris, 1986). A description and explanation is thus likely to be less well structured than a narrative of what was done.

11.9.2 Andy reading Richard

Richard's section of text differs in a number of ways from Clive's, several of which may contribute to the differences in Andy's readings. Firstly, it is more clearly defined than Clive's: it is on a separate page with a heading (although there is some ambiguity about whether the heading for the page is "Working out" or "Stars"). There is no explicit reference in this section to any other part of the work although when seen within context its structural similarity to other pages contributes to the coherence of the text as a whole. Also unlike Clive's text, there are few cohesive ties within the section; the labels on the large star, S.H and T, are repeated in the formula but apart from that it is left entirely to the reader to make sense of the connections between the various parts of the page. There is a line drawn apparently connecting one of the "working out" star diagrams to the table which might be interpreted as a cohesive link suggesting that the table contains data taken from the diagram; it might also, however, be read as a line drawn in error that has been partially covered by the paper containing the table that has been stuck over it. As in the rest of Richard's text there is a complete lack of any verbal language other than disconnected headings and labels. While this characteristic of his work was heavily criticised by his teacher-readers (see chapter 10 and section 11.2 above), this does not seem to have detracted from the ease with which Andy makes sense of this section.

- A *Oh, some stellations here! What have we got for this? . . . Slant length times perimeter . . . aha! Yeah, now here he's made a jump which needs to be made clear because the unit triangles have now changed. The actual size of the unit triangle has now changed from . . . the unit is now a two by two by two triangle*
- I *Oh, is it? I hadn't noticed that.⁴*
- A *We've got, on this one here, the perimeter's twelve, he's counted that as being one here*
- I *I see, yes*
- A *And the slant height as well, it's just the length of one side*
- I *Yes*
- A *And so he's actually taken different size unit triangle which I think he should make clear about. Whoever's marked this should have made a comment about it [said with irony]*
- I *So do you think that this star corresponds to the first one rather than to that*
- A *I think he's done that. That's my first [...]. Because the perimeter's twelve and it's, and I think he's taken that as one unit so that would seem . . .*
(Andy: 218-234, Richard)

In contrast to his method of reading Clive's text, Andy is making strong connections between the diagrams and the table. In particular, he is making inferences about the nature of the correspondence between the smallest diagram and the first entry in the diagram. In assuming that the first entry (perimeter 12 and slant height 1) refers to this diagram (perimeter

⁴Although the interviewer's interventions lead Andy to expand on and clarify his interpretation of Richard's method, his first utterance clearly indicates that he had already formed his belief that the size of the unit triangle had been changed.

24 and slant height 2) he is forced to conclude that Richard is using a different scale for his diagrams. The necessity of this assumption might be questioned: there is no labelling on the diagrams to indicate such a change of scale; there are no explicit links between the diagrams and the table; while there are four entries in the table, there are only three diagrams and two of these diagrams could be taken to correspond directly to the second and fourth entries while the third could be read as a generic star being used to demonstrate the reference of the variable names rather than as a specific example. However, the information structure provided by the left to right orientation of the page (Kress & van Leeuwen, 1990) may contribute to the assumption that the diagrams (left hand side of the page, hence 'given' as fundamental data) have given rise to the table and formula (right hand side of the page, hence 'new'). They are thus read as different, perhaps progressively more sophisticated, representations of the 'same' data.

This section of Richard's text, while lacking the 'writing' expected by the general assessment requirements, is nevertheless consistent with another of the conventional expectations of the coursework genre. Its three parts (diagrams, table, and formula) may easily be identified with the investigation algorithm: generate data, tabulate it to look for a pattern, generalise. Within the expectations of the genre, the page can thus be read as a narrative of what Richard did, i.e. he drew three examples, he put the values derived from these examples into a table, he formed a fourth set of values based on the pattern he observed in the table, he derived a general formula from the pattern. As the text appears to conform to this algorithm it is not surprising that Andy should make sense of it within this convention.

11.9.3 Comparison of the two readings

In reading both these sections of text, Andy has constructed a story about what the author was attempting to communicate, identifying in each case a difficulty or lack in the text but expressing no doubt about his own interpretation of the student's thinking. The amount of use he makes of the text, however, differs considerably between the two texts. In the case of Cl've, the first diagram is interpreted by itself while the verbal text and the second diagram are passed over with little comment; apparently no attempt is made to use the diagrams to make sense of the verbal part or vice-versa. When reading Richard's text, in contrast, Andy clearly makes an assumption that the diagrams and the table are intimately connected and, indeed, works hard to establish that connection himself. The degree of integration of diagrams into their context during the reading of these two sections actually appears to be in inverse proportion to the amount of integration in the form of explicit cohesive links provided by the authors.

As suggested above, the coherence that Andy constructs in Richard's text may arise from its close adherence to the form of the 'generate data - tabulate - generalise' investigation algorithm (see Appendix 15, section 3). This investigation algorithm itself provides cohesion

in a student's text by providing a conventional assumption that the juxtaposition of the various parts implies equivalence in their content. Thus the table is read as a re-presentation of the information contained in the diagrams and the formula as a re-presentation of the pattern formed within the table. The section of Clive's text does not conform to this convention either in the mathematical processes he is writing about or in the relationship between the various parts of the section. His verbal text does not merely repeat and reorganise information provided in the diagram; rather it attempts to explain how the diagram may be used to gain further information. Andy's reading, however, firstly assumes that Clive is only trying to describe a pattern (which is an appropriate thing to do within the conventions of the genre) and then identifies the second diagram as a repetition of the first. There appears to be a mismatch between what Clive is attempting to do in his writing and the way in which his teacher is reading it.

In spite of the fact that Andy and the other teachers who read Richard's text criticised his lack of writing or 'explanation', none had any difficulty in making coherent sense of his work. He is given credit for mathematical understanding, achievement and even ability. Clive's work, on the other hand, appears to be more difficult to read and he is given credit for effort rather than achievement. While 'explanation' is explicitly desired by the teachers, its linguistic complexity creates difficulties both for students and their teacher-readers. Thus, in spite of the cohesion within Clive's text, Andy focuses on the diagram as a self contained entity and does not attempt to construct a coherent reading of the whole section.

11.10 The teachers' 'ideal text': a summary

All the teachers interviewed expected to see 'words' or 'sentences' in the coursework texts. This appears to be a generic requirement, irrespective of the teachers' need to understand the text, and is in some cases justified by appeal to the authority of the examination board. There is some tension expressed between this requirement and a recognition that students' levels of proficiency in writing may not match their mathematical achievement. A major part of this writing is expected to form a narrative of the student's thought processes; examples provided and approved by teachers suggest that the processes which should be displayed include in particular 'noticing' and 'predicting'. The narrative serves a number of functions for the teachers: it is read as providing evidence for assessment purposes that such thought processes have taken place; it may be seen to provide evidence of more complex reasoning by justifying further actions, although in this case a more complex syntactic form is required that indicates logical links between actions rather than merely concatenation; it also serves to construct the text as a coherent whole. Non-specific actions such as 'I found' or 'I discovered', unless elaborated by some indication of the processes giving rise to the discovery, are not likely to be valued and may be considered 'suspicious' or taken as a sign of immaturity. In a few cases, it is recognised that non-verbal forms such as diagrams or

calculations may be acceptable or even preferable to verbal forms where it is perceived that the message is difficult to express using words only. On the whole, however, such forms are valued as an additional form of communication and are not generally an acceptable substitute for words.

While reading the texts, tables were generally marked with approval. Their inclusion was also frequently mentioned in a more general way during the interview as an expectation or as advice for students. For some teachers, tables are explicitly used as a sign of fulfilment of the assessment criterion 'working systematically'. Where the systematic nature of the work is identified with the order in which the data was gathered, this has the consequence that only a one dimensional table will provide suitably ordered evidence. Other forms of table may be recognised as evidence of possibly 'higher' levels of thought by some teachers, but it seems possible that, if two dimensional tables are used, other additional forms of evidence of systematic working need to be included.

Similarly, diagrams were generally considered to be a 'good thing' but there were also indications that in some circumstances the use of diagrams might be considered a sign of a more concrete way of working and hence of a lower level of achievement. In particular, the presence of naturalistic diagrams may influence a teacher's reading of the student's activity.

An introduction of the problem task is marked with approval by most teachers if it is perceived to be written in the student's 'own words'. While any introduction may serve the function of helping to create coherence for the reader, a teacher who is familiar with the problem may not require this. (The extent to which teachers expect to be treated as 'naive' readers varies between individuals.) However, an introduction which is read as being a direct copy of the words of the task given on the question paper is not likely to be valued and may even be condemned. An introduction read as being in the student's 'own words', on the other hand, appears to be used as a sign of fulfilment of the assessment criterion 'understanding the task' or as a sign that the examples provided on the question paper have been acted out by the student. Whether such a section of text is read as a copy or as 'own words', however, apparently does not depend entirely on its degree of similarity to or difference from the original text. It is possible that student ownership of the task (i.e. understanding and involvement) can be signalled by the quality of the visual presentation as much as by the choice of words.

Explanation of the reasons for a result is highly valued. This is consistent with the official place of proof in the assessment of coursework, defined by the London examination board:

The ability to explain or prove these generalisations, relate the generalised form to the geometry of the experimental mode (a proof) should **always** be deemed to be an **extension** and - if correct - worthy of a very top grade A.

(ULEAC, 1993: 21, original emphasis)

As a 'correct' explanation is only expected of a small minority of students, those who are aspiring to provide explanations appear to be judged on the form of their argument as much as on its content. Although the data available here is very limited, there are indications that explanations are likely to be more highly valued by teachers if they contain explicit claims about causality, that is, if they make use of terms such as 'because', 'the reason is', etc.

Although conventionally correct use of mathematical vocabulary and symbols is generally valued by teachers it is not necessarily required by all. There may be different expectations for students perceived to be working at different levels. The use of algebraic symbolism, in particular, is taken as a sign of the 'more able' student. (This issue is addressed in greater detail in chapter 12.) The analysis of possible sources within extracts No. 2 and No. 3 for Dan's assessment of their authors' levels of understanding suggests that the degree to which the section of the text containing the generalisation is abstract or concrete, signalled by the amount of reference to objects and by the tenses used, may be significant in affecting a teacher-reader's general impression of a student.

In contrast to the impersonal formality of academic mathematics writing, the requirement for narrative discussed above expects students to include a record of their individual actions in their coursework text. Other intrusions of the personal into the students' texts, however, in particular the expression of attitudes, appear to strike teachers as incongruous, although they are not explicitly condemned. It is not clear what effect this may have on teachers' assessments of the text and of the student.

It is relatively easy to identify the major features (i.e. tables, narrative, statement of the problem, etc.) that the teachers paid attention to during their reading of the texts, and hence to construct a picture of the gross form of an 'ideal' coursework text. This is essentially what some of the teachers themselves had done in providing advice to their own students to, for example, draw tables and explain what they had done. The evidence discussed in this chapter, however, suggests that the detailed form of these features significantly affects the interpretations that teachers make of them and may consequently affect their assessments of the students' achievement. Moreover, the contrast between Andy's readings of two longer sections of text suggests that the extent to which the text as a whole conforms to the stereotypical 'investigation' format may have an important effect on a teacher's ability to make coherent sense of the text.

12 'Algebra' in coursework

Because of the high status of generalisation and algebra within school mathematics and in the assessment of coursework within the 'data-pattern-generalisation' paradigm (see Appendix 15), it was anticipated that this would be a significant theme in teachers' reading and assessing of the coursework texts and the nature of generalisations was thus one of the features taken into account in selecting the student texts to be used in the interviews with teachers. All of the selected student texts on both tasks included a generalisation expressing a relationship between the variables of the task situation or describing a procedure for generating the dependent variable from the independent variables. The way in which this relationship was expressed varied from a sentence describing the procedure without algebraic symbolism, through a formula with descriptive variable names, to an entirely symbolic equation relating single letter variables. In addition to the whole texts, extracts from three further 'Inner Triangle' texts were given to all the teachers to compare. These extracts consisted of the section of each text in which the generalisation was expressed; they were chosen to display different ways of embedding the formula into the text as well as a range of forms of expression of the relationship between the variables. A summary of the characteristics of the generalisations in the students' texts is provided in the first section of this chapter; the detailed analysis is in Appendix 14. The teachers' responses to these generalisations and to other algebraic aspects of the students' texts are then examined.

12.1 Characteristics of 'algebra' in the students' texts

Two main features of the algebraic aspects of students' texts will be considered here: the way in which the variables are named and the extent to which the generalisation is expressed as a procedure or as a relation between variables. The use of single letter variables is conventional in the context of higher school algebra and clearly signals to the reader that the author is 'doing algebra'. Many of the student's texts, however, include generalisations which use verbal descriptions of the variables. These may suggest to the teacher-reader that the author is working at a lower level of abstraction. This is not, however, a simple dichotomy as verbal descriptions also vary in their degree of abstraction. For example, "the length of the rod that first makes the pile topple" is more closely linked to the concrete context of the problem than is "topple no.". The use of capital letters can also make a variable name appear more independent of the context. It seems reasonable to hypothesise that a higher degree of abstraction is likely to be more highly valued by mathematics teachers¹.

¹An example of a teacher's comment confirming this hypothesis may be found in chapter 11.

Some of the generalisations found in the student texts are in the form of sets of instructions providing procedures for calculating the value of the dependent variable, given the values of the independent variables. Many of these are addressed to a general reader in the imperative or using a general 'you'. Of course, formulae written as symbolic equations may also be interpreted procedurally. However, such a form additionally allows them to be interpreted as objects in their own right; in particular, it makes it possible to manipulate the symbols in order to gain further insight into the nature of the relationship between the variables and to solve a wider range of problems. Where such manipulation has not taken place, there is no indication of whether a student-author conceives of her formula as (or intends it to be read as) a relation or as a procedure. However, the equation form, interpreted as a relation between variables, is more valued in higher mathematics and may be valued more highly by teachers.

In addition to these two main features, some of the student texts involved other aspects of algebra, for example, the use of brackets and the manipulation of symbolic expressions. In several cases, brackets were omitted or used in unconventional ways within a student text. Teachers' readings of such 'errors' are discussed in chapter 13.

Each of the three complete 'Inner Triangles' texts read in the interviews contained a generalised expression for the number of unit triangles within a trapezium given the dimensions of the trapezium. Apart from the omission of brackets, all these generalisations are correct. Clive and Richard both also included generalisations for other figures. However, since these largely shared the characteristics of the students' original generalisations, they are not considered separately. The analysis of these generalisations is summarised in Table 12.1 below.

Table 12.1
Algebra in the 'Inner Triangles' texts

<u>student</u>	<u>variable names</u>	<u>type of generalisation</u>	<u>brackets</u>
Clive	succinct verbal	procedural	missing
Steven	mainly symbolic	equation format but used procedurally	missing
Richard	symbolic	equation	correct

The three 'Topples' texts also each contained a generalised expression for the length of the rod which makes a pile of rods topple, given the length of the bottom rod in the pile. In addition, Sandra and Steven included an expression of the inverse. All these generalisations fit the data collected and reported by the student. Sandra's data was different from that collected by Steven and Ellen; hence her generalisation is not equivalent to theirs. The ways

in which teachers responded to these differences between the texts is discussed in chapter 14. The analysis of these generalisations is summarised in Table 12.2.

Table 12.2
Algebra in the 'Topples' texts

<u>student</u>	<u>variable names</u>	<u>type of generalisation</u>	<u>brackets</u>	<u>inverse</u>
Steven	symbolic	equation format but used procedurally	correct but unconventional	similar
Ellen	mainly verbal	procedural with imperatives	n/a	none
Sandra	verbal	procedural with human agent	n/a	implicit

The three extracts from further 'Inner Triangles' texts consist of the section of each text containing the expression of the generalisation for the number of unit triangles in a trapezium. The extracts were chosen to display a range of contrasting features but all of them, including their use of brackets, were technically correct. The analysis of these generalisations is summarised in Table 12.3.

Table 12.3
Algebra in the 'Inner Triangles' extracts

<u>student</u>	<u>variable names</u>	<u>type of generalisation</u>
No.1	symbolic	equation with manipulation
No.2	succinct verbal	procedural followed by equation format
No.3	verbal followed by symbolic	procedural

In the rest of this chapter, the teachers' readings of these sections of the students' texts will be examined and the place of 'algebra' in the discourse of coursework will be considered.

12.2 Teachers reading 'algebra': symbols and words

For mathematics teachers, the most easily identified difference between the various generalisations described above is probably the presence or absence of an expression involving single letter variables. This appears to play an important role in the assessment process although several teachers expressed an ambivalence towards it. In spite of the value placed on symbolism, however, a completely non-verbal text is not considered adequate.

12.2.1 Use of symbols as a key assessment criterion

The use of symbols as variable names in the expression of generalisations is stated as one of the assessment criteria for achieving higher grades in GCSE coursework in both the general criteria and the task specific performance indicators issued by the examination board. It has, moreover, become part of the folklore that, in order to achieve one of the top grades (A - C)

for an investigation in GCSE coursework, it is necessary to have used 'algebra' (Wolf, 1990). In this context, 'algebra' normally appears to be interpreted as the use of symbols rather than verbal variable names. Most of the teachers interviewed made some use of this criterion during their reading of the students' texts. Jenny's assessment of Ellen's text, for example:

She ain't got no algebra in this one so if she hasn't got no algebra she can't have her 7a cos it's definitely not new mathematics if it hasn't got algebra.
(Jenny: 97-99, Ellen)

shows that, even with the introduction of the use of National Curriculum criteria which make no mention of algebra², the requirement to include a symbolic generalisation is still strongly identified with a particular grade level. Not all the teachers made explicit use of grades or official criteria but, even in these cases, symbolic algebra appeared as a key indicator in forming their assessments both when making decisions about ranking actual student texts:

This one [number 2] perhaps may come towards the bottom, maybe the third one, because they've just shown a couple of examples and then written it out in words whereas the other two have used the letters.
(Grant: 282-284, Nos. 1-3)

and when stating the general principles of their assessment practice:

perhaps coming up again eventually at the other end with algebraic formulas, things like that, um I think one of the dividing lines which I generally look for when I'm assessing something like this
(Charles: 7-9)

Some teachers may even be led to overlook other aspects of a student's text in their search for algebraic expressions (see the discussion of Jenny and Charles reading Steven's 'Topples' text in chapter 10)

In spite of this general recognition of the role of algebra as a decisive assessment tool, its application appears problematic. Even the two teachers quoted above have hedged their statements with the qualifier *perhaps*. In other cases, the importance of algebra was seen to conflict with other assessment criteria, causing apparent discomfort. Charles, for example, found it difficult to resolve the tension between his assessment of 'understanding' and the use of the algebra criterion. Thus, having identified the lack of algebra in Ellen's text, he initially claimed that it was irrelevant:

C I think we've got very similar to the last one [Steven] except obviously we don't have an algebraic way of representing the data at the end, of representing the formula at the end but we've got a very good way of expressing it in words.
(.....)

I Does the fact that she hasn't used algebra matter?

²From 1994, GCSE coursework assessment has been officially referenced to the National Curriculum Attainment Target 1. A National Curriculum level 7 is equivalent to a grade C. The statement of attainment 7a that Jenny refers to states:

Follow new lines of enquiry when investigating within mathematics itself or when using mathematics to solve a real-life problem.
(DES, 1991)

- C *Umm . . . probably not cos she understands the problem as much as the other person. She understands how to get the answers just as well as the other person who has been using algebra. So at the end of the day, no it doesn't really. Umm . . . no [laughs]* (Charles: 123-138, Ellen)

but, when pushed to rank the two texts, decided that algebra was nevertheless crucial to the assessment process:

- Well I don't know cos I think these two are very similar. I think probably Ellen understands it a bit better than Steven does but the Steven's got his little bit of algebra in there which . . . I mean you're always looking for your algebra aren't you so [laughs]* (Charles: 188-190, Ellen v Steven)

His nervous laughter is symptomatic of the difficulty he appears to be experiencing in coming to an assessment and the use of *you* in the final sentence serves to distance himself from his decision by passing responsibility to an external (but unspecified) authority³.

Several other teachers, like Charles, indicated some degree of conflict with the authority of the criteria provided by the examination board, claiming themselves to be unconcerned by the difference between verbal and symbolic variable names. Andy appears to feel his own conception of 'algebra' to be in conflict with that required by the examination board when evaluating extract No. 2

- I *Would they get extra marks for writing it as t plus b rather than top length plus bottom length?*
- A *The exam board seems to think so. They often make the point that hasn't got algebraic - well I think that's algebraically quite correct and I see no real difference between that and t plus b times s. Once you start putting brackets in you start making it algebraically correct - I think that's right. And it's quite concise. I mean if it was in words they would say something like 'I would take the top length and add the bottom length to it and then I would multiply by the [. . .]'. (Andy: 310-318, No. 2)*

For Andy, the key distinction between algebra and not-algebra is between a relational or equation format and an explicitly procedural generalisation, whereas he sees the examination board to be making a distinction between verbal and symbolic variable names. Similarly, Dan sees little difference between No. 2's formula and one expressed using algebraic symbols and suggests that he might interpret the examination board's criteria (perceived by him to demand such notation) flexibly:

- If every other criteria [was fulfilled] and the only thing that was stopping it being a particular grade was the fact that it wasn't expressed algebraically I wouldn't mind. To me, the difference between that and sticking letters in instead is so minimal that it doesn't really matter.* (Dan: 354-357, No. 2)

The opposite problem is experienced by Carol who sees Steven's (Topples) text to be lower in quality than might be expected from the fact that it contains symbolic generalisations:

³Charles' reading of Steven's work is considered in full in chapter 10.

it's strange because this student has in a few places tried to use algebra which would make you think well people who try to use algebra are generally sort of the good Ds and the Cs but other things that you're looking for in good Cs and Ds don't seem to be there so there seems to be a sort of inconsistency when I'm assessing this as a teacher. Um . . . I'm wondering if it's not so much using what you'd call real algebra but just using a letter in place of a phrase
(Carol: 358-363, Steven)

Having identified a mismatch between the 'algebra = good grade' criterion and her other, unspecified and probably less easily identifiable criteria, she resolves it by devaluing Steven's symbolic generalisations, suggesting that they are not really algebra at all. It is not clear what she would consider to be "real algebra"; it is possible that this is a context-bound concept and that her reading of a section of text as algebra or not-algebra will depend on her reading and evaluation of the rest of the text within which it is embedded.

Where there is tension between presence or absence of symbolic algebra and the evaluation of the rest of the text, some teachers will resolve the tension by appealing to the authority of the examination board, while others will respond by redefining the nature of 'algebra' in order to fit their overall evaluation. These two methods of resolving tension are associated with different positionings in relation to the activity of assessment. Deferring to authority suggests a position as examiner/employee with relatively little power, using externally determined criteria. Redefining the terms of the criteria suggests a more autonomous position for the teacher as a professional able to determine the criteria herself. The different resolutions are also likely to give rise to different ultimate evaluations of the student's work.

12.2.2 Reasons for valuing symbols

In the examples provided above, no reasons were given for the importance accorded to the use of algebraic symbolism by the teachers or by the examination board. Algebra appears as an unquestioned and unjustified 'good thing'. A few of the teachers did, however, indicate reasons for preferring generalisations expressed symbolically.

Both Joan and Fiona appeared to find it easier themselves to read such generalisations. Fiona claiming to have had difficulty understanding Clive's text, which used verbal variable names:

He hasn't used algebra for his formulas. So obviously, you know, that makes it a bit more difficult. But I found this, you know it took me a second to understand what he meant, slant and top, but that was probably me not reading it properly.
(Fiona: 177-180, Clive)

while Joan, on reviewing the same text, did not at first recognise that Clive had formed a generalisation at all. Grant, although not apparently experiencing any difficulty in making sense of Ellen's verbal generalisation, also suggested that a symbolic form would be easier:

Ellen has got to the stage of finding a formula but hasn't tried to put that in a more easy way for people to use.
(Grant: 254-255, Ellen)

There is ambiguity here in that it is not clear who the "people" are or what they might wish to use the formula for. While Grant may be referring to his own reading in order to make sense of the generalisation, it is also possible that he is considering other possible uses of a formula, perhaps including evaluating or manipulating the expression (although, given the coursework context, it is unclear who might wish to do this).

Other teachers related the value they placed on algebra to their apparent perceptions of the nature of mathematical ability. Jenny expressed this bluntly:

I mean I think that people with a reasonable mathematical brain ought to be able to think algebraically a bit. (Jenny: 107-108, Ellen)

suggesting a simple absolutist view of ability. Similarly, Grant displays different expectations of students he perceives to be at different levels of ability:

I But you said that this one, number two might come third. Because it hasn't used letters. You think that's important.

G Yeah. I think that, well with higher ability pupils it is. We are looking for more of the good algebraic work from them. If this had come from a low ability pupil, writing this, which is algebraically correct and well written even down to using top length instead of the top of the trapezium, it cuts it down what they've got to write down for the formula and shows some knowledge . . so if I was to get that from a low ability pupil I would be well pleased

I But if you thought that was from a higher ability pupil you wouldn't be so pleased?

G No. I'd go for letters. (Grant: 296-304, Nos. 1-3)

Others, while displaying more developmental views of mathematical learning and attainment, also linked using symbolic algebra to higher levels:

I This one you mentioned that she hasn't abbreviated her top length, bottom length. Would it make .. I mean, would you rather that she had written t plus b?

H I think so, I think so because yes I think substituting you know reasonably longwinded [...] for letters shows you that they're comfortable with that, even at that sort of reasonably low level. So it's something that you know I'd definitely recommend at this next stage. (Harry: 304-308, No. 2)

Although Harry approves No. 2's ability to evaluate her formula by substituting numbers into it, he nevertheless does not value this highly. In his examiner role he assesses her to be at a "low level", while his recommendation for the "next stage" suggests that he is adopting a teacher/adviser role, considering the student as a learner working towards a higher level.

Reasons for valuing symbolic generalisations over those expressed in words were not addressed deliberately during the interviews with teachers; the reasons deduced from the extracts quoted above were given by the teachers spontaneously as justifications for their judgements. A more explicit inquiry into teachers' beliefs about algebra might well discover a

variety of different rationalisations for its value, including ones which related more to the place of symbolism within mathematics itself. In particular, the distinction between procedural and relational generalisations that was used in analysing the student texts did not arise explicitly in the teachers' discourse. Although those generalisations identified as potentially relational were more highly valued, the reasons given for this seemed to be associated with the use of symbols rather than with the structure of the statement. It is not possible to determine whether the teachers were reading generalisations in the form of equations as statements of relations between variables or as procedures for performing calculations.

Whatever the reasons for valuing algebra, the apparent close association between use of algebra and teachers' perceptions of ability is significant in the coursework assessment context. As was shown in chapter 10, some teachers seem to form their assessments of a piece of work by building up a picture of the personal characteristics of the author. A perception that a student is 'able', based on the existence of symbolic algebra within her text, would thus be very influential in forming a final assessment.

12.2.3 The need for words as well as symbols

In spite of the value placed on a symbolic generalisation, however, it is not considered to be adequate in the coursework context without some verbal elaboration. Richard's 'Inner Triangles' text was a particularly extreme case of a lack of words. His section dealing with the generalisation for trapezia consisted only of a table of results, a symbolically expressed formula and a diagram labelled with the variable names used in the formula. Most of the teachers took this to be a sign of relatively high achievement or ability. For example, Fiona praised the algebraic notation as "quite advanced" (Fiona: 97, Richard)⁴, while Dan, who had at one time taught Richard, described him as "quite an intuitive mathematician" (Dan: 137, Richard). It is interesting to note that both Fiona and Dan hedged their praise with the qualifier *quite* as they, like all the teachers who read these 'Inner Triangles' texts, also criticised Richard for not including more 'writing'.⁵

The reasons given by the teachers for requiring words as well as algebraic notation are multiple, arising from their simultaneous occupation of a number of (potentially contradictory) positions in relation to the activity. Teachers, in their role as examiners and as teachers responsible to a range of external authorities, are naturally concerned that their judgements of students should be seen to be valid. They feel the need, therefore, to find evidence within a student's script that the work 'belongs' to the student. For example, Dan commented that:

⁴Fiona's reading of Richard's text is considered in full in chapter 10.

⁵Teachers' expectations about the inclusion of 'writing' in a coursework text in general are discussed in chapter 11.

the formula . . . did sort of appear out of nowhere as though he did have a sly look round somebody's arm or something. (Dan: 283-284, Steven)

It is not clear why a symbolic formula should be considered more likely to be copied from another student than any other part of the text. It is, however, true that Steven's 'Inner Triangles' formula is not integrated into the rest of his text either by a narrative of its provenance or by an argument for its validity.

Andy's reading of Richard's text is particularly interesting because of the multiplicity of reasons he provides for demanding a generalisation in words. Andy, who actually taught Richard, acknowledged the lack of evidence in his text but was prepared to act as teacher/advocate on his behalf:

I'm very confident that, although there's no evidence of it, what he's produced is right and he's done it. It definitely wouldn't be a copy. . . . That's interesting, that can only be a teacher's inside knowledge. Somebody marking that cold wouldn't be able to state that. (Andy: 181-186, Richard)

In spite of the fact that the other teachers reading this text did not have similar personal knowledge of the student, he was usually given the benefit of the doubt on this point. Demonstrating ownership is not, however, the only function of a generalisation in words. While satisfied that the formula was Richard's own, Andy nevertheless went on to give a number of other reasons for wanting to see the generalisation expressed in words:

I would like to see that he can generalise in words first of all. It kind of gives the understanding, I think, putting it into words the patterns which they see. Then I think it underpins the algebra which they produce later. (Andy: 188-191, Richard)

Initially, Andy claims that he wants to see that Richard "can generalise in words"; he is looking for evidence of a skill. In the next breath he is suggesting that this would also provide evidence of "understanding". The expression that he uses is, however, ambiguous. "It kind of gives the understanding" to whom? Do the words give evidence to an examiner who needs to know that the student understands, or is the understanding given to the student himself by the process of expressing the generalisation in words? This ambiguity marks a shift in Andy's reading between an examiner role, looking for evidence, and a teacher/adviser role, suggesting ways in which students can be helped. In the final sentence of the extract above, the role of generalisation in words has clearly shifted from being evidence towards assessment of the student to being a pedagogic device for helping the student to gain understanding. At the same time Andy has shifted from referring specifically to Richard to using a non-specific *they*; he is now stating general pedagogic principles. He went on to describe the assessment methods used by the teachers in his school:

When we moderate, we usually do it as a group in the same room usually working in different corners on the scripts that we're moderating and frequently we ask across the room to the teacher concerned 'where did this come from, is that alright?', and . . . on we go. But there's usually a check of that kind. We try very hard to tell the children generalise in words first of all and we say if you know a pattern, can you tell us about it, tell your friend

about it. When they can explain the pattern in words and they write those words down then they're ready to produce the algebra. (Andy: 195-203, Richard)

Again, he moves from a description of the way the teachers behave as examiners looking for evidence to a description of the pedagogical device they use to help the students to become "ready to produce the algebra".

Andy and the other teachers interviewed appear to be using an unwritten criterion that any algebraic generalisation appearing in a piece of coursework should be preceded by a verbal statement of the same generalisation. Stated like this, it is a simple matter to judge whether or not a student has fulfilled the criterion. The justification for its use, however, is not so simple but takes a number of forms: it is evidence that the student has the skill to write a verbal generalisation; it shows that the student understands the algebra; it proves that the formula 'belongs' to the student; it provides evidence of the processes that the student has gone through in order to arrive at the formula; it helps the student to understand the pattern; it prepares the student to "produce the algebra". The discourses of assessment and of pedagogy are intertwined here.

Advising students to generalise in words as a step towards using algebraic symbols is a common pedagogic strategy. The process is described by Mason as "a necessary part of the struggle towards meaning along the spiral [of symbolising]" (1987: p.80) and has been incorporated into the official discourse of the mathematics curriculum in statements such as this from the HMI (1987):

It is damaging to pupils' mathematical development if they are rushed into the use of notation before the underlying concepts are sufficiently developed and understood. At all stages the teacher needs to stress the translation of words into mathematical symbols, and the reverse, so that pupils may develop a facility in the use of symbols and an understanding of the meanings attached to them (p.10)

It seems for these teachers, however, that a principle related to facilitating students' learning has been transformed into a prescriptive algorithm for 'doing investigations'. Regardless of the individual student's actual facility with symbolising, she or he is expected to have gone through the entire "struggle towards meaning". As Amy put it, criticising Richard's (correct) symbolic generalisation:

He needs to explore. There's something needed before he could generalise. (Amy: 36, Richard)

Although Richard is clearly able, at least in the context of this problem, to generalise symbolically without such aids, because he has failed to comply with the conventions of the coursework genre he is judged harshly; the 'need' is a requirement of the examination, expressed in the language of pedagogy.

12.3 Algebraic 'content' skills

While the formation of a generalisation is clearly a fundamental component of the standard 'investigation' process, a number of other algebraic skills were also demonstrated in some of the students' texts and drew comments from teachers. These aspects, which might be found as 'content' in the traditional school mathematics curriculum, included the presence of worked examples positioned after the statement of a generalisation (substituting values into the formula or demonstrating how the generalised procedure works) and any manipulation of algebraic expressions, including the presence of an expression perceived to be an inverse. The presence of brackets in an algebraic expression was also generally approved where they were seen to be used correctly. Teachers' ways of dealing with formulae in which brackets were seen to be missing or to be used inappropriately are discussed in chapter 13.

12.3.1 Examples

A number of the teachers approved of or expressed a wish to see worked examples after the generalisation. Several reasons were given for this. Fiona, for example, wished to use the examples as evidence of skills related to the evaluation of formulae:

Just, the purpose would be to show that they knew how to use this formula themselves. (. . .) Can they work out brackets, things like that. So its a test of their algebra as well.

(Fiona; 344-350, No. 3)

While Fiona seems quite clear about her intention to use the examples to assess students' content knowledge, Charles has more difficulty in articulating his reasons for expecting to see examples. Although he eventually comes up with several purposes, his struggle to produce a rationale during the interview suggests that this is a post hoc rationalisation and that his original expectation arose primarily from his understanding of the features of the coursework genre.

I Why do you want to see examples after the formula?

C Yeah, exactly, why? [laughs] It's a good question, actually. Umm, I want to see perhaps that they understand what the formula is and how they use it. So I think [...] you can see, you know, I got this formula and I can put these numbers in I can see it gives me the right answer so it's like a check that it works as well. Umm but it .. I can actually recognise then that they see how to use a formula, so it's like er, what's the word, evidence that they understand what they've got. *(Charles: 294-301)*

Like Fiona, he wishes to see that the student can evaluate the formula. He also wishes to see whether "they understand what the formula is"; it is not clear from the interview text whether this understanding is completely identified with an ability to evaluate the formula or whether it contains some further component. A suggestion that substituting values into a formula in order to provide answers to further questions provides evidence of some more general understanding of the nature of formulae and their status as generalisations is made by Grant:

She's used her formula here again . . and stated she's using her formula and it seems obvious in her brain that because she's found the formula it's going to work for everything. Which would again gain credit in terms of once you've got it it's there for ever and even though this is the next question on the sheet it still works.
(Grant: 231-233, Ellen)

These reasons are all related to the assessment of the student's skill or understanding of the 'content' of the topic of algebra rather than the quality of her investigative processes. In the extract from Charles above, he suggests additionally that the examples serve as a check on the validity of the formula itself. In using the first person at this point, the teacher appears to be taking on the role of the student-author, suggesting that this check is for the use of the student as part of the problem solving process rather than for his own benefit as an assessor.

12.3.2 Manipulation

No.1's text was the only one read by the teachers which unambiguously displayed algebraic manipulation. This was commented on positively by several of the teachers:

I think number 1 is very good, is good the way, you know, they've simplified their algebra as well.
(Fiona: 315-316, No.1)

Number 1 has brought in algebra - factorisation, substitution - that's good.
(George: 91, No.1)

These teachers' praise of the display of manipulative skill suggests that they are applying criteria from more traditional assessment modes. They suggest no function for the manipulation but merely approve its presence. The manipulation is an end in itself and the student is displaying her skill in performing a routine procedure rather than using that skill to further the mathematical investigation. This seems to be recognised by Andy, who, while approving the manipulation, nevertheless does not appear to find it relevant in the context.

I mean although the little bit of extra algebra, the simplification of it of the equation is clearly different from the other two I wouldn't try giving much extra credit for that but it's nice to see.
(Andy: 326-328, No. 1)

There is a tension here between the value placed on the display of content knowledge and the fact that the manipulation of the formula plays no role in furthering the student's work on the investigative task.

One aspect of both tasks in which manipulation might legitimately have played a role in the solution of the problem was the inclusion of questions which could be considered as 'inverse' problems. In 'Inner Triangles' this question occurred at an early stage in the task and appears in all cases to have been solved by trial and error methods rather than by applying an inverse formula. In 'Topples', however, the question was positioned after the formation of a generalisation, suggesting that this might be expected to be used in order to obtain an answer. As was seen in section 12.1.2 above, Sandra and Steven both provided generalised methods for solving the inverse problem. Sandra's use of the word *opposite* and her example, which may be read as simply reversing the order of the operations involved in her original generalisation, strongly suggest that she is aware of the inverse relationship between

the two procedures and is even making use of the first in order to derive the second. This suggestion is not, however, good enough for Grant:

Now she's given a formula for working out how to work backwards on the problem but hasn't said why. Just the mention of the word backwards would gain marks there I think.

(Grant: 94-96, Sandra)

The word *backwards* was also used by several of the other teachers when discussing the inverse relationship. It may be that it is so commonly used to designate *inverse* that Sandra's alternative *opposite* is not recognised. None of the other teachers who read her text gave any suggestion that they saw any evidence of inverse in her work. The fact that she has not used algebraic notation here may have contributed to this lack of recognition.

In contrast, all the teachers who read the 'Topples' texts identified Steven's second formula as an inverse formula, although they were not all convinced about its validity. As was seen in chapter 10, Jenny and Charles both assumed that Steven had formed the inverse by manipulating the original formula, Jenny even performing a manipulation herself to check that he had done it correctly. Grant also appears to make an assumption that Steven has performed a manipulation but he is suspicious of its validity:

He's changed his formula slightly, in fact quite a lot hasn't he, he's changed his formula quite a lot. Um to what to me looks like fiddle the answer . . . I think he's obviously doing something here that he knows what he's doing but I'm not so sure. Perhaps with a bit more explanation it would work. So I would perhaps gloss over that, not give that too much credit at the end there.

(Grant: 176-180, Steven)

Steven's second formula is, in fact, valid and does "work" although there is no obvious indication that it was derived by manipulation of the first formula rather than being derived directly from the data. It is, however, relatively complex in format, involving brackets and fractions. It seems likely that it is this unconventional format that gives rise to Grant's identification of a "fiddle". Carol was also unhappy with this formula:

I'm looking at this alternative formula for going backwards which is intriguing me . . . um, I'd have to look at that much further but my sort of intuitive feel is that probably this strange alternative rule for going backwards only works in that situation, I mean I'd have to try it out on some other numbers and I don't want to hold things up at this stage.

(Carol: 345-349: Steven)

It is not possible to know whether, in a genuine assessment situation, she would have taken the time to test the formula or whether confirming its validity would have overcome her intuitive suspicions.

The difference between the ways in which Sandra's and Steven's 'inverse formulae' were read may be related to the teachers' different expectations about the two students based on their readings of the whole texts. The lack of symbolic algebra and the 'errors' (see chapter 13) in Sandra's text give rise to an impression of a 'less able' student. Since 'inverse' is considered to be a relatively advanced concept, its presence may not be recognised in her

text. Steven, on the other hand, because of his use of algebraic notation, is seen to be likely to gain a high grade and hence to be capable of more advanced work. His second formula is thus identified as 'inverse' in spite of a lack of direct evidence. The contrast between this identification and the difficulty that the teachers have in making sense of the formula appears to give rise to a tension which may be resolved either by accepting the 'inverse' formula and its supposed derivation unreservedly (as Jenny did, claiming that Steven's algebra was better than her own) or by rejecting it as invalid or as achieved in an inappropriate way.

12.4 Summary

It is clear that teachers' perception of the presence of algebra in a text is a decisive assessment criterion. The nature of this perception, however, varies between teachers and is a source of tension for some. While in some cases 'algebra' is identified solely with the use of symbols, for others, any generalisation in the form of an equation will be accepted as algebra. Those who take the latter view may feel themselves to be in conflict with the authority of the examination board, which is alleged to demand the symbolic form. In some cases, there may be tension between the perceived presence or absence of algebra in a text and other characteristics of the text. Teachers may resolve this tension in different ways: some by taking algebra as the decisive criterion, others by redefining their perception of 'algebra' in order to make their evaluation of the whole text consistent. It seems that the presence or absence of algebra in the text, by contributing strongly to the general impression of the student, may influence the ways in which teachers interpret other aspects of the text, including in particular the expression of an inverse relationship. Reasons given for the value ascribed to algebra include:

- it is easier to read than words;
- it is easier to use (although it is not clear what use is intended);
- it is a sign of high mathematical ability.

A symbolic generalisation, although highly valued, is not considered sufficient to stand alone in the coursework genre. A procedural description in words is expected to precede it. There are several reasons given for this:

- it provides evidence that the formula 'belongs' to the student-author⁶;
- it provides evidence of 'understanding';
- it demonstrates that the student is capable of providing such a verbal generalisation;

⁶Mathematics teachers seem to have a faith in the authentic nature of the written word that contrasts strikingly with their suspicions that students may copy numerical results and symbolic equations.

- it serves to help the student to produce the symbolic formula.

The pedagogic belief that generalising in words is a helpful step towards making use of symbolic notation has become transformed into an assessment requirement. Even those students who have no need of the support provided by describing a verbal procedure are expected to include one in order to fulfil the expectations of the genre.

The primary, and in most cases only, use made of symbolic notation in the set of coursework scripts examined is to express a generalisation. This generalisation appears to be seen by both students and teachers as the end point or solution of the task and is in many cases underlined, presented on a separate page under a heading *FORMULA*, or otherwise signposted as the 'answer'. While forming such symbolic generalisations is an important mathematical process, outside the school context it would not normally be seen as an end in itself. Symbolisation is not merely a process of translation from one language into another but is the starting point for developing new ways of looking at a problem and for enabling manipulations that may lead to new discoveries and further generalisations. As Mason points out, 'Classification is for the purpose of formulating theorems, not simply to achieve superficial classification' (1987: p.77).

Where evaluation or manipulation of a formula is found in a coursework text, it appears to be used by both students and teachers as a means of demonstrating proficiency in decontextualised algebraic skills rather than as part of the investigative or problem solving process. While the formation of a generalisation may be seen as part of the investigative process and the use of symbolism may be seen as assisting communication, other algebraic aspects, including substitution and manipulation of algebraic expressions appear to be valued primarily as demonstrations of 'content' skills, although substituting values into the formula may also be valued as a part of the problem solving process, serving as a check on the validity of the generalisation and helping the teacher-reader by providing evidence that the generalisation is correct. The manipulation of an algebraic expression was particularly highly valued by some teachers, perhaps as a sign of even higher ability. In at least one case, however, there was some conflict between this perception of the value of manipulative skills and a recognition that performing such manipulation was not relevant within the given problem solving situation. Algebra is clearly one area which gives rise to tension between the value placed on process within the discourse of investigation and that placed on content in traditional modes of curriculum and assessment.

13 Dealing with error - is 'Practical' different?

The two tasks 'Inner Triangles' and 'Topples' may be distinguished from one another by reference to the nature of their subject matter, although, as has been shown in chapter 7, the structures of the tasks and the processes that students are expected to undertake are very similar. The subject matter of 'Inner Triangles' is essentially 'pure' in the sense that all numerical results and relationships between variables (if correct) are entirely determined by the definitions of the basic objects (although variations may of course occur in the forms in which results and relationships are expressed and in the particular relationships pursued by individual students). In 'Topples', on the other hand, because the basic objects are physical and the student's initial activity is experimental, numerical results may be affected by factors other than the defined lengths of the rods (e.g. other properties of the rods, the stability of the surface on which the piles are built, the student's manual dexterity); the nature of the resulting relationships will be affected correspondingly by the quality of the experimental data. 'Topples' may thus be seen to be a 'practical' or 'scientific' activity rather than a 'pure' mathematical activity.

One area in which mathematical and scientific activity vary significantly is in the attitude to error. In pure mathematics, errors do not arise if procedures are followed correctly; scientific activity, in contrast, when making use of experimental data, must take account of the possibility of error arising from uncontrolled variables in the experimental situation. It might be expected, therefore, that the teachers' readings of the two tasks would differ in respect to the ways in which they respond to students' errors. While the differences between teachers' readings of 'pure' and of 'practical' tasks are of interest here, the fact that none of the teachers read complete student texts on both tasks means that direct comparisons based on the practice of individual teachers reading both types of task are not possible.

The incidence of error was not taken into account in the selection of students texts to be used in the interviews; there is thus considerable disparity between the number of teacher comments on error and accuracy related to each of the two tasks as one of the 'Topples' texts contained substantially different results from the other two texts on the same task, drawing attention to the possible presence of error, while all three 'Inner Triangles' texts achieved approximately comparable results. Three of the six teachers who read the 'Inner Triangles' texts made no reference to the ideas of error or accuracy while all those who read the 'Topples' texts dealt with these issues to some extent. Alternative explanations of this are available: given that those who did mention error in the 'Inner Triangles' task also placed some emphasis on accuracy in general as well as specific cases, it is possible that those who did not do so were relatively unconcerned with accuracy and that the absence of major differences in the students' texts did not challenge this lack of concern. A further

consideration that must be taken into account, however, is the interview context itself. Whereas when assessing their own students' work teachers are likely to be very familiar both with the task and with the answers that might be expected, the teachers in the interview situation had a far less detailed awareness of the 'correct' answers. Although all were requested to familiarise themselves with the task before the interview took place, their degrees of engagement with the task varied considerably. The teachers' ability to identify errors must have been affected by this and, while some asked the interviewer to confirm whether results were accurate, others may have avoided the issue completely in order to avoid losing face in the interview situation. The interview data is not adequate to resolve this point. Nevertheless, in spite of these limitations, the data that is available raises interesting issues about the ways in which teachers deal with error in investigative work.

The cases of error identified by one or more teachers during the interviews are shown in Tables 13.1 and 13.2 below.

Table 13.1

Errors identified in 'Inner Triangles'

<u>error</u>	<u>student</u>	<u>commented on by</u>
brackets missing from formula	Clive	Andy
brackets missing from formula	Steven	Joan, Andy
incorrect value given for one trapezium	Steven	Fiona

Table 13.2

Errors identified in 'Topples'

<u>error</u>	<u>student</u>	<u>commented on by</u>
experimentally inaccurate results	Sandra	all
inconsistency between data and formula	Sandra	Jenny, Carol
wrong formula	Sandra	Harry
unconventional use of brackets in formula	Steven	Harry, Charles, Jenny, Carol
inconsistency in experimental results	Ellen	Harry, Charles, Carol

In addition, there were further incidents in which teachers expressed general concern with the accuracy of numerical results and formulae without identifying specific errors in students' texts. There are three main areas of concern that may be seen in relation to both tasks: the accuracy of the data arising from the initial stage of the task, the correspondence between this data and the formula or other description of patterns within the data, and compliance with the conventions of algebraic notation.

13.1 Accuracy of data gathering

For two of the teachers who read the 'Inner Triangles' texts, checking the accuracy of the answers to the initial questions was an important first step in their process of forming a judgement of the student's text. (These two questions require only that the student should give numerical answers to questions about the dimensions of specified trapezia.) Thus Andy, responding to a general request to describe the way he would go about assessing work on the task, stated:

So I think it's obviously accuracy in the first few parts here, can they count the number of unit triangles in the trapezium. Are there any particular problems with that, generally speaking all the children at all the levels got that extremely well.
(Andy: 6-9)

Although accuracy is "obviously" important, Andy simultaneously limits the scope of its ability to differentiate between students at different levels. Fiona defines this scope more clearly by equating accuracy in the first two questions with achieving a grade F:

So he's got question one done correctly. He obviously understands the problem for, that's been given. He's also got number two all done correctly which was more difficult. So you know, if we were grading he's certainly moving up to a grade F at this point.
(Fiona: 38-41, Richard)

A further function that Fiona ascribes to this level of accuracy is as a sign of understanding of the nature of the problem. She commented on this aspect when starting to read each of the three texts. 'Understanding the problem' is one of the general assessment criteria proposed by most published assessment schemes for investigational work. The criterion is not usually, however, elaborated in a way that makes it immediately applicable. In the context of this task at least, Fiona has operationalised the criterion by relating it to the more familiar (and more easily identified) idea of giving correct answers.

In practice, all three 'Inner Triangles' texts gave correct answers to this data-gathering section of the task; it is not possible, therefore, to see how these teachers would have reacted to errors in this section, although the priority that Andy gives to accuracy and his suggestion that errors would indicate "particular problems" suggests that he at least would not find them acceptable.

In contrast, the teachers reading the 'Topples' texts presented a very different attitude towards the accuracy in the data gathered. Thus Harry, although expressing a belief that Sandra's results were wrong, simultaneously claims that this should not affect the evaluation of her work:

So I'm not sure that that's - I'm not sure that it would topple then. But if she's convinced that it did topple - again we said about the accuracy of the actual modelling of it the setting up of it. If it toppled it may be something to do with

it but she's convinced that it toppled. Then we have to accept that.

(Harry: 220-224, Sandra)¹

The practical nature of the task is explicitly seen to change the nature of teachers' judgements of error. For example, Grant, discussing his general criteria for assessing 'Topples', states:

whether it's actually correct in the end or I've got the correct answer is besides the point I think (. . .) That's for this investigation though. I say for others I'd go for a proper correct answer.
(Grant: 40-43)

Presumably the other investigations he refers to are those which do not start with experimental data gathering. In this practical task, however, having data that appears 'too' accurate may even cause the teacher to be suspicious. Ellen's original set of data could be fitted exactly to a linear formula:

H Um, I'm not convinced that will be right, I don't know. . .

I Why not?

H Um, I think the real working, from our when we were working out, we produced a table and it wasn't always constant. It wasn't always two point five. Um, sometimes it actually came out to be more than that. It was reasonably consistent. It's hard to actually work it in practice in ideal conditions because sometimes the way that you've set up the blocks and so it needs really some constant sort of method of putting it into practice. But I mean I, I mean she's used, she's formulated her own theory in her mind and she's made a prediction which I think is, which is all valid to be honest about it.
(Harry: 57-65, Ellen)

For Harry, being 'right' in this context involves having the messy sort of data that arises from a scientific experiment. The absence of error leads him to suspect that Ellen may have worked backwards in order to produce a 'clean' set of data. On the one hand accuracy is valued, while at the same time it may be taken as a sign of inappropriate problem solving strategies. Harry's discomfort with this is resolved by the 'hedge' at the end of the above extract which allows him to accept Ellen's data without penalty, although he goes on to suggest that commenting on the inconsistencies in her data (for whose existence there is no evidence in the text) would have allowed her to be assessed at a higher level.

she should have found that unless she was perfect in the way that she set up the blocks, there would have been a certain amount of inconsistencies in the actual two point five times. Um so I think some sort of mention of that here rather than saying it is always two point five times. I think to go to a higher level she maybe should have mentioned something about that, even if she couldn't say why.
(Harry: 69-75, Ellen)

Charles suspecting a similar strategy in Sandra's work, resolves the problem in the opposite direction:

But having had a go at it myself earlier, I think probably I'd have realised well it doesn't quite work out quite as neatly as that. Umm so think to a way she's changed to .. I think she's made things fit the pattern that she wants. Umm

¹Harry's reading of Sandra's diagrams as a sign of the nature of her practical activity is discussed in chapter 11.

so for that reason I think probably you'd have to mark it down slightly if you were going to do a marking.
(Charles: 176-182, Sandra)

In this case, Sandra's data did not even coincide with Charles' beliefs about the correct results; this seems likely to have influenced his decision to "mark it down" rather than declare the strategy valid.

The student working on such a practical task is thus placed in a situation in which she is expected to gather data that is accurate but not too accurate. She must then comment on, and preferably explain, inconsistencies in her data (even if there were none). It appears that these teachers are looking for an ideal level of accuracy appropriate to a practical task. It is likely that many students do not share a knowledge of this ideal level or an appreciation of the differences in teacher expectations of accuracy between pure and practical tasks. While working backwards from the formula in order to generate data may be a successful (if not teacher approved) strategy in a pure task it may be detected and hence condemned more easily in the practical task. The way in which both pure and practical tasks have been fitted into the same 'investigation' format may make it more difficult for students to distinguish between them and between the different sorts of expectation that teachers may have.

13.2 Correspondence between data and formula

Whether or not the original data was seen to be correct, teachers reading both tasks expressed concern that the formulae produced should 'work', fitting the data gathered at the beginning of the task. Thus some, like Joan, spent time checking the formula with the data provided:

he's leapt into a formula . . . so let's just see if it works . . . so he's given the top the bottom, the slant height and number of triangles, top plus bottom times the slant height . . . let's try one of these [referring to the data in Richard's table] . . . 3 and 5 is 8 . . . it seems to . . . yeah, so that seems to work just as a quick check.
(Joan: 72-77, Richard)

Other teachers, however, were more prepared to take the correctness of the formulae on faith or to make a more rapid judgement. Interestingly, both Andy and Fiona, who had expressed most concern about the accuracy of the original (Inner Triangles) data, were willing to accept the resulting formulae without thorough checking.

As was seen in the previous section, teachers reading the 'Topples' texts were faced with the dilemma of what to do with Sandra's work which was based on a set of data which they all believed to be wrong but which contained a formula which fitted the data set reasonably well. Charles, before reading the texts, identified this as a potential issue:

I think I'd be looking for some kind of statement based on their observations and their tables of results. So even if the table of results may be wrong, if they could generalise their own table of results, so long as it wasn't too trivial, I'd be quite happy to actually give them something for that. . . Um . . . how important is it that they get the right result? I think it's . . . I think it's more

important that they go through the right thought processes that they achieve the right result than actually get the right result itself as long as they're aware of how they should be tackling the problem, or aware of strategies that they can use to tackle the problem rather than actually getting the right answer would be more important.
(Charles: 36-44)

His focus on process rather than content allows him to take a relativist position towards the accuracy of results. Thus he decides that consistency between the generalisation and the data is more important than correctness, although he tempers this with the caution that the generalisation must not be too "trivial", indicating that the process-content distinction is still problematic for him. It is this idea of triviality that allows Harry eventually to justify assessing Sandra's work to be at a lower level:

- H . . . from what she's seen um using her formula the maths is correct. (. . .) and she's presented the table for results so again you know there's no there's no criticism of that so far based on what she's - it just makes the investigation very very simple. (. . .)*
- I You said that the fact that she'd got a different set of figures from the other ones meant that anything [...] made the investigation very simple. Does she then get penalised for that, or to what extent would you see her mark being lower at the end?*
- H Firstly the maths I think, the numeracy skills there, they're definitely being affected. And so is the formula. So she's not actually getting up to very high level in that respect. So that's that's straight away going to be putting a limit onto how high a grade you can give her. But at the same time what she's actually seen as evidence herself she's actually articulated quite well so that in terms of presentation and what she's actually found from her results it's on a par with the other people but the numeracy there isn't as much . . .*
(Harry: 230-262, Sandra)

The difference between the degree of complexity of Sandra's relationship (of the form $2n+2$) and that of the relationship found in the other students' texts (of the form $2.5n$) seems hardly significant enough to make such a profound difference to the evaluation of the work overall. It is, however, sufficient to provide grounds for Harry and others to justify their low assessment of Sandra's work without making overt use of the fact that her results are wrong.

13.3 Conventional use of algebraic notation

The students' texts on both tasks included examples of formulae expressed in algebraic forms which did not conform to conventional norms. Teachers reading both sets of texts commented on these formulae, suggesting that there is some tension between the notion of 'correct' use of algebraic notation and the idea that such notation is useful for communication.

Speaking in general terms, Joan identifies 'working' as the primary criterion for assessing the validity of a generalisation:

a lot of the less able ones certainly find the mapping fairly easy to deal with - the idea of an input and an output - whereas as soon as you say something like $3x+1$. . . the order, the fact that the 3 is before the x is very confusing, so . . . I wouldn't be saying it's got to be a specific solution so long as it's one that works. It does make the marking harder of course cos it means you've got to check every single variation
(Joan: 72-78)

She suggests that conforming to convention is not important to her assessment (although unconventional forms may make it more difficult). At the same time, however, she identifies this as an area in which “the less able ones” are likely to have difficulty. An unconventional form may thus serve to mark a student as “less able” and hence affect Joan’s judgement of the whole text. In practice, this ambiguity about the value to be placed on convention is reflected in her discomfort about Steven’s formula for the number of ‘Inner Triangles’ in a trapezium ($y + x \times z = \text{Unit No.}$) which fails to include brackets to indicate the correct order of operations:

the use of the algebra unfortunately is still not absolutely brilliant as he hasn’t used the brackets in the right place but [it could] have just been an error so . . . that’s probably not a fair thing to say . . . The use of um, how could I put it, the symbolic communication isn’t quite there. (Joan: 262-266, Steven)

It is interesting that she hedges her condemnation of Steven by suggesting that he may have “just” made an error. A judgement that he had made an error would presumably be considered more lenient than a judgement that his use of algebra suggests him to be one of the “less able”. In contrast, Andy seems to read the same formula in a rather different way:

The formula is accurate needs a bracket in it but it’s quite clear that his intention and he’s given a nice example which clarifies his thinking, so although algebraically it’s not that strictly correct, it’s quite clear he knows what he’s doing. (Andy: 100-103, Steven)

Unlike Joan, who condemns Steven’s “symbolic communication” and hence his ability, Andy judges Steven to have communicated his intention and his competence effectively, taking into account the example accompanying the formula as well as the formula itself. I would suggest that these two teachers are both resolving (in their different ways) a tension between valuing the student’s self-consistent solution and valuing the use of conventional algebraic forms - a tension which once again seems to arise from the process-content dilemma in the coursework discourse.

The question of conventional use of brackets was raised again in readings of the ‘Topples’ texts by Steven’s formula: $(a) + (a) + \left(\frac{a}{2}\right)$. Although this formula is technically ‘correct’, it is not conventionally concise², making unnecessary use of brackets and failing to simplify the whole expression by ‘collecting like terms’. The teachers’ readings of this formula echo the tensions described above in the ‘Inner Triangles’ context. Harry, for example, while eventually accepting that the formula “does sort of work” takes its form as a sign of a lack on Steven’s part:

*H Um, a lot of brackets around from this formula. . .
I Is that good or bad?*

²As Pimm (1987) remarks, the mathematicians “aesthetic for symbolic expressions” includes valuing “brevity over clarity” (pp.126-7)

H . . . Well I'll have to see what it's like from the figures. It's - my initial reaction to this formula is that he doesn't really know how to use brackets. But maybe that's not too much of a worry in this case cos he's got three lots of A divided by two. He's tried to get this two point five which, you know, is the point of the thing, but . . it may need a quiet word about when to use brackets and when not to maybe.

(Harry: 131-138, Steven)

The main conclusion that Harry draws from this is related not to Steven's achievement on this particular piece of coursework but to his need for further teaching. Charles, on the other hand, while expressing similar dislike of the unconventional form, nevertheless considers its value within the context of an assessment of Steven's understanding and problem solving processes:

I didn't think it's expressed very well. I think it could be expressed better if I was looking at the algebra but saying that, he's obviously used the formula and obviously understands the formula and gets reasonable results from it.

(Charles: 93-96, Steven)

Like Andy reading Steven's formula above, Charles considers using and understanding to be more important to his assessment than "the algebra".

There are clearly differences between the ways in which different teachers resolve their readings of students' work containing unconventional algebraic forms. The examples discussed above suggest that such 'errors' give rise to similar tensions and possible resolutions for teachers reading both types of task.

13.4 Summary

Despite the lack of direct comparability between the sets of texts used and the teachers' readings it is clear that there are differences between the issues related to error and accuracy that arise in the contexts of the 'pure' and the 'practical' tasks. In the practical situation there is a tension between the value placed on accuracy both in the data and in the formulae arising from the data and the simultaneous belief that data arising from a practical situation is likely to be inaccurate. This gives rise to problems in assessing students' texts both in cases where the experimentally derived data is 'incorrect' and in cases where it is too perfect. A valuing of 'process' over 'content' suggests that the relationship between data and formula is more important than the relationship between either and any sort of objective reality. It appears that 'incorrect' data and formulae may thus be acceptable in the practical setting. If the resulting formula is perceived to be too simple, however, this conflicts with the value that is placed on demonstrating mathematical skills. At the same time, while there is a concern in both types of task that the formulae should 'work' for the data, if the correspondence is too perfect in a practical task this may raise suspicions that the student has used an inappropriate method, working backwards to generate her data from the formula rather than generating the data experimentally. While the data available in the present study does not provide evidence of teachers' reactions to inaccurate numerical results in a 'pure' setting, it

does appear likely that a student producing a perfect set of results would be given credit for working in an appropriate way. From the student's point of view, the general similarity in the structure of investigational tasks and in the forms of reasoning expected may make it difficult to distinguish those tasks which the teacher-assessor may designate as 'practical' and hence to know what levels of accuracy and of match between data and formula is desirable.

In almost all the cases in which errors are identified by teachers, there is a suggestion of anxiety on the part of the teacher in deciding how to judge the student's work. There is repeated shifting between condemnations of the errors and playing down their importance in coming to an assessment of the student. Part of this shifting may be related to the multiple roles that the teacher has in relation to the student and the student's text, in particular the potentially incompatible roles of examiner and advocate on behalf of the student. I would suggest, however, that an important role is also played by the tensions that exist within the discourse of investigation and coursework and between this and the traditional discourse of external examination. As was indicated in chapter 4, the discourse of investigation and coursework within which the teachers and students are situated (including the publications of the examination boards) officially values diversity and, in particular, places emphasis on the use of general mathematical processes rather than specific pieces of content knowledge. Simultaneously, however, the discourse contains ambiguous messages about the importance of accuracy. On the one hand, there is stress on the idea that investigative work does not have single right answers and that there is even positive benefit for students in being wrong, while on the other hand students producing coursework for examination purposes are also expected to demonstrate mathematical skills and knowledge. (The details of this analysis are presented in Appendix 15.) In this respect, the examination of coursework is not clearly distinguished from the traditional paradigm of assessment in mathematics in which great emphasis is laid on accuracy and hence great significance on the identification of error. It is hardly surprising, therefore, to find such tensions and evidence of anxiety reflected in the way the teachers read students' texts containing errors.

14 Teachers' responses to 'unusual' features of students' work

The introduction of investigational work in school mathematics and of assessment by coursework had as one of its aims to encourage students "to create their own mathematics: actively taking part in mathematical thinking rather than passively receiving mathematical thought" (Pirie, 1988: p.7) (see also Appendix 15). There is, however, a tension between the aim of encouraging creativity and the traditional values of assessment practices which aim to make the final grade appear valid and 'objective'.

Early in her interview, when asked what she would look for in students' answers to the 'Inner Triangles' investigation, Joan, a head of department, described her attempt to resolve the tension between a perceived need for standardisation and a recognition of variation in students' work:

. . . gradually as we're going along we're actually building up a bank of these [guidelines for 'levelling' work on particular investigations according to the National Curriculum]. Now the big problem with that of course is that before you've actually seen the children's work you're not always sure what you're going to come up with and they do sometimes surprise you, come up with something brilliant or just something that you haven't thought of. And so the idea, as I say, is that we build up a bank of these levelling guidelines and then review them in the light of what the children have done. That's the first thing perhaps to explain to you. Um the reason we started doing it really was for standardising because that's been the biggest problem . . . (Joan:11-20)

The fact that Joan chose to describe this general process before addressing the specific question she had been asked, together with the repeated statement that this is a "big problem", point out the importance of this issue for her.

While creativity or coming up with "something brilliant" in coursework may be rare (and how indeed may it be recognised?), the nature of extended, relatively independent work ensures that there are many variations between students' responses to the same starting point. These variations include different lines of inquiry (including 'extensions'), strategies, interpretations of results and forms of communication; variations identified as errors are dealt with in chapter 13. In this chapter, teachers' identifications of and responses to such variations in the students' texts they were asked to read and assess will be examined.

14.1 What is different?

The texts which were read by the teachers during their interviews were selected to be different from one another in a number of ways related to their linguistic and other textual features (see chapter 6). However, these were not on the whole the differences identified explicitly by the teachers during their reading. There was generally little direct comparison made between the texts read, but comparisons were made between each individual text and an imaginary norm. For those teachers who were familiar with the tasks this norm may have

been based either on their general expectations related to the specific task or on their recollections of other students' work on it, including knowledge of particular common results or ways of working. Joan's collection of "levelling guidelines" described above suggests a formalisation of such norms.

Although all of the teachers interviewed had been given the statement of the task several days in advance and most had taken some time to work on it and become familiar with it, few had actually used either 'Inner Triangles' or 'Topples' with students or seen students' work on these tasks before their interview. They were, however, able to identify some aspects of the students' texts as being 'different' and thus worthy of comment. In these cases, the identifications cannot have been made on the basis of concrete knowledge of what students usually produce in response to these particular tasks but rather on the basis either of their own work on the task or of more general expectations and experiences of how 'typical' students respond to this *type* of task.

When the teachers were reading the students' texts, much was read quickly and either passed over without comment or evaluated by applying a common category. This was often done by making a comment about the ways in which students or teachers work in general rather than by referring in detail to the characteristics of the individual student's text. Much of the students' work thus appears to be read in a routine way as typical non-problematic examples of the type of work usually produced by students. When a particular feature of a text was commented upon during an interview it was often in the context of making a general point about common characteristics of students and their work. Where a teacher has dealt in an individual way with a feature of a specific text, this has been categorised as an instance of identification of 'difference'. These instances are identified in the interview transcripts by one or more of the following indicators: an explicit statement that this student's work is different or unusual; a statement that the student has had an idea or has thought about the problem; an expression of interest, surprise, or difficulty; an effort to make sense of the mathematics involved; the construction of a narrative to explain how the student might have arrived at a result. The features identified as 'different' are shown in Tables 14.1 to 14.3 below.

Table 14.1
'Different' features in 'Inner Triangles' texts

	<u>difference</u>	<u>identified by:</u>
Clive	use of a two-way table rather than a list to show his results a table containing zeros indicating the impossibility of drawing certain triangles use of a diagram to demonstrate a number pattern	all (this is discussed in chapter 11) Andy Joan
Richard	investigation of the area of a star in terms of its perimeter format of formula for the area of a triangle	Joan, Andy, Dan Andy

Table 14.2

'Different' features in 'Topples' texts

	<u>difference</u>	<u>identified by:</u>
Steven	presentation of a method of scaling up earlier results as an alternative to applying his algebraic formula inverse formula	Harry, Grant, Jenny Grant, Jenny, Carol (this is discussed in chapter 12)
Ellen	extension	Harry, Charles, Jenny, Carol
Sandra	extensions	all

Table 14.3

'Different' features in 'Inner Triangles' extracts

	<u>difference</u>	<u>identified by:</u>
No.1	format of formula	Andy, Dan
No.3	putting the formula before the table	Dan

14.2 Difference is desirable but difficult

Doing something that is different may be judged to be good in itself without examination of the details of the difference:

*He's done it in a different way. That's good, they can see it in a different way
[. . .] I might give him credit for doing it in a different way. (George: 58-60, Clive)*

George's switch between the singular *he* and the plural *they* suggests that his approval of Clive's novel method is not specific to this single case but is a general approval of any "different way" of seeing or doing. The quality of the specific case is not significant in making this judgement. Similarly (also in the context of commenting on Clive's work), Dan expresses general approval of difference for its own sake:

I always like it when kids come up with something that is . . . at least to some degree different, you know. They've thought, you can see that they've thought it out themselves rather than just sort of going along the usual trail and doing the obvious things. (Dan: 203-206, Clive)

Differences and unusual behaviour do, however, cause problems for teachers. Extract No.1 contained a generalisation for the number of inner triangles in a trapezium in the form $ab + ac^1$ rather than the more commonly found $a(b + c)$. Although the performance indicators provided by the examination board actually gave the formula in this form, both Andy and Dan, who had used the Inner Triangles investigation with their students, commented on this as unusual.

¹Where a = slant height, b = length of base, c = length of top.

it seems the formula which they produced where they multiply the top by the slant and multiply the base by the slant and add them to find the area. It's perfectly correct. It's an unusual way of finding the area. It's more common to add top and bottom first of all. From a geometrical point of view it's easier to do it that way. These things happen, and it's nice that children do work in different ways and I'm not at all surprised that somebody's found that way of doing it from the geometry angle. (Andy: 292-299, No. 1)

Andy's reaction is complex. He first interprets the formula by reading it using the full referents of the variables rather than the single letter variables used in the student's text, thus marking the formula as non-routine and in need of interpretation. He then validates the formula; the qualification of his judgement of its correctness with the word *perfectly* indicates that there is a particular need to validate this formula because of its unusual format. The usual formula would be more likely to be passed over either without comment or with a bare 'that's right' - the verbal equivalent of a tick. His validation of the student's work is then elaborated: not only is the formula correct but it is given some additional value because it is in general "nice that children do work in different ways". In using several bases - both specific to this case and general principles - for judging the unusual formula to be acceptable, Andy appears to be displaying some discomfort with his judgement which needs to be resolved by creating an extended narrative to justify himself. He elaborates further by suggesting an explanation of how the student might have found the formula. There is no evidence in the extract of the student's text presented to Andy of how the formula was arrived at or, in particular, that any consideration was given to the geometry of the situation; he appears to have constructed this explanation from his own understanding of the 'Inner Triangles' problem and from his need to justify his judgement of the work.

Dan, on the other hand, is not able to construct the same narrative as he appears not to have considered the possibility that students might work on this problem "from the geometry angle". Nevertheless, he too constructs a story of how the student might have arrived at the formula, stressing the unusual nature of the result. The agency for the discovery this time is "curiosity" rather than geometry

I mean I think it's unusual that it's that way round . . . I actually think that most children anyway would see that way round first [pointing to the form $a(b + c)$] and see those two added together multiplied by that far easier than they would see . . . I mean unless there was a table there that existed before that actually did that $[ab]$ and did that $[ac]$ and then did that $[ab + ac]$ just by curiosity and saw that it came out like that (Dan: 335-341, No.1)

Unlike Andy, however, Dan treats his explanation as merely hypothetical and is not satisfied that it establishes the validity of the student's result.

I mean I find this a very difficult thing. I think I'd have to talk through this . . . I find this an odd way round of doing it and I don't think I'd give them straight advice would be difficult I'd have to actually talk to the children involved and discover why they did that and how they came across that, so I'm a bit curious about how that comes up. They never explained how it works but anyone can get a formula as long as they understand algebra and they can get a formula and show how it works but that's the wrong way round. You

want to know how it's actually derived. If there wasn't enough information I'd be a little bit worried about that. (Dan: 388-397, No.1)

The unusual structure of the formula makes him “a bit curious” and “a little bit worried” and he finds the interviewer’s request to give the student advice about how to improve her work “difficult”. The modality of this passage suggests the conflict that Dan is experiencing in attempting to deal with unusual student behaviour.

The other two extracts presented to the teachers simultaneously with No. 1 gave the formula in the more usual structure (although using different notations). Although neither of these two extracts contained any real evidence of the origins of the formulae, this was not commented upon by either Dan or Andy; in these cases they were prepared to accept the formula as given without questioning or attempting to justify its validity. The ‘usual’ way of doing things is thus naturalised by these teachers. Although explanation even of the usual method is desired (see chapter 11) and even required for its own sake, the usual result or formula is likely to be accepted as valid and unproblematic even without any explanation. The unusual, on the other hand, is suspect and needs further validation and the construction of a narrative to explain its origin.

In his ‘Topples’ text, Steven presented a method for finding results for large lengths of rods by scaling up his results for small lengths. This method caused difficulties for some of the teachers because they were unsure of its meaning or its validity.² Grant, for example, while expressing interest in the method, is clearly uncomfortable with it:

It's interesting that the next part works, I don't know if it works for everything or it just works for this but he's spotted it and again he hasn't really looked into it any further. He's done it for one case but whether it would work for any other case is er I don't know, he hasn't looked into it. . . And he's used it in the next part er used the this multiplying section in the next part and it's just a knowledge of number that's got him there I think intuition whatever. He may have guessed at a few and found one that works for it for that. It's worth putting in . . . (Grant: 169-176, Steven)

Like Dan explaining No.1’s formula above, Grant creates a narrative to explain how Steven might have arrived at the method. Grant himself is unsure whether the method would work in general. Perhaps because of this uncertainty, his narrative devalues Steven’s achievement, suggesting that the processes involved were perhaps not really ‘mathematical’: ‘spotting’ the method, not looking into it properly, guessing, using “just a knowledge of number” or “intuition”. Steven is clearly not being given credit either for the result itself or for the processes he may have gone through in order to arrive at it. Nevertheless, it is stated to be “worth putting in”, perhaps simply because it is different and hence may be taken as a sign of an attempt towards originality.

²Given that Steven’s data and his algebraic formula (which was accepted by all the teachers) indicated that the quantities were in direct proportion, this scaling method is completely valid.

While the teacher reading as mathematician or as a teacher personally interested in the student may express pleasure and interest in novel student behaviour, this causes tension with the teacher's role as assessor, both in deciding what credit to award and in the effort required to validate a non-standard result, particularly if it is not accompanied by extensive verbal elaboration. Different teachers, constructing different narratives of student behaviour to explain the text, resolve these tensions in different ways.

14.2 'Good mathematics' v 'good coursework'

The tension between the value placed on difference and the difficulty in validating it is also reflected in conflict between teachers' expressions of what is good mathematics or good mathematical behaviour and their judgements of what is good coursework. For example, while all those interviewed regarded evidence of working systematically and explaining how results were achieved as important criteria for assessing coursework, there were several indications that these were not necessarily considered to be sufficient criteria for identifying good mathematicians. Steven's work on 'Inner Triangles' was described as

a good sound piece of work from not particularly able mathematician in my analysis. But somebody who certainly does work systematically, can explain the tables they've put together with no extension (Dan: 313-316, Steven)

It is not clear on what basis Dan is judging Steven to be "not particularly able" although this may be related to the routine nature of his work on the 'Inner Triangles' problem, none of which was identified as unusual by any of the teachers. The requirement to work systematically is not even necessary for producing good mathematics although it is necessary for good coursework. Joan presented a theory about the differences between adults and children to explain the differences between her own way of working successfully and her expectations of students' work:

It just so happens that if I was doing an investigation at my own level I wouldn't do it like that I would jump about a bit, but probably you see I'd lose marks for that . . . So I don't happen to build things up like that but it's a good way of doing it especially at that age basically cos there's something, not just the mathematical ability but there's the maturity of the person as well, so I think that's got to be taken into account (Joan: 282-288)

Pedagogic advice to "build things up" in a systematic way that might be offered to help students to get to grips with investigative work has been converted into an assessment requirement to demonstrate that the work has been done systematically.

The 'Inner Triangles' coursework text which caused most difficulty for the teachers was Richard's, which consisted almost entirely of diagrams, tables and symbolic formulae without any accompanying verbal elaboration (a comparison of two teachers reading this text may be found in chapter 10, see also chapter 12). This text clearly failed to fulfil the criteria for good coursework while its author was nevertheless judged to be an able mathematician. Richard's text provided one of the few instances of student's work which provoked interest in the

mathematics from the teacher readers. He extended the 'Inner Triangles' problem to look at the areas of six pointed stars drawn on isometric paper. Andy analysed this section in detail, commenting on the change of scale that had been used in the examples given (see chapter 11). Dan also spent time on this section:

Now this one he's gone on to stars and he's sort of gone on to stars hasn't he? I don't quite know what he means by this . . . Perimeter times slant height . . . mm! . . . Blimey, I would definitely have liked a bit more explanation of that, cos that's quite interesting isn't it. The perimeter multiplied by the slant height gives you the area inside, that's quite an interesting find which I doubt if many other people did so it's quite innovative and something a bit different but given us absolutely no . . . not even 'oh gosh look at this' would have been . . .
(Dan: 162-169, Richard)³

His approval of the interesting and unusual is stressed by repetition but is simultaneously qualified (quite interesting; quite an interesting find; quite innovative; a bit different) as if to reconcile this approval with the simultaneous disapproval of Richard's lack of explanation. It is interesting to note that the 'explanation' required does not appear to be any form of mathematical proof but is merely a commentary on the surprising nature of the finding.⁴ This 'innovative' section is also the key to Dan's final evaluation of Richard's whole text, although he still indicates that the lack of "English skills" is a problem:

I think I would probably go for putting Richard's first . . . If nothing more than he's worked on the star, which although he didn't develop, I mean that's, I mean Richard's would easily be the top if there was more development, if there was more explanation in there.
(Dan: 323-327, Richard)

Good mathematical behaviour may thus include working in a non-systematic way and providing little in the way of verbal explanation as well as setting oneself unusual and interesting tasks. These behaviours, however, all cause problems for teachers as assessors. The first two are explicitly disapproved of in coursework; this disapproval being justified in terms of the characteristics of immature students or the requirements of the external examination board.

³The way in which Dan shifts between a position as interested reader and a position as examiner in this passage is discussed in chapter 9.

⁴The suggestion that Richard might have written "Oh gosh look at this" assumes that his ideas of what is interesting and unusual must coincide with the teacher's ideas. This suggests a stereotyped idea of the nature of the problem and even of mathematics. Stars (unlike trapezia, triangles and parallelograms) are not figures whose areas are normally considered in school mathematics; they are thus interesting. This does not take into account the fact that, when working on isometric paper as these students were, stars are highly regular and easy to draw. They might thus be considered by the student to be a normal and natural variation rather than an innovation. Using the perimeter as one of the variables in an area formula may also be considered as unusual but this could only be identified by someone with a knowledge of a wide variety of standard area formulae and a general view of their common features.

14.3 'Interesting' extension

Both the 'Inner Triangles' and 'Topples' tasks asked students to do an "Optional Extension", extending the investigation in a way of their own choosing. This is the part of the task that would appear to provide most opportunity for originality on the part of the student as neither a task nor ways of working are specified (although the 'Inner Triangles' extension does restrict the student to using shapes drawn on isometric paper). Of the selected student texts, Richard and Clive included extensions of 'Inner Triangles', considering several different shapes each, while Ellen and Sandra extended 'Topples', building piles of rods according to different rules.

Neither of the extensions of 'Inner Triangles' were remarked upon as unusual, apart from Richard's stars which were discussed above. Andy, who was familiar with this task, commented that "extending it is fairly limited I think on this one" (Andy: 58). The restricted nature of the context of this task means that the teachers are likely to find the extensions predictable and hence are unlikely to consider them interesting.

In contrast, the 'Topples' extensions were remarked on frequently, either as being interesting or because the teacher appeared to have difficulty understanding what had been done by the student. Indeed, the very fact that the teachers had difficulty in making sense of Ellen's extension seems to be associated with the interest they displayed:

I don't understand what's happened . . . I don't really understand that, what she's put there. But I would presume that that's just the number of units at the bottom and um yeah this it looks interesting but there isn't actually a clear explanation as to what this new task is.
(Harry: 82-84, Ellen)

although the difficulty in understanding also makes it difficult to validate this section of work:

I'm not sure what's happened on this extension here. I think she's just put some more [. . .] numbers in. Quite an interesting extension . . . I'm not sure, obviously again I haven't done this so but I'd probably think about checking some of those results and I'd probably, if she was doing the work I'd probably be asking her to go back and check them and see if there was any mistakes.
(Charles: 128-131, Ellen)

Since these teachers claim to be unable to understand what Ellen's extension is about, its 'interest' must lie in its novelty rather than in any intrinsic quality.

Sandra's first extension, which was illustrated by a diagram showing two piles of rods set at right angles against each other, also gave rise to difficulties for the teachers in understanding the nature of the problem. This extension seems, moreover, to be judged to be inappropriate. Harry labelled it as physics rather than mathematics:

Very interesting extension but I think there's more, there's more sort of physics than actual maths in here to start with because there are things balancing against each other and reacting against each other in different directions so I think that's far more complicated than it actually seems.
(Harry: 238-241, Sandra)

and Carol appeared relieved to find Sandra's second extension which returned to building a single pile with the rods increasing by two's:

Yeah this second extension is more what I'm, is the more predictable sort of extension where instead of going up in unit at a time decided to go up two units at a time

(Carol: 161-162, Sandra)

Similarly, Jenny found Sandra's first extension 'interesting' but simultaneously rejected it in favour of her own idea of what an extension ought to be:

I What would you hope to see in an extension?

J Well I think this one is probably reasonably interesting, mm, the idea of having them joined together. I'm not sure how it works. I think I'd want, I mean I'd expect them to start with something other than two units at the bottom and go up in ones is what I'd expect. Or to go up in, to start with two units and go up in, no you can't go up in threes you got to go up. No I think that's all I can think of really is starting with a different base point ... yeah and I would expect anybody who was going to get anywhere seriously to do that.

(Jenny: 141-147, Sandra)

There is a tension here between the value placed on originality within the discourse of investigations and these teachers' apparently clear ideas about the ways in which an investigation may be appropriately extended. The 'predictable' extension is applauded, although it may not be seen as 'interesting'. Jenny's expectation that "anybody who was going to get anywhere" would pose the same extension question as she would herself suggests a view of the nature of problem solving that allows little room for alternative lines of inquiry. Moreover, given that Jenny admitted to having spent very little time working on the problem herself, it seems likely that she had not pursued her expected extension and seen it to be useful in some way but was judging its appropriateness on the basis of general expectations about the formation of extension problems. In general, little effort appeared to be made to make sense of the problems posed by the students, suggesting that value is placed on their existence rather than on their content or possible solution.

14.4 'Open-endedness': pedagogical and mathematical opportunities

Difference or originality in students' work can only occur without being identified as error when the task set as well as the assessment criteria allows some room for alternative responses to be acceptable. The teachers' treatments of difference and originality described above have been largely from the point of view of teachers reading as assessors, adding to or subtracting from their evaluation of a piece of work or an individual student without explicit attention being paid to the nature of the task. Andy, one of the teachers who was familiar with the 'Inner Triangles' task in the classroom, also discussed the task itself and the students' work from a pedagogic position. The following passages illustrate his shifting between these different positions and the tensions between them. Firstly, in response to the request to say what sort of things he would look for when assessing work on this task, he responded with descriptions

of the nature of the task itself and of his own pedagogic behaviour in the classroom as well as of signs he would look for in students' work.

5 And then question two is a more entertaining question in that it's slightly more
open ended and it gives the children opportunity to try to find dimensions for
themselves. In the class, quite often I've done it, they'll have one answer for
eight units and one answer for thirty two units and then the question that you
pose is are there any other trapezia which you could make which give you
eight and thirty two. In a way . . . if they say they've done it then they have
because the question's quite a straightforward one: give the dimensions of a
trapezium rather than all trapezia. But that gives you the opportunity to get
10 them to think about different types of trapezium, different sorts of sizes and
that then obviously leads them to part three. So on part two in the class I'm
looking for open the question: how many different types of trapezium give
them eight and thirty two. Question three then, then becomes a little more
open ended again and you're sort of starting the investigation properly. I
think with that I think there are two things. First of all can they calculate a, the
15 area of a triangle just by looking at the the dimensions given for any particular
trapezium without drawing them and counting. Can they get to that stage.
And then I suppose it would lead us to the second part would be, given an
area, how many different trapezia can be drawn and is it true that a
trapezium can be drawn for any given area. So under what circumstances,
20 for example, I don't know, an area of thirteen? An area of thirteen. Under
what circumstances can you draw a trapezium of area thirteen. And that sort
of thing. Tackling those sorts of questions, really provoke them in the job of
the work required for that particular investigation. (Andy: 13-37)

Being "open ended" is approved as being "more entertaining" (line 1) and a "proper" quality of an investigation (line 13). Similarly, Dan dismissed the closed questions at the start of the 'Inner Triangles' task with the words "I wouldn't actually call that any kind of coursework" (Dan: 101-102). Most importantly for Andy, being open ended provides "opportunities"; the importance of this idea is signalled by its repetition both in this passage (lines 2 and 8) and subsequently when reading Clive's work (see the two extracts below). These opportunities are identified by the teacher, based on Andy's own ideas of what is mathematically "entertaining" or possible within this problem. It seems also that they are likely to be missed by students and that they thus become opportunities for the teacher to pose questions (lines 4-5) and "to get them to think" (lines 8-9) rather than opportunities for the student. Andy points out two such missed opportunities in Clive's text:

He has an opportunity there . . . yeah, he has an opportunity which he's missed which would have been quite useful. He's actually given two different trapeziums with an area of twenty four so from his table he could have actually seen that two different trapezia could be drawn. But to be fair the question doesn't say that. It just says give every dimension. The work is there but, the data's there to make that connection if he wants to.
(Andy: 137-143, Clive)

And interestingly there he's got a lot of zeros showing and he's indicated that these obviously cannot be drawn. And nicely the ones which can be drawn are all square numbers but he's missed that opportunity to make that clear.
(Andy: 156-159, Clive)

There is a tension here between Andy's role as a teacher and his role as an assessor. As a teacher, he is wanting his students to take the opportunities he has identified for them. As an assessor, on the other hand, he is expecting them to answer very specific questions. Thus, in

the first passage above, having said that question three is “open ended”, he immediately goes on to list the precise outcomes that he expects to see in students’ answers (lines 14-21), although after this he shifts to a teacher/adviser position, suggesting that these are questions he would pose to “provoke” the students to come up with what is “required”. In either case, however, being open ended does not appear to mean that genuinely alternative or unanticipated approaches would be valued. The task for the students has become ‘guess what the questions are’. When his students fail to take the opportunities he has identified as provided by the task, Andy then acts as advocate on their behalf; they may not have done good mathematics but they have nevertheless done what is required by “the question” posed by the external examination board and they should not therefore be penalised.

14.5 Summary

In reading students’ coursework texts, teachers seem to have clear expectations about what a text is likely to contain. The individual text is compared with this imaginary norm and deviations are noted. The characteristics of such a ‘typical’ text seem at least in part to be related to the characteristics of standard ways of working, including in particular the data-pattern-generalisation process. This leads to expectations about the order of presentation of work and the type and form of results. For example, inspecting values in a table arranged in a standard way is likely to give rise to a formula expressed in a standard format; deviations from that order or format (e.g. placing a table after the formula or using different bracketing in the expression of the formula) may be seen as unusual. Teachers’ expectations relate both to the content of the work and to the way in which it is written. Expectations about the writing were discussed in chapter 11; deviations from the ‘ideal text’ constructed by these expectations are likely to be condemned. In contrast, deviations from expectations related to the results achieved, questions posed and paths followed may be read more positively and may even be taken as signs of high mathematical ability.

While some features of the students’ texts were noted as unusual by all or most of the teacher-readers, others were less generally identified (although not mentioning a feature does not necessarily imply that the teacher considers it ‘typical’ - indeed, one response to the unusual may simply be to ignore it). It seems likely that familiarity with the task and with students’ work on the task would play a large part in forming teachers’ expectations. However, it is clear that even those teachers interviewed who had never used the task with students and had spent little time working on it themselves were nevertheless able to compare the students’ texts to an imaginary norm, presumably basing their comparison on their experience of other tasks.

As might be anticipated, given the place of ‘creativity’ within the discourse of investigation, features of students’ texts identified as unusual were generally welcomed by teacher-readers.

While in some cases this approval seemed merely to be of the fact that the feature was unusual, a number of reasons may be inferred or were explicitly stated in relation to specific features. It seems that the unusual:

- is 'interest'ing';
- is evidence that the student is thinking for herself rather than following a routine algorithm;
- may be a s'gn of a good mathematician.

However, the fact that the teachers are reading the texts in order to assess them means that their appreciation of the unusual is not unproblematic. Furthermore, even a favourable evaluation of the student's thinking or ability may not be converted into a comparably favourable assessment of their text.

The assessment context brings with it assumptions about the importance of standardisation and 'fairness' that are in tension with the valuing of 'difference'. Standardisation of the assessment of students' work is sought through the use of criteria and performance indicators such as those provided by the examination board and through the use of exemplar material. The use of exemplars, as described in the extract from Joan's interview at the start of this chapter, does not address the problem of how to assess unusual work; it is likely, however, to contribute towards the construction of imaginary 'typical' texts. Similarly, performance indicators for individual tasks contribute towards teachers' specific expectations of student behaviour on the task without providing guidance for dealing with unusual work. The general criteria, while stated in terms which could in principle be applied to any piece of work, appear in practice to be associated with particular features of a 'typical' text. Teachers, therefore, experience some difficulty in applying them to more unusual texts.

In reading and assessing sections of students' texts which they have noted as unusual, the teachers appear to need to decide whether the work:

- is correct;
- was achieved by using appropriate processes (i.e. not by copying, guessing, etc.);
- fulfils assessment criteria.

These three aspects all caused problems and are in tension with the reasons for valuing unusual work listed above.

At least part of the source of 'interest' in an unusual section of text lies simply in its novelty and the consequent need for a reader to struggle to make sense of it. Teachers' expressions of interest were often accompanied by expressions of difficulty in understanding or by time spent making sense of the text. This was in contrast to the speed with which more 'typical' sections of text were read and evaluated. Although the interview context within which this data arose could not reproduce either the work patterns of assessing large numbers of texts

in a restricted time period or the 'high stakes' nature of authentic assessment practice, it appears likely that, where the effort required to make sense of an unusual section of text is too great, the decision about correctness will be deferred or made on superficial grounds. Because of doubt about its correctness, the value placed on the section of text on the basis of its interest becomes less significant. This may be particularly the case with 'extensions', which, potentially allowing more scope for deviations from the norm, appear to be valued merely for their presence rather than for any mathematical quality.

Although the unusual may be taken as a sign that the student has been 'thinking' rather than following routine procedures, the teacher nevertheless has to decide whether the student has achieved the unusual result by 'appropriate' means. Whereas the routine result, even without the support of accompanying verbal explanations, may be taken as evidence that the routine processes have been gone through, the unusual result is treated with suspicion. This is seen in the narratives constructed by teachers to explain how the student might have been working. Whereas originality in the content of the investigation is valued, teachers' expectations about students' processes are more rigid. The imaginary 'typical' text embodies the approved processes within its content; the unusual text must make the processes explicit if they are to be recognised.

Some of the characteristics of unusual sections of text that were taken as signs that the student-author was a 'good mathematician' are in direct opposition to the standard interpretations of the assessment criteria. In particular, achieving results without providing evidence of working systematically and stating these results without providing an accompanying verbal narrative to explain them were identified as characteristic of some students with high 'ability' and yet contravene the assessment requirements. There is tension for the teacher here between her role as an examiner, applying externally determined criteria, and her role as a teacher/advocate, making use of all available (or constructable) evidence in order to arrive at an assessment that reflects the quality of the student rather than just the quality of the text.

Both the coursework tasks used in this study, while providing a strong structure for students' work including closed questions, also contained some more open questions which allowed variation in the student responses. In particular, they both allowed the student to 'extend' the task in a non-specific way. Although the 'opportunities' provided by such openness may be applauded and students' attempts to pose and work on their own questions may be welcomed as 'interesting', it seems that teachers own ideas about what might be appropriate responses to such open questions are strongly developed. The 'predictable' extension is seen to be more 'useful', probably because the teacher already has a clear idea of where it might lead. Again, the need to ensure that there is an assessable outcome to the student's work is in tension with the rhetoric of encouraging creativity and following blind alleys.

While the difficulties experienced by teachers in reading and assessing unusual student texts have been considered, there are also difficulties for students in attempting to produce a text that will be valued by its readers. It is clear that, although any evidence of originality is likely to be welcomed by teachers, the amount of value placed on it will vary according to the ease with which the teacher is able to make sense of it, the extent to which the processes are communicated, and the extent to which characteristics which fulfil standard criteria such as 'working systematically' are displayed. In reading the 'normal' coursework text, teachers are able to make assumptions about meanings and about the student's processes; similar assumptions are not available when reading the 'unusual' text. The student is thus required to support unusual work with additional elaboration of the question they have posed and of the processes they have undertaken in achieving their results. The problem for the student, of course, is in knowing what is likely to be identified as unusual.

15 Conclusions

In this final chapter, I shall summarise the main findings, first from the review of the literature, and then from the results of the empirical study: the analyses of mathematics coursework texts and of interviews with teachers reading and evaluating those texts. A number of issues and implications will then be discussed. Finally, the limitations of the present study and suggestions for further research in this area will be considered.

15.1 Theoretical research

Mathematical language is not a unitary phenomenon but includes a wide variety of genres of text. However, the review of descriptions of and research into mathematical writing identified features which make it likely that a particular text will be identified as mathematical. These included the presence of mathematical symbols, specialist vocabulary and locutions, including the use of nominalisations, contributing to the formality and “distant authorial voice” (Strube, 1989) that characterise academic texts in scientific disciplines. A human presence was, however, found in the use of *we* and imperatives. Deductive argument is valued in mathematics and was found in some cases to be marked by a thematisation of reasoning and references backwards within the text although forms may vary between different fields of mathematics.

School students’ experience of mathematical text is largely structured by their experience of text-books. Features identified as characteristic of school text books included, as well as their symbolic content, a substantial graphic element, some of which was seen as ‘decorative’. There was found to be a prevalence of questions in text books, not only in exercises but also in ‘exposition’. Again, there was some variation between texts intended for different groups of students. A review revealed little description of students’ written production but a typical text in the traditional classroom may involve a repetitive transformational sequence and a labelled answer. This study set out to create a description of the writing produced by students in the particular context of reports of investigative work and to consider how this relates to other writing in mathematical genres.

In spite of recent interest in the use of ‘Writing to Learn’ in mathematics, there has been very little attention paid either to the forms of writing actually produced by students in mathematics or to possible ways in which students may be taught to write in ‘mathematical’ ways. It was found that, although it was acknowledged that students need support in developing their understanding and use of algebraic symbolism, other forms of mathematical writing appeared to be considered unproblematic and hence largely unexamined.

Where programmes explicitly addressing the question of improving the quality of students' writing in mathematics were reported these suggested that drawing students' attention to a small number of linguistic features might improve the quality of their writing within the specific genre required by their course. A review of the limited number of previous studies of student writing in mathematics education suggested that features that might be significant in influencing the acceptability of a student's mathematical text include: the use of recount or dialogue forms; the expression or obscuring of agency; the role of the author within the text through the use of personal pronouns. The nature of any such influence must, however, be strongly dependent on the particular genre concerned.

As well as the assumption that student production of mathematical text is, on the whole, unproblematic, past research has generally assumed that the reading and interpretation of students' texts is a straightforward task. This study has set out to challenge the assumption of a transparent relationship between students' writing, their mathematical thinking and their teachers' reading of the texts.

A reader, rather than gaining direct access to the author's meaning, constructs her own meaning from the text. A consequence of adopting this position is the problematisation of the concept of 'audience'. One of the sources of difficulty for student-writers identified in the research on school writing was the fact that their readers are teachers who, while possessing 'expert' knowledge of the subject matter, expect to see detail in the text. This creates a problem for students in determining both the amount of knowledge that needs to be displayed and the way in which it should be communicated. Even when a non-teacher audience was specified, it was found that it could not be assumed that students would adopt this audience or would share the teacher's perception of the characteristics of the proposed audience or of the linguistic features likely to be judged appropriate to it. It may even be the case that the most successful school writing does not result from adopting and addressing the assigned audience but from making use of knowledge of what the teacher is likely to value highly. This study has sought to develop such knowledge of teachers' preferences through examining their reading practices.

An important aspect of the context within which the student texts used in this research were produced was the fact that they were to be formally assessed. The 'common sense' view of assessment makes two assumptions: that there is a direct relationship between curriculum aims and assessment practices, and that the object of assessment may be unambiguously identified and objectively evaluated. The review of previous research, however, challenged both of these assumptions. Firstly, the expressed curricular aims of investigative work, including aspects such as creativity, diversity of outcome and the encouragement of risk taking, were found to be in conflict with the traditional value placed on reliability in assessment. Secondly, although there was some evidence that teachers tended to be

consistent in their assessment of mathematics coursework, the bases upon which evaluations were found to be made were neither 'objective' nor easily communicable to students. In spite of official listing of specific assessment criteria, several studies suggested that teachers made more general 'intuitive' judgements of students' work based on a construct of the 'ability' or 'level' of a student or on a comparison with an imaginary ideal or norm text.

There has been little previous research into teachers' assessment practices in the context of investigative work in mathematics. Research in other curriculum areas, however, suggested that there were problems in disentangling the 'content' of a text from its form. Similarly, where the object of assessment was the student's processes, there were problems in distinguishing processes from content knowledge and skills. Teachers' evaluations of the worth of the content of a text were found to be influenced by mechanical aspects such as spelling or handwriting, by the use of features of conventional academic writing, and by interpersonal aspects of the text. This indicated another theme for this research: to identify those aspects of the forms of writing used by students that may influence mathematics teachers' evaluations of coursework texts.

15.2 Empirical research

This part of the research centred on the writing and reading of a set of students' text written in response to two coursework tasks. In order to produce adequate descriptions of coursework texts, it was necessary to develop a set of analytic tools to identify significant features of the texts and a means of interpreting these features. This development made use of Halliday's functional linguistics, which provided not only a grammar to describe the verbal parts of the texts but also the framework of three meta-functions, the ideational, interpersonal and textual, to structure the analysis of both verbal and non-verbal parts.

While the linguistic and non-linguistic tools served to identify features that might be significant to the analysis, their interpretation depended on reading as a "self-conscious insider" (Fairclough, 1989) to the discourse within which the texts are produced and consumed. The characteristics of students' texts identified in the analysis were thus related to my own experience as a practitioner involved in GCSE coursework with students and with other teachers as well as to the issues identified in the review of the literature and, in particular, to my analysis of the values and tensions identified in the discourse related to investigative work and its assessment.

The study focused on two coursework tasks provided by the London examination board (LEAG) for students entering the GCSE examination in 1991. These were chosen to include a 'pure' task ('Inner Triangles') and a 'practical' one ('Topples', which made use of data derived from manipulation of physical objects). However, textual analysis of the tasks and the

performance indicators for their assessment revealed substantial similarity between the structures of the tasks, both of which appeared to expect inductive generalisations based on empirically generated data to be the primary means of reasoning. The 'practical' aspects of the 'Topples' task were marginalised, particularly for those students expected to achieve at higher levels, and the major part of the task presupposed that the situation being investigated was a uniform abstract system. The analysis of both tasks and performance indicators revealed clear differences between the pictures of students of different 'levels' constructed within the tasks: gathering data accurately was of much greater importance for lower attaining students, while only the very highest attaining students seemed to be expected to make any attempt to reason theoretically rather than empirically. Students were instructed to 'investigate' and to extend each task. However, in spite of official declarations to the contrary, it was clear both from the wordings of the tasks themselves and from the assessment guidelines provided for teachers that little variation either in methods or in results was likely to be considered to be acceptable. The valuing of student autonomy and diversity was thus in tension with the need to validate assessment decisions and with the absolutist view of the nature of mathematics constructed within the tasks.

15.2.1 Analysis of students' coursework texts

A sample of coursework texts produced by eighteen students in response to the two tasks was analysed and, on the basis of this, three student texts on each task displaying a range of characteristics indicating differences in their ideational, interpersonal and textual aspects were selected for detailed case study. These included texts produced by one individual student on both tasks.

The analysis of each of these case study texts identified the main linguistic and non-linguistic features of the texts and related these to the ideational, interpersonal and textual meanings that might be constructed by a reader. There were some similarities between the texts, in particular:

- a focus on product rather than process in the case of the three 'Inner Triangles' texts;
- a picture of mathematics as essentially procedural in the case of the three 'Topples' texts,
- in all cases, an overall structure determined at least in part by structure of the statement of the task provided by the examination board.

There were also, however, substantial differences between the texts.

In relation to the ideational, both the subject matter of mathematics and the nature of human activity varied across the student texts. Whereas some texts focused on the material objects given in the tasks (shapes or rods) and the properties or behaviour of those objects, in others the subject matter was more abstract, focusing rather on number patterns, relationships and

formulae. Similarly, the picture of human mathematical activity presented in the text varied from concrete material action to more abstract mental action. In one text, human activity was entirely obscured.

The relationship between the author, reader and subject matter constructed in the texts also varied considerably, both in the degree of intimacy with the audience and in 'ownership' of the task. In the small number of texts considered here, the most impersonal and formal texts on each of the tasks were also those which constructed the most concrete picture of their subject matter.

A summary of the textual aspects of the students' texts was more difficult to achieve as most of the texts contained a number of inconsistencies and discontinuities. All the texts were to some extent presented as 'school maths' tasks, structured as a set of 'answers' to the given task. Most also had 'school project' features, including decorated covers and headings. While some texts appeared merely to display a set of discrete products, parts of others included a few explicit cohesive links and chains of connected themes to construct passages of descriptive report or of explanation. It would appear that there are difficulties in constructing a coherent coursework text because of the tensions between the specific requirements of the given task and the attempt to fulfil the more general expectations related to the doing of extended investigative work.

Consideration of the texts produced in response to five different tasks by a single student showed his writing to be remarkably consistent. Similar pictures of the nature of mathematics and mathematical activity and of his relationship to his reader were constructed in all five texts, in some cases using very similar wording. The main differences between the texts appeared to be related to the subject matter of the tasks or to the ways in which they were presented and structured.

15.2.2 Teachers reading and assessing coursework texts

Task-based interviews were undertaken with a sample of eleven teachers from six schools. Each teacher read and assessed the three selected student texts on one of the two tasks. Transcripts of the interviews were analysed in order to describe the teachers' reading practices and to determine the ways in which teachers identified, interpreted and responded to characteristics of the texts.

The interpretation of teachers' readings of coursework texts was informed by an understanding of the contextual factors which influence their practice and of the possible positions that they might adopt within the institutional setting in which coursework assessment takes place. The positions adopted by teachers during the reading and assessment process included: examiner, using externally determined criteria; autonomous examiner, setting and

using the teacher's own criteria; teacher/advocate, looking for opportunities to give credit to a student; teacher/adviser, suggesting ways in which a student might meet the assessment criteria; teacher/pedagogue, suggesting ways in which a student might improve her mathematical understanding; interested reader; imaginary naive reader; and, given the interview context in which the reading and assessment was situated during this study, some teachers occasionally were seen to adopt the position of interviewee. There are clearly tensions between some of these positions that appeared to cause some difficulty for teachers in attempting to form a single judgement of an individual student. The ways in which such tensions were resolved differed between teachers; strategies of resolution included: resignation to the external authority of the examination board; appeal to the possible existence of missing 'evidence' of the students' work; redefinition of the criteria for accepting the validity of a piece of work. It was found that the imaginary position of 'naive reader' was consciously adopted by some teachers as a device to signal the amount of detailed 'explanation' they considered to be appropriate in a student's text.

Comparisons of pairs of teachers reading and assessing the same student texts showed that, while the same features of a given text were identified as significant by each of the pair, the interpretations and values placed upon these features differed. These differences appeared to be associated both with different dominant positions adopted by teachers as readers and with different orientations towards the text and its author. The teachers who read primarily as examiners appeared to form a comparison between the student's text and an imaginary 'ideal text', seeking for those features which would constitute such an ideal and judging the particular text on the extent to which it matched their expectations. Other teachers, reading from positions less strongly influenced by the externally-determined assessment criteria, appeared to interpret features of the text in order to build up a picture of the characteristics of the student-author and to use the resulting profile of the author in order to arrive at a judgement of the text.

These differences in practices did not necessarily result in different assessments of an individual text. It seemed that in many cases the constructed picture of the student-author would lead to an assessment that largely coincided with that reached by comparing the text with an 'ideal'. However, an aberrant text was found to give rise to a mismatch between the teachers' construction of a picture of the mathematical understanding or ability of its author and the extent to which the text was seen to match an 'ideal text'. This mismatch was associated with difficulties for individual teachers in coming to an assessment of this aberrant text and major variations between different teachers' final rankings.

When teachers were asked to state what characteristics they would expect to see in a good piece of coursework, the typical response provided a simple list of features such as tables, diagrams, explanation, without any elaboration either of the forms these features might take

or of the functions they might serve within a coursework text. The use of task-based interviews in which teachers engaged in the activity of assessment of student texts made it possible to identify not only those features that teachers actually paid attention to during their reading but also the particular forms that were approved, the interpretations ascribed to them and the purposes that they appeared to serve for the teacher-readers. A list of the features attended to looks very similar to those provided by the teachers themselves: 'writing' (specifically a narrative of what the student did), an introduction, 'explanation', tables, diagrams, algebra. Indeed, each of these features appeared to be, at least to some extent, approved for its own sake, regardless of its specific form or of its role within the text. In each case, however, some forms were valued more highly than others. Moreover, in many cases, teachers experienced some difficulty in reconciling the value placed on a particular feature with other aspects of their reading of an individual student's work (for example, a general construction of a picture of the student's 'ability') or with values associated with the discourse of 'investigation' or with more traditional assessment practices.

narrative

There was a strong expectation that a coursework text should not be entirely non-verbal. In some cases, however, the teachers' need to 'understand' what the student had done (used as a rationale for the demand for 'writing') was actually met by non-verbal forms of communication. Moreover, where a text contained correct results without verbal elaboration, the requirement for 'writing' was in tension with the traditional paradigm of assessment in mathematics in which correct results are most highly valued. One major form of writing that was expected was a narrative of the problem solving process. This appeared to serve a number of purposes for teachers: to display what the student had done or what she had been thinking; to provide reasons for the student's actions or a basis for her conclusions; to provide evidence of the student's ownership of the results; to fulfil examination criteria; or to provide coherence in the text. Not all forms of narrative, however, were equally valued by teachers. The inclusion of statements of mental processes such as 'I notice' or 'I predict' was approved, especially in so far as these appeared both to provide insight into the student's thoughts and to suggest that the student was making links between results and conclusions. Less specific statements such as 'I found', on the other hand, did not fulfil these functions and were, indeed, parodied by some teachers and even suspected of covering up the possibility that the student had 'found' the result by copying from a friend.

introduction

Providing an introduction to the problem seemed to be an optional part of the coursework text, although when there was an introduction that was perceived to be 'good' it was generally approved. Any introduction had to be seen to be written in the student's own words rather than copied directly from the statement of the task provided on the question paper. When this

was the case, it was taken as a sign that the student had understood the problem or had actually worked through the examples provided. Determining whether an introductory section would be taken to be in the student's own words, however, was not unproblematic; identifications which appeared to contradict the evidence of the words on the paper were frequently made by teachers and the same section of text was interpreted differently by different teachers. It seems likely that aspects such as the level of care taken in the presentation of the text or more general impressions of the student's 'ability' may have influenced the ways in which teachers read the introductory sections.

'explanation'

Any 'explanation' of the reasons for a result was highly valued although a 'correct' explanation was largely considered to be beyond the capability of most students at GCSE. Text providing the form of a coherent argument appeared to be valued for its own sake, regardless of its mathematical 'correctness' or even of the teacher-reader's ability to make sense of it. On the basis of limited evidence it appears likely that the use of terms such as 'because' or 'the reason is' that make explicit claims about causality served to make it more likely that a teacher would recognise that a student had provided an 'explanation'.

tables

As well as being valued for its own sake, a table appeared to serve the following functions for teachers: to fulfil the assessment requirements; to help communication; to provide evidence of the student's thinking and problem solving processes; to signal that data had been gathered systematically. This last function was particularly significant although most teachers appeared to recognise that the texts presented to them were 'write-ups' rather than transparent recordings of the problem solving activity. In at least one case, the identification of the criterion 'working systematically' with a particular form of one-dimensional table had become so close that the teacher was unable to see any value in a two-dimensional table that displayed a relationship between three variables.

diagrams

The reasons given by teachers for their wish to see diagrams in coursework texts were to demonstrate the student's ability to use a variety of forms of visual representation (and hence fulfil assessment criteria) and to assist the student in the solution of the problem. While diagrams seemed generally to be considered useful and hence ascribed a high value, being able to solve a problem without resorting to a diagram was also taken as a sign of higher achievement. Indeed, using the wrong kind of diagram (in particular a naturalistic diagram) or too many diagrams was interpreted to be a sign that the student was working at a lower, more concrete, level. Such an interpretation served to lower the teacher's evaluation of the whole of the student's work.

algebra

The presence of a generalisation recognised as 'algebra' played a decisive role in the assessment of coursework texts. There was general acceptance among teachers that the use of 'algebra' was a definitive sign of both a high value piece of work and a 'high ability' student. The boundary between what might be considered algebra and what was not algebra was not, however, a clear one; this was an area of tension for teachers and an area of conflict with their perception of examination board requirements. This tension was exacerbated when the teacher's overall evaluation of the student's text was in conflict with the level of algebra displayed, whatever the direction of the difference. In some cases the tension was resolved by acquiescing to the supposed authority of the examination board and taking the presence of symbols as the only decisive criterion; in other cases, teachers redefined their perception of algebra in order to make their evaluation of the whole text consistent.

While the presence of symbolic algebra was clearly very highly valued, it had to be incorporated into the coursework text in a way that was consistent with the expectations of the genre. It was expected to be preceded by a procedural description of the generalisation expressed entirely in words. This verbal procedure fulfilled a number of functions for teachers: to provide evidence that the symbolic formula 'belonged' to the student (i.e. was not copied); to provide evidence that the student 'understood' the formula; to demonstrate that the student was capable of producing such a verbal generalisation; to help the student to produce the symbolic formula. Indeed, if a verbal procedure was not present, the value of the formula itself was sometimes called into question.

features perceived to be incongruous

A number of features of individual student's texts were noted by teachers in such a way as to suggest that they were felt to be inappropriate or incongruous in some way. These included:

- the unconventional use of specialist mathematical vocabulary;
- the use of the equals sign as an operator (in situations where the expressions on either side of the sign were not strictly equal);
- lengthy variable names, maintaining a close relationship to the concrete referent;
- explicit expressions of confidence or other attitudes;
- reference to the context within which the task was done (i.e. mention of other members of a group working together).

The inclusion of personal statements of the form 'I noticed . . .' was expected, but other intrusions of the personal in the form of expressions of attitude or explicit acknowledgement of the social context appeared to be judged inappropriate. While there was no clear indication that teachers' judgements were explicitly affected by such incongruous features, they seem likely to contribute to a teacher's construction of the characteristics of the student-author, in particular her 'ability', and hence to affect the ultimate judgement of the work.

15.2.3 Error and creativity

The ways in which teachers coped with the tensions between the value placed within the 'investigation' discourse on creativity and the acceptability of diverse outcomes and the traditional value placed within school mathematics on 'correctness' and the assessment-driven requirements for reliability and comparability of evaluations of different students' work were explored through examining the sections of students' texts that were identified as being 'unusual' or erroneous and the ways in which the teachers read and responded to such sections.

Even those teachers interviewed who had very limited familiarity with the task or with the specific characteristics of students' work on the particular task appeared to have clear expectations about the probable content of students' work. Where the form of the text differed greatly from the 'ideal', this was normally condemned. Deviations in the results achieved, questions posed or processes used tended to be greeted more positively. Such unusual aspects appeared to be valued because they were 'interesting', were evidence that the student was thinking for herself rather than following a routine algorithm, or because they might be a sign of a good mathematician.

The unusual nevertheless posed problems for teachers in coming to an assessment of the piece of work as a whole. While the components of the 'usual' text conformed closely to those of the teacher's imaginary ideal and could be directly compared with this, the unusual might have other characteristics that would not be so easily measured either against an imaginary ideal or against standard criteria. Thus there was conflict between the reading of some aspects of texts (e.g. extreme conciseness) as signs of the student's 'high ability' and a simultaneous lack of fulfilment of specific criteria, leading to difficulty in arriving at an evaluation of the coursework as a whole. There even appeared to be a tension for some teachers between what they perceived to be 'good mathematics' and what they believed could be assessed as 'good coursework'.

Where the paths followed by the student differed substantially from those anticipated by the teacher there was the additional problem of validating the results. While the standard result was assumed to have been achieved by the standard, approved processes, this assumption could not be made for the unusual result. A text lacking narrative and explanation yet achieving standard results was accepted as valid, but a text or section of text with non-standard content that similarly lacked narrative and explanation was treated with suspicion or even dismissed as worthless.

Whereas the unusual was generally welcomed, perceived errors in students' work were, as might be expected, generally condemned. The condemnation of error was not, however, a simple matter and frequently appeared to cause the teachers anxiety. A focus on process led

some teachers to take a relativist position towards the accuracy of results, the fit between data and generalisation being more important than the formal correctness of either. However, where incorrect data gave rise to a generalisation that was considered to be too 'trivial' the processes used, although 'correct', were less highly valued.

In spite of the general marginalisation of any 'practical' element in the coursework tasks, the teachers' attitude towards errors in the collection of data was the main area of difference between the ways in which the 'Inner Triangles' and the 'Topples' tasks were assessed. In the practical situation, where the data was collected experimentally, a tension arose between the value placed on accuracy and the simultaneous perception that 'real' experiments give rise to inaccurate data. A perfect correspondence between data and formula in the 'Topples' task was thus treated with suspicion as a sign that the student may have worked backwards from the formula to the data - a process not considered to be appropriate. The student was, moreover, expected to comment on the accuracy of her results in the practical situation.

15.3 Discussion of issues and implications

15.3.1 How 'mathematical' is the coursework genre?

As was argued in chapter 2, there is no single type of language that may be called mathematical. Rather, a variety of genres of text arise out of various mathematical practices. Nevertheless, some features that are characteristic of high status types of mathematical texts are likely to be identified as mathematical and hence to serve to identify texts containing them as more or less mathematical. In this section, I consider the extent to which the coursework texts analysed here contained such features and the values placed upon them by the teacher-readers.

The use of algebraic symbols¹ rather than words to designate variables in general formulae was one of the features used in the analysis to distinguish between the various student texts considered in this study. It also proved to be very significant for the teacher-readers in arriving at their evaluations of the texts. Mathematical language is commonly identified with the mathematical symbol system. Clearly this is an inadequate characterisation but it is nevertheless true that both school and academic mathematics texts contain symbolism. The role of symbolism in these coursework texts, however, appeared to be as an 'answer' rather than a means of communication or a problem solving tool, as it might be in other mathematical genres. Any identification of the texts as 'mathematical' on the basis of their

¹I am considering only algebraic symbols to be relevant to this discussion. While all the texts contained numbers, this does not distinguish a text as mathematical to the same extent. Many texts in non-mathematical genres also contain substantial amounts of numerical symbolism (e.g. railway timetables, newspaper articles, shopping catalogues, etc.).

symbolic content was thus made on relatively superficial grounds. The place of algebraic symbolism is discussed further in section 15.3.5 below.

All of the student texts made some use of specialist vocabulary in the sense that they named the objects that formed the subject matter of the task. However, the extent to which this usage may be considered mathematical varied. The naming of objects in both school and academic mathematics texts is characterised by consistency and conciseness, including the use of nominalisations. These characteristics were found in the statements of the tasks analysed in chapter 7. The use of concise names for variables was also valued by teachers and read as a step on the way to using 'algebra'. Some student texts, however, made use of a variety of forms of name for the same mathematical object. Such variety in the vocabulary might be interpreted as fluency in some other subject domains, suggesting that students may be making decisions about their writing that are based on a knowledge of values that are not those of the mathematical domain.

The impersonal and highly symbolic style that is typical of some academic mathematics text and of much of the writing expected of students in the traditional, non-investigative mathematics classroom was found in one of the student texts. This text appeared to be recognised as mathematical by its teacher-readers and its style appeared to act as a mark of its author's high mathematical 'ability'. Nevertheless, the impersonal and non-verbal aspects of this text were clearly not considered appropriate within the coursework genre. The demand for a narrative of the student's processes is in conflict with the characteristics of other high status mathematical genres.

Although some of the student texts attempted to construct arguments or explanations, these did not generally display the features of mathematical arguments identified in chapter 2. Moreover, it appeared that the teachers often found it difficult to make sense of passages of argument in the student texts and either did not recognise them as such or passed over them quickly without attaching value to them. Those student arguments that were recognised seemed to be those which made use of explicit expressions of reasoning such as *because*. Given the high status of explanation and argument in mathematics and in GCSE coursework assessment it would seem that this is an area to which some attention should be paid in order to develop students' use of language to construct coherent and explicit mathematical arguments.

The procedural nature of much of the coursework texts may be seen to echo the procedural emphasis of many school text books and mathematics lessons. The influence of traditional school student writing could also be seen in the use of question numbers and the labelling of answers. This is not surprising, given that the statements of the tasks were similarly structured around specific questions and that answering these questions was used by some

of the teacher-readers as a sign of a basic level of student achievement. In some cases, a repetitive structure of series of similar symbolic statements or calculations was found, similar to that identified by Ernest (1993a) as characteristic of traditional school mathematics writing. The value placed on this by teachers, however, varied according to its position within the whole text and the role that they thus ascribed to it. Where repeated substitutions occurred after a symbolic generalisation this was seen as demonstration of understanding or algebraic skill. Where repeated transformations of data occurred before the formation of an explicit generalisation, this was not highly valued.

While the forms of language in coursework texts produced by students and approved by teachers shared some characteristics with other mathematical genres, there were also areas of difference. In particular, the demand for the inclusion of a personal narrative is not found in other mathematical writing. This lack of models may go some way to explaining the general failure of the students whose texts have been analysed to produce forms of narrative that would be approved by their teacher-readers. On the other hand, the construction of argument was another aspect in which the student-writers were not generally seen to be successful. While there are many existing models of mathematical argument, it is acknowledged that students (at university level as well as at school) have difficulty making sense of them (Alibert & Thomas, 1991). The question of how students may be helped to write in ways that are considered appropriate to the genre is considered further in section 15.3.8 below.

15.3.2 The conversion of pedagogy into assessment

The essential duality of the teacher-assessor's role was reflected in the shifting of individual teachers between pedagogic and examining positions and in the conversion of pedagogic knowledge into assessment guidelines. In particular, procedures for helping students to achieve particular learning objectives or to make progress in a particular problem situation themselves became assessable outcomes. In this section such pedagogic procedures are identified and the implications of their conversion into indicators of student achievement are considered.

'see-say-record'

As was seen in chapter 12, the teachers expected any symbolic generalisation to be preceded by a verbal description of the generalisation. A number of reasons were given for this demand, including the suggestion that it would help the student to achieve the symbolisation. This idea of a progression towards algebraic symbolisation, characterised as 'see-say-record' (James & Mason, 1982; Mason, 1987), was intended, both by its original advocates and by the subsequent curriculum materials which made use of it, as a support for learners, making symbols meaningful. At the same time, it was stressed that gaining such a comfortable familiarity with algebraic symbolism is intended to lead the student to an ability to manipulate symbols freely, independent of their reference. However, the development of this

pedagogic support into an algorithm for forming generalisations and hence into an assessment requirement, demanding that the student include a verbal description of any generalisation before symbolising, negates its pedagogic intent: the student is never able to operate entirely symbolically because she must always reconstruct the concrete referents in order to demonstrate that she has gone through the required processes.

ask 'what if...'

While the presence of an 'extension' was generally welcomed by teachers, it appeared to be the posing of the problem rather than its solution that was valued. Where the extension problem posed by the student was unusual, this caused difficulties for the teacher-reader. One consequence of the tensions between the ideals of 'openness' and creativity and the values of the assessment system is the conversion of advice to teachers about working in 'open' ways into conventions for creating 'extensions', effectively restricting the acceptable possibilities available to students. The Cockcroft report suggested that investigative work could start from questions such as "could we have done the same thing with three other numbers?" or "what would happen if . . . ?" (Cockcroft, 1982: p.74). Similarly, articles in professional journals and advice in guides for teachers (e.g. Walter, 1989; Hunt et al., 1988) have exemplified ways in which questions may be posed to begin or to extend investigative work. While the Cockcroft report suggested that such questions might arise naturally from children's curiosity and be followed up spontaneously during a lesson, the GCSE requirement for all students to produce 'extended' work to be assessed has meant that posing such questions has become institutionalised² and assessable. A potentially infinite number of such questions might be posed; within the context of a particular situation, however, only a few of these are likely to allow a student to achieve an assessable outcome (i.e. to work systematically, to generalise, etc.). The art of judging such a question for its potential for a given student is one that is difficult for teachers, let alone for students themselves. Although Cockcroft suggested that there might be profit in following and subsequently discussing "false trails", in the context of investigative work done for examination purposes posing and following up the 'wrong' sort of question is likely to lead to a substantial expenditure of effort for little reward. Consequently, the types of extensions students are advised to try (by their teachers and by the statements of the LEAG tasks analysed in this study) may be characterised as 'change one of the variables in the situation and repeat the same investigation processes'. There was little evidence either in the students' texts considered here or in the teachers' responses to these texts that an extension might build on or

²A further contradiction between the standard practice of 'investigations' and the value placed within the discourse on student ownership and autonomy is pointed out by Wells (1993), who criticises the institutionalisation of the 'extension' for preventing students from making their own decisions about when a problem is completed.

generalise from the results achieved earlier in the task. This may in part reflect the types of task originally set, which may not have lent themselves to such extensions. It does appear, however, that teachers have clear ideas about what is expected in an extension and that deviations from this routine are unlikely to be valued. The pedagogic intent of encouraging independent mathematical thinking and creativity is thus negated.

make a table

The presence of tables in student texts was generally approved by the teachers and was included in their lists of desirable features of coursework and in their reports of the advice provided to their students. In some cases, tables appeared to be used by teachers as a way of operationalising the assessment criterion 'works systematically'. The table's presence in a student's coursework text has thus become highly significant. As observation of the processes used by individuals working within a class is likely to be difficult, the table is taken to be a written sign that data has been collected systematically. Where a student has followed the routine inductive path through the problem, this may be a valid inference. By making the table the only acceptable sign, however, other ways of working are devalued. In particular, those students who seek insight into the structure of the situation and hence achieve a generalisation without gathering large amounts of data or spotting number patterns are disadvantaged. Although teachers may recognise that such students are 'more able' and are working in a valid mathematical way, they are nevertheless seen to be failing to fulfil the assessment criteria. A table in an acceptable format needs to be included even if it serves no purpose in helping the student to achieve a solution to the problem. In the words of one teacher, "The system says they should jump through hoops" (Morgan, 1992b).

Like the advice to describe a generalisation in words before attempting to symbolise it, the advice to students to make a table is intended to play a role in helping them towards an algebraic generalisation. From a pedagogic point of view, the table serves to encourage the student to organise her data while simultaneously providing visual cues to assist pattern spotting, in particular the use of difference patterns. The table is, however, firmly allied to an inductive mode of reasoning and is unlikely to support a structural analysis of the situation being investigated. The institutionalisation of investigative work subsequent to the Cockcroft report and the introduction of GCSE coursework has been associated with the routinisation of such an inductive approach, strongly criticised by, among others, Wells (1993) and Hewitt (1992). From being a tool to be used for helping to solve some problems but not for others, making a table has become part of a routine algorithm for 'doing investigations'. By becoming a routine fulfilment of an assessment requirement, the table may cease to be a useful tool in the student's problem solving repertoire, particularly in the light of the possible rejection by some teachers of more complex tables which organise data in ways which, while possibly helping the student to form a greater understanding of the relationships between the

variables, do not demonstrate a systematic varying of the variables during data collection. In the criterion “uses . . . appropriate visual forms” (LEAG, 1989), the term *appropriate* must be read as ‘appropriate to the teacher’s view of the standard investigative process’ rather than ‘appropriate to the student’s approach to the problem’³.

make sense of the problem

Where a text included an introduction to the problem that was identified as being written in the student’s ‘own words’ this appeared to be interpreted as a sign of the student’s ‘understanding the problem’ and hence as fulfilment of one of the general assessment criteria specified by the examination board (LEAG, 1989) and by the practical discourse of investigations, coursework and their assessment (e.g. Pirie, 1988). Where an introduction was identified as copied from the question paper, however, this was dismissed as irrelevant and a waste of the student’s time.

A pedagogical reason for linking a written introduction to ‘understanding the problem’ may be found in the arguments of those concerned with using writing to support learning across the curriculum. The initial stages of making sense of an unfamiliar problem may involve rewriting the question or trying a small number of specific examples. Even copying the question may play a role in providing the problem solver with time and a structure within which to make sense of the words. This role for writing in organising thought is recognised by some of those concerned with ‘Writing to Learn’ (e.g. Emig, 1977) although the specific role of copying does not appear to have been addressed. In the assessment context, however, the role of such tactics as tools for the problem solver has been transformed into a role as signs of the extent to which she ‘understands the problem’. In doing this, the concept of ‘understanding the problem’ is itself trivialised.

15.3.3 Reading a student’s ‘ability’ from the text

At least some teachers appeared to make judgements about a student’s general level of mathematical ‘ability’ on the basis of their reading of features of a single coursework text. This judgement of ‘ability’ may have an effect on the teacher’s ultimate judgement of the piece of work, allowing the teacher, for example, to ignore errors or missing ‘evidence’ on the basis that they are merely ‘slips’ by a ‘high ability’ student. This raises the question: what is the GCSE examination assessing - the person or the piece of work? The variation in teachers’ practices suggests that this is not clear in the case of coursework. While in some cases teachers insisted on finding ‘evidence’ within the text itself, in other cases they were willing to make assumptions about the student’s ‘understanding’ based on beliefs about the personal

³See Fairclough (1992b) for a critique of the use of ‘appropriate’ as a descriptor of forms of language used in similar criteria within the English National Curriculum.

characteristics of the student. This duality is also reported in Broadfoot's (1995) study of primary teachers administering and assessing National Curriculum standard assessment tasks.

Features of texts which seemed to be associated with judgements of 'high ability' included:

- the use of 'correct' terminology;
- the presence of 'algebra' - in particular the use of single letter variable names and any manipulation of algebraic expressions;
- 'abstractness', including an absence of deixis, the use of the present tense, an absence of reference to practical apparatus or diagrams;
- an attempt to 'explain' results;
- the presence of 'unusual' forms of representation or posing 'unusual' problems;
- the absence of evidence of process (seen to be characteristic of some 'high ability' students, though not approved).

Several of these features are similar to those identified in chapter 2 as characteristic of academic mathematics texts. They are, however, only indicative and some were not necessarily recognised by all teachers or in all contexts. Moreover, there was no simple connection between identifying a student as 'more able' and valuing his or her text highly. Although errors might be treated indulgently, some teachers were not tolerant of a text which lacked evidence of processes, despite identifying the author as 'more able'.

Features that seemed to be associated with judgements that a student is 'less able' included:

- 'concreteness', including the use of concrete variable names, the use of tenses other than the present tense, and naturalistic diagrams;
- the use of unconventional vocabulary or other forms of communication;
- writing that was difficult for the teacher-reader to make sense of (extensive deixis may have contributed to this).

The unconventional or unusual appears in both lists and may be interpreted either as a sign of originality (and hence high ability) or as error or lack of facility with conventional forms (and hence low ability). The way in which a specific instance is interpreted is likely to be dependent on other signs of the student's 'ability' within the text. While the student judged to be 'more able' may be indulged to the extent of excusing some errors and the absence of a narrative of processes, the student judged to be 'less able' is likely to be given less credit for her achievements.

In addition, features that were interpreted by the teacher as not contributing to the problem solution, including copying the question (or at least being perceived to have done so) and elaborate presentation, were sometimes taken as an indication that the student, who might have been capable of better work, had wasted her time, diverting her energies to aspects which would not contribute to her grade. Some of the teachers remarked that such features

were typical of girls' work; the sample of texts used, however, was not wide enough to allow comparison with teachers' reactions to similar features occurring in texts written by boys, although Spear's (1984; 1989) studies of teachers reading student texts in science suggests that there are likely to be differences. Such readings of 'good presentation', while not contributing directly to the teacher's perception of the student's 'ability', nevertheless form part of the teacher's characterisation of the student; this appears likely to affect the ultimate assessment of the student's achievement.

Previous studies that have observed teachers' use of a general construct of a student's 'ability' to affect their assessment of achievement (e.g. Ruthven, 1987; Filer, 1993; Broadfoot, 1995) have considered only cases in which the teachers had personal knowledge of the students and hence made use of 'evidence' beyond that offered by the particular piece of work being judged. In this study, the teachers had, in most cases, no personal knowledge of the student-authors involved and no evidence beyond that presented to them in the coursework texts. Any judgements about the students' 'ability' must therefore have been made entirely on the basis of the texts in front of them. The use of a construct of 'ability' in teachers' assessments of coursework cannot, therefore, be located solely in the bias or preconceptions that might arise from familiarity with the previous achievement or behaviour of known individual students. It is clear that characteristics of the text itself affect the ways in which teachers will read and interpret it and the weight that will be accorded both to negative aspects such as errors and to apparently positive aspects such as 'good presentation'.

Although one of the teachers interviewed had taught the students whose texts were used, his recollection of the students' characteristics was distanced by some time and hence this study has not substantially addressed the issue of the influence of teachers' personal knowledge of students on their assessments. It seems likely that there would be some interaction between such personal knowledge and the textual features associated with 'ability' and other personal characteristics identified above.

15.3.4 'Investigation' and 'good mathematics'

The students' texts used in this study largely conformed to a data-pattern-generalisation (DPG) format but there were some indications in the teachers' readings of these texts of a recognition of a tension between this and other perceptions of mathematics and mathematical thinking. This tension echoes criticism in the professional discourse of the stereotyped 'investigation' as restrictive of mathematical content and thinking (Hewitt, 1992) or even as non-mathematical (Wells, 1993).

Although Richard's text conformed closely to the DPG format (indeed this appeared to contribute to the way in which parts of his text were read - see chapter 11), the conflict between teacher perceptions of his 'ability' and his lack of conformity to other expectations of

the coursework genre raised the issue for some of the teacher-readers of apparent contradictions between 'good mathematics' and the expectations of investigative work. It prompted Dan, for example, to relate anecdotes about 'able' students who had achieved generalisations without going through the processes seen to be required by the examination criteria. Although it is not clear what processes such students had actually used (in particular, it is not clear whether they had developed a structural analysis of the situation rather than deriving the generalisation inductively), the conflict recognised by the teacher indicates strongly that what is seen to be 'good mathematics' will not necessarily be valued by the examination system. In an earlier study which looked at teachers' readings of an investigation solution using a structural approach (labelled BS) as well as one using the DPG approach, this conflict was explicitly stated by one teacher:

"I did originally put BS first. I have great sympathy with BS - you can see where the results come from. Some high level children do do it like this. This is the way I would do it. I would hate to have to write all this out. But children should be aware of the criteria." (Aileen)

(Morgan, 1992b: p.5)

Whether or not the examination board would officially agree with such rigid interpretation of its assessment guidelines and performance indicators, the fact that teachers appear to perceive them in this way must have a strong influence on their practice in the classroom as well as on their own assessment techniques. This influence has been noted by, among others, Lerman (1989) and William (1994), who lay the blame for the prevalence of the DPG investigation at the door of the assessment system. The teachers' need to help their students to be successful in their public examinations is perhaps more immediate than the desire to help them to be mathematical. This analysis suggests that the two are not necessarily compatible.

One way in which the high value placed on the DPG approach is justified is the suggestion that it provides a supportive structure that is necessary for learners and will (at least for the more 'able') eventually lead to more independent and 'real' mathematical behaviour, including (for some) deductive proof. This suggestion was implicit in the tasks and performance indicators provided by the board, which reserved any consideration of explanation or proof making use of the structure of the situation rather than observation of patterns in the data to the end of each task and, apparently, for only the most 'able' students. One teacher even stated explicitly that the lack of 'maturity' of students made the systematic DPG approach necessary (see section 15.2). While it is widely recognised that the development of the concepts and methods of deductive proof is problematic (Alibert & Thomas, 1991), there is no evidence that extensive experience of pattern spotting is likely to support it. Indeed, it seems more likely that the emphasis on 'scientific' induction within the DPG paradigm may make recognition of the importance of theoretical reasoning even more difficult.

15.3.5 The place of algebra

It has been clear from the analysis of the teachers' readings of coursework texts that the identification of the presence of 'algebra' in the text played a very important if not decisive role in influencing the way in which the teacher-reader would ultimately value the text. The privileged position of algebra in mathematics coursework raises questions about the relationship between the assessment of coursework and the content of school mathematics. Is the use of algebraic symbolism a piece of mathematical 'content' (it is normally taught as such within the mathematics curriculum)? If so, it is the only 'content' that appears explicitly within the examination board's assessment criteria as well as in the general guidance on coursework offered to students (e.g. Bull, 1990). The tension between the valuing of content and of process in the discourse on 'investigation' (see chapter 4, Appendix 15) appears to be resolved in this case alone by singling out the use of algebraic notation as a specific assessment criterion.

One of the reasons given for the demand for a verbal description preceding an algebraic expression was to provide 'evidence' that the formula itself belonged to the student. This echoes the suggestion found in practical advice for teachers and students (Pirie, 1988; Bull, 1990) that the presence of algebra in a student text without other supporting 'evidence' may be considered suspect. The singling out of 'algebra' as subject to copying suggests that, as a piece of 'content', its use is conceptualised within a more traditional paradigm of assessment. Ball & Ball's (1990) claim that 'cheating' is irrelevant in the coursework context appears not to apply in this case.

The place of algebra in the corpus of coursework texts considered here, while crucial to the assessment process, was limited almost exclusively to the expression of generalisations. Where manipulation of algebraic expressions took place it appeared to play no role in the solution of the problem or in furthering the investigation. It was, rather, presented as the sort of display of skills that might be performed in a more traditional examination setting. Indeed, whereas using algebraic manipulation might in other circumstances be interpreted as a more advanced type of 'process' acting to further the solution of the problem, in these coursework texts it appeared only as 'content'. The formulation of a symbolic generalisation has become the culmination of the stereotypical DPG investigation and algebra is seen as an endpoint rather than as a tool.

The 'race to a formula' in students' work criticised by, among others, Hewitt (1992) was paralleled by a similar race by the teacher-assessors. This was illustrated in the reading of Steven's 'Topples' text by Jenny and Charles (see chapter 10). Not only did Charles pass over a whole section of Steven's text containing his 'explanation' because he was looking for the formula to satisfy his expectations of the genre (in which the algebraic generalisation should immediately follow the table and pattern spotting), but both teachers paid very little

attention to Steven's 'alternative' methods of solving the inverse problem. In reading the page containing two different methods, they seemed to 'home in' on the algebraic formula, focusing their attention on the correctness and supposed derivation of this, without considering the (non-equivalent) alternative which was expressed in words rather than symbolically. It may be that their generic expectations led them to believe that the sentences at the beginning of the page were merely a verbal description of the formula symbolised later on the page and hence to ignore them as relatively insignificant or to pass over them without paying attention to the words because their 'meaning' was already apparent. Whatever the explanation for the particular ways in which these teachers read this page, it seems to point to another way in which the privileging of the symbolic form is impoverishing the range of mathematics that may be valued.

15.3.6 Consistency and diversity in teacher assessment

While reading the student texts, the teachers adopted a number of positions in relation to the texts and their authors. This diversity reflects Purves' (1984) "anatomy" of the teacher-reader. However, while Purves focused on the problems that such multiple teacher roles might cause the student-writer in determining her audience, this study has suggested not only that teachers themselves may experience tensions between their various roles but also that they may resolve them in different ways. The comparison between two teachers reading Richard's text indicated two fundamental, but related, differences between their practices: one read primarily as an examiner while the other read primarily from a position as teacher/adviser or teacher/advocate; one sought to compare the student's text with an imaginary 'ideal' text while the other built up a picture of the personal characteristics of the student, including his ability. In the case of this aberrant text, the evaluations reached by these approaches varied; the different approaches to assessment, however, were also apparent for this pair and for other teachers in cases where there was ultimate consensus on the final rankings of the students texts. While I would hesitate to assert an exclusive or exhaustive categorisation of two types of teacher-assessors, there would appear to be two general approaches to reading coursework texts in use among the sample of teachers considered here: while some searched the text for indicators that particular processes had been gone through and criteria fulfilled, others, at least on their initial reading of the text, sought to discover the intentions of the student-author and to understand her arguments and results. These two orientations will lead to different readings of the same section of text. For example, a table may be read as a sign of systematic working or as a communicative device to display results; a diagram may be read as a sign that data has been gathered or as an attempt to explain the structure of the solution.

A theme that has recurred throughout this study has been the tension between the value placed on 'creativity' and diversity in students' investigative work and the value placed on

consistency and reliability in teachers' judgements of that work. The largely anecdotal evidence available in the literature (Banwell, 1987; Gill, 1993; Pike & Murray, 1991; Wiliam, 1994) suggested a substantial degree of consensus in teachers' assessment of mathematics coursework. The sample of teachers in this study confirmed that, even where the texts all achieved comparable results, teachers were likely to make roughly similar rankings (the question of whether they would give the same grades was not addressed). Richard's text (which was specifically chosen for study because of its relatively unusual cluster of characteristics, including its very formal and impersonal aspect) was, however, a major exception to this apparent consensus. It prompted extreme reactions from the teachers, who ranked it either first or last. It is clear from the teachers' readings of this text that they too considered it to be unusual and that this caused difficulties for many of them in forming their evaluation of it. This difficulty in dealing with an aberrant text suggests that the apparent consensus reported in the literature is based on an aggregation of assessments and does not take into account the assessment of unusual texts. It is the unusual, and possibly 'creative', texts which are likely to pose the greatest challenge to the search for reliability in teacher assessment. However few such unusual texts there may in general be, the wide variations between teachers' assessments in this one case point to differences in their reading and assessment practices that need to be explored.

15.3.7 Who is to blame for the mismatch between rhetoric and practice?

It has been clear from the analysis of teachers' assessment practices that there are substantial mismatches between the rhetoric and the reality of 'investigation', at least for students in the two years of preparation for the GCSE. At the same time, however, it cannot be claimed that the teachers studied were in general inexperienced, incompetent, or hostile towards the ideals of investigative ways of working. Most of them had been involved with GCSE coursework since the early years before it became compulsory and in most cases their status as competent teachers has been recognised by promotion and additional responsibilities within their schools (although it is, of course, possible to challenge this notion of competence). With only one exception they participated uncritically in the discourse of investigation, making use of concepts such as 'openness', 'ownership', diversity and the valuing of mathematical processes. It is not possible to suppose, therefore, that further experience or attempts to convince them of the value of investigative work would radically change their practice. While there were occasions when teachers appeared anxious or lacking in confidence when attempting to form judgements of students' work, these occasions seemed to arise, not from lack of experience or expertise, but from the fundamental tensions within the discourse of investigation itself and, in particular, from the contradictions between the values of investigative work and those of the assessment system.

There is, however, a recurrent theme in the professional discourse that many teachers are not using investigative work well or do not understand how it ought to be done. This is apparent not only in the early years immediately following the Cockcroft report (e.g. Fielker (1982), Whitney (1981), Melrose (1982)), when it might have been expected that teachers would lack experience of such ways of working, but also in more recent publications. These locate responsibility for the inadequacies of the classroom reality of investigations in teacher inexperience (Tall, 1990), insecurity, or lack of understanding of the idea of coursework and of the nature of mathematics (Ball & Ball, 1990). Responses to criticism of weaknesses in the implementation of investigative work have also tended to focus on teachers, suggesting that they need support rather than criticism (e.g. Andrews (1993), Ollerton (1992)). Such defences of teachers suggest that the defenders themselves share the perception of teachers' practice as inadequate. The results of this study, however, bring into question the suggestion that teachers' inadequacies may serve to explain deficiencies in the implementation of investigative work.

The discourse of 'investigation' incorporates the beliefs that whatever is valued within the curriculum should be assessed and that the introduction of the assessment of investigations into high-stakes public examinations would encourage 'good practice' in the classroom (see Appendix 15 and, for example, Cockcroft, 1982; ULEAC, 1993; Burkhardt, 1988). These beliefs have, to a large extent, been echoed in the professional literature, not only in relation to GCSE but also, more recently, in relation to Attainment Target 1 of the National Curriculum (e.g. ATM, 1994). It appears that most curriculum developers consider it to be a simple matter to separate the role of assessment as a means of social control from its role as a facilitator of learning. However, the effects of the tensions between the ideals of the advocates of investigative work and the constraints of the examination system bring into question the use of assessment as a means of 'leading' the curriculum. The evidence of the conversion of pedagogic advice into assessment guidelines discussed above (section 15.3.2) supports Ernest's suggestion that the incorporation of investigation into the official curriculum serves "to routinize strategic mathematical thinking" and hence to rob it of its "emancipatory power" (1991: p.292). While Lerman (1989) argues for the development of an alternative paradigm of genuinely open work in the mathematics classroom, I would suggest that the strength of the assessment system as the regulator of teacher and student activity makes any such development impossible for any but a maverick few who are willing and able to challenge its values.

It is clear that the tensions within the discourse cannot be resolved by more experience. What may be achieved by experience is the development by teachers of an ability in different circumstances to switch between different positions within the discourse in order to achieve local coherence. This might, for example, enable a teacher to describe and justify her

classroom practice in terms of the ideal characteristics and pedagogic intent of investigative work, while using an alternative set of characteristics when assessing students' work in order to fulfil her need to comply with official assessment requirements, using the apparent requirements of the examination board as a device for absolving herself of responsibility for decisions which resolve situations of tension.

15.3.8 How may the student learn to produce effective coursework texts?

The findings of this study demonstrate clearly that, as Hodge & Kress (1988) argue, 'transparency' is not a viable theory of the relationship between thought, writing and reading. Not only did different teacher-readers make different interpretations of the meanings of the same passages of texts but there were a number of indications that teachers' interpretations were strongly influenced by their expectations about the investigative process itself. Parts of the students' texts were interpreted in terms of their fit with the stereotypical investigation process or, where this was difficult to do, the section of text was likely to be neglected or evaluated unsympathetically. This brings into question the claim of curriculum developers and researchers (e.g. Miller, 1992a; Borasi & Rose, 1989) that students' writing may enhance mathematics teachers' awareness of their students' understanding. This claim is based on a naive assumption that the text produced is a transparent representation of the writer's thoughts and that these are simply transmitted to the reader.

While the validity of the use of written work as an assessment tool at GCSE rests on the assumption that the writing in some sense 'represents' the mathematical activity and achievement of the student there is a simultaneous perception, both by the teachers studied here and in the literature on 'investigation' (e.g. MacNamara & Roper, 1992b), that there is interference between this 'true' representation and students' lack of language skills or lack of judgement about what should be included in their reports. It appears, therefore, that there is a mismatch between the teachers' readings of the forms of written language used by students in investigation reports and their readings of other aspects of students' work in the mathematics classroom. While the teachers' perception of this mismatch allowed them to some extent to compensate for weaknesses identified in the writing, they experienced tension between such an advocate role and their role as examiner. It is clearly of importance to students that their coursework achievement should be assessed as highly as possible. Teachers are also concerned that their students should be seen in a favourable light but at the same time are concerned to be seen to be competent professionals and to assess, in some sense, 'accurately'. If students are to achieve the highest possible evaluations of their investigative coursework, attention must be paid to the forms of written language they use in order to ensure that these are likely to be read positively by teachers as signs of high achievement or at least to match teacher expectations about the genre.

The analysis of a single student's complete set of texts, written over the two year period of the GCSE course, showed no obvious change in the form of the texts over time that might have been the result of interaction with a teacher. For example, the set of five texts consistently included the use of the phrase "I have found a formula" without any further elaboration of the student's mental processes. This suggests that the student was not aware that this phrase and a general lack of a narrative of mental processes were specifically condemned by teacher-readers or, if he was aware of this, that he did not have adequate resources to construct a more acceptable text. Any feedback he may have received about his writing had not effectively addressed this aspect. The review of the literature related to writing in the mathematics classroom revealed a general lack of attention to forms of language and an assumption that effective forms of communication develop 'naturally' through experience. This view of natural development of writing is challenged by Martin et al. (1987) who argue that writing, unlike speech, is unlikely to develop 'naturally' because of the lack of the possibility of 'immersion' in a written language environment. While curriculum developments involving continuous, long-term written interaction between teacher and students (e.g. Powell & Ramnauth, 1992; Clarke et al., 1993) may show some development in students' writing, leading to more positive teacher-researcher evaluations (i.e. 'Improvement'), it appears unlikely that the present UK mathematics curriculum allows enough time and space for such development to take place in the writing of reports of investigative work.

It is sometimes argued (e.g. Andrews, 1993) that the use of highly structured tasks is likely to help both students and teachers to develop ways of working independently. The differences between Steven's responses to relatively highly or weakly structured tasks, however, suggest that 'desirable' characteristics of the language used in structured tasks may not be adopted. For example, the formal naming of mathematical objects and consistent use of such names is valued within mathematics and was encouraged by the more structured tasks; moreover, there is some evidence from the teachers' readings that they particularly valued concise types of names. While Steven used a very consistent vocabulary when responding to the structured tasks, where he was allowed more independence he devised and used multiple names for the same mathematical objects. This sort of overlexicalisation may be interpreted as 'fluency' in some other genres of writing but is not likely to be valued in mathematical text. The structure provided by some of the tasks clearly did not help this student to develop this aspect of conventional mathematical language as part of his independent repertoire. In general, it is unlikely to be clear to students which characteristics of the structure provided by a particular task are those which are likely to be valued if used in other contexts unless these characteristics are deliberately attended to in the classroom.

There is currently some debate in the domain of literacy education about whether students should be explicitly taught the characteristics of specific genres of writing (see, for example,

Reid, 1987). Most of the examples drawn upon by both sides in this debate seem to be from the earlier years of education when, it might be argued, the consequences of deviations from the expected genre may not be so significant. Moreover, it appears to be largely assumed that this is an issue for language teachers, concerned with general language development, rather than for teachers in other curriculum areas, concerned with communication within their subject area. An exception to this is Kress (1990), who illustrates his argument in favour of teaching specific genres with examples of student writing in school leaving examinations in economics. Here, as is the case in GCSE mathematics coursework, the use of forms of language that will be judged 'appropriate' by the teacher and that will construct meanings that conform to the expectations of the particular academic discipline has great significance for the individual student, for whom success or failure in the examination may have life-long effects. As Kress points out, subject teachers are unlikely to be aware of the ways in which their judgements are affected by students' use of particular forms of language. If teachers were more explicitly aware of the forms that are highly valued within their discipline and of the effects that may be achieved by various linguistic choices and could pass this awareness on to their students, this would not only help students to conform to the conventional expectations of the genre but would also empower them to make informed choices to break the conventions in order to achieve deliberate effects, including to demonstrate 'creativity'.

The analysis of teachers' responses to students' writing has suggested that, while a simple list of highly visible features of writing appears to be the only language available to mathematics teachers to describe the language of coursework texts, such a list is not adequate to describe the characteristics of a text that will be highly valued in the assessment process and is thus unlikely by itself to help students to produce such highly valued texts. Moreover, one of the effects of using such a list to guide the writing of investigation reports is likely to be to reinforce the hold of the stereotypical 'investigation', stifling any possibilities of creativity, as Dixon (1987) argues in his attack on the idea of 'teaching genre'. On the other hand, as Kress (1990) argues, knowledge about the different effects that various linguistic choices can achieve could provide students with the power to manipulate their own use of language to produce such effects deliberately. Students need to be aware of the ways in which their texts will be read and assessed. Such awareness does not necessarily arise simply from being informed of the criteria. Love & Shiu's (1991) study of GCSE students' awareness and understanding of the criteria for assessing mathematics coursework suggests that, while some students who already have a relatively sophisticated understanding of the discourse and of the power of manipulating language will be able to make use of such lists of criteria, others are unlikely to find them helpful. These authors also identify a tension between the provision of explicit lists of criteria which may result in the "routinisation of producing work for assessment" (p.356) and the attempt to "elicit" the required behaviour in

less explicit ways, which may result in some students failing to discover what behaviour is required.

Another device used by some of the teachers during their assessment of student texts and apparently offered as advice to their students was the invocation of an imaginary naive reader. This appeared to act as a guide to the amount of detail that needed to be included and to be an attempt to overcome the problems for student-writers of addressing a teacher-as-examiner audience (Britton et al., 1975). The review of the research on the effects of 'audience' on student writing, however, suggested that such advice to imagine a non-expert reader was not guaranteed to be helpful to students, particularly as their lack of experience of non-teacher readers in mathematics may leave them with little awareness of the needs or preferences of such an audience (Morgan, unpublished; 1992a). Indeed, Gilbert (1989) suggests that the most successful student-writers may be those who ignore such advice and focus on their experience of the preferences of their teacher-reader.

This study has gone some way towards identifying those features that may be significant to teachers' interpretations and evaluations of students' coursework texts and some of the forms that are particularly likely to lead to high or low evaluations of the student's achievement or ability. Any attempt to use the knowledge about language and coursework derived from this study is likely to have to contend with teachers' and students' existing beliefs about writing in mathematics. As Langer & Applebee's (1987) case-studies of science and social studies teachers show, writing activities are only likely to be successfully introduced into the classroom when they fulfil "important pedagogic functions" (p.87) that are either familiar or obvious to the teachers themselves. Moreover, teachers and students may have different perceptions of the areas of their writing that need development. For example, Weir (1988) found that while science teachers perceived their students (A level and undergraduate) to have great difficulty with the accuracy and appropriateness of their grammar, the students themselves did not recognise this difficulty but considered that using an appropriate range of vocabulary was a greater problem. By focusing on the different effects that may be achieved through different linguistic choices rather than on absolute categories such as 'accuracy' or 'appropriateness', the knowledge achieved by this study should provide a basis for explicit discussion with teachers and students of the ways in which language and other forms of communication may be used most effectively to produce reports of investigative work that are likely to be judged to display highly valued mathematical processes.

15.4 Limitations of the present study and suggestions for future research

The texts used in the interviews with teachers were selected to display a range of characteristics. The scale of this study, however, has been necessarily limited. In order to enable the detailed level of text analysis needed to provide insight into the ways in which

coursework texts influence readers it has only been possible to consider a small sample of texts responding to a restricted set of tasks, representing possibly only a part of the range of possible forms of investigative coursework texts. While the reactions of the teacher-readers to these texts in no way suggested that they were unusual, it cannot be claimed that they were representative in any broader sense and it is likely that some significant features of student texts have not yet been identified. To gain a fuller knowledge of the genre of reports of investigative work it would be necessary to consider student texts drawn from a wider range of sources. In particular, given the contestation within the discourse of investigation about what constitutes an investigative task, analysis of texts and teachers reading texts responding to tasks with different degrees of structure would be likely to provide further insight into the relationships between the forms of language used and the texts' status within the discourse.

Even where significant features of student texts have been identified, further investigation may be needed in order to produce knowledge at a level that may be useful for teachers and students. In particular, while it is clear that a narrative of mental processes is required that involves more specific descriptions than a generic 'I found . . .', the alternatives (such as 'I noticed . . .') that were recommended by teachers did not actually appear in the sample of texts considered. There is thus no evidence of the ways in which teachers might react to a text containing such formulations - merely their own reconstruction of a hypothetical text. The characteristics of the 'ideal' narrative of processes still remain to be investigated. Similarly, the amount of 'explanation' contained in the texts was very limited and, while there were indications that explicit statements indicating causation may make a passage more likely to be recognised and valued as 'explanation', more investigation is needed of the forms of language that are likely to be considered effective in this context.

It is clear, however, that the language commonly available within the discourse to describe the features of students' mathematics coursework texts is inadequate to characterise those forms that are likely to be valued highly during the assessment process. This study has gone some way towards establishing such a characterisation and, moreover, provides tools with which to describe the ways in which student-writers' choices of linguistic and other forms of communication may affect the ways in which their texts are read. In order to make use of this knowledge, teachers will need support to develop more explicit awareness of the characteristics of the genre and of the effects that may be achieved by various choices.

The focus of this study on the written texts has enabled the analysis to capture features affecting teacher-assessment of coursework that reside in the text and hence are potentially available to be manipulated by student-writers as they gain greater control over the genre. In taking this focus, the characteristics of the individual student and the personal relationship between student and teacher-assessor have been neglected. Nevertheless, there are clear

indications that these aspects may also affect a teacher's reading of a text and may interact with the interpretation of the writing. The study of the reading and assessment of coursework texts through interviews, while enabling comparison between different teachers reading the same texts, has abstracted the process from its normal context. In most cases, the teacher-assessor will know the individual student-author, will have observed at least some of their work on the task and will be likely to have a pre-existing perception of the student's 'ability' level and other personal characteristics. Any or all of these factors may influence the way in which the text itself is read. Indeed, some of the teacher-readers studied here constructed their own picture of the student and his or her activity from the text, perhaps in an attempt to substitute for missing personal knowledge. Further study of teachers' reading and assessment practices would need to take into account interactions between personal knowledge of the student and characteristics of the text, perhaps in more 'naturalistic' settings in which teachers are assessing their own students' work or moderating within their department.

While differences between teachers' readings have been identified, the possible sources of these differences have not been addressed by this research. Different readings of the same text arise because the readers bring different sets of resources (Fairclough, 1989) to bear on the texts. In order to account for the specific differences between teachers' readings of sections of student text as well as for different general orientations, I would suggest four areas of difference in the teachers' resources that would appear to impinge on the readings and the judgements made.

- *beliefs about the nature of the investigative process* The degree to which the stereotyped DPG investigation structured the teachers' readings varied. This suggests that, while some take the investigative process to be a deterministic algorithm, others take a broader view of the types of ways in which students may approach a problem and hence of the types of meanings that may be ascribed to features of the texts.
- *beliefs about the nature of coursework assessment* Here there are two related questions that appear to divide teachers. Firstly, is assessment of coursework concerned with the processes gone through or with the product achieved? Secondly, what is the object of assessment - the text, the student's problem solving activity or the student herself? Those teachers who appeared to read by comparing the student's text to an imaginary 'ideal' seem to have an orientation towards product. Those who used the text to hypothesise about the student's actions or about her 'ability' and other characteristics seem more oriented towards process.
- *beliefs about the nature of communication* The search for indicators of student processes suggests that the text itself is seen as a transparent record of the student's activity (in which case the process/product problem does not arise), while seeking for understanding

of the mathematics and of the student's intentions suggests that the text is a possibly opaque report of that activity, in need of interpretation.

- *relationship with the authority of the examination board* The tensions that teachers experienced during the assessment process were often expressed in the form of a conflict between their own values and those ascribed to the examination board. The resolution of such conflict by acquiescing to the external authority was associated with the search for indicators of the fulfilment of criteria demanded by that authority, while those who were concerned to discover the student's intentions were more likely to assert the authority of their own judgements.

Investigation of teachers' belief systems in relation to these issues might further illuminate their assessment practices and provide further help for both teachers and students in coming to terms with the demands of coursework assessment.

As with the sample of student texts, the sample of teachers interviewed has been small, although here it seems likely that a broader range of practices has been captured through considering teachers drawn from several schools with different previous experiences of coursework. It cannot be claimed, however, that the picture of teachers' assessment practice constructed here is definitive. Moreover, as the structure and regulations of GCSE and of the National Curriculum change, it seems likely that the details of teachers' practice will also change. There were indications of this within the study itself as some of the teachers made use of National Curriculum statements while others referred only to examination board criteria, although reference to these different sets of criteria did not seem to make fundamental differences either to the teachers' reading practices or to their ultimate judgements of the texts. The variation in teachers' practices found in this study is itself an important indication of the complexity of the assessment process and of the inherent tensions involved in it.

One major aspect of the discourse of investigation that has not been addressed by this study is the student's perspective. The students' texts have been considered only from the point of view of the ways in which they may be interpreted by a reader. No attempt has been made to determine the students' intentions or their knowledge and beliefs about the forms of writing they chose to use. Any attempt to devise means of using the knowledge of coursework texts derived from this study in order to inform teaching and to help students to produce more effective coursework texts would need to address the nature of students' practices as writers.

While recognising the relatively limited scope of the detailed findings in relation both to the form of student texts and to the specific character of teachers' assessment practices, this research has demonstrated that a view of text as a transparent representation of an author's (or speaker's) intentions or thought processes is not tenable. The variation between teachers' readings of the same passage of student text and the recognition that apparently minor

differences in language between texts can lead to significant differences in interpretation bring into question assessment practices and research methodologies which make use of students' linguistic production as unproblematic evidence of their state of understanding. Moreover, it is clear that the use of written language in the mathematics classroom demands attention from both teachers and researchers. The method of text analysis developed here provides a set of tools with which it is possible to interrogate texts produced within the context of mathematics education in order to inform our understanding of the nature of communication between students and teachers and to provide a critical perspective on curriculum and assessment innovation.

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Appendix 1

Grade descriptions for assessing coursework

from LEAG (1989), pp.61-62

The grade descriptions given are for the award of grades A, C and F.
Teachers should use their professional judgement in determining the other grades.

GRADE	ITEM
F	<p>Shows an understanding of the tasks. Strategy poorly defined. Uses fairly routine and/or elementary methods. Usually explores a situation by experiment or by trial and error. Processes some data. Some simple calculations complete. Uses the information provided. Recognises some simple patterns. Attempts to relate results to the original task. Summarises the results and makes some valid observations. Describes some patterns or features of the results. Produces some sketches and graphs, and, where appropriate, computer output. Can give short, fairly clear responses to questions when prompted. Able to make limited use of mathematical terms. Gives single or obvious reasons for choice of strategy, apparatus, method but cannot sustain argument. Brief, not always relevant responses when questioned. Rarely initiates discussion.</p>
C	<p>Shows good understanding of the task. Applies some reasoning to plan the strategy. Adopts a systematic approach though not necessarily an efficient one. Orders and categorises information. Selects appropriate variables. Uses appropriate methods. Generally processes data accurately. Applies some variety of skills, knowledge and procedures to a task. Recognises patterns. Makes conjectures about patterns, etc. and tests them. Attempts to formulate some general rules. Devises simple formulae when generalising. Attempts to verify and justify results. States results achieved and relates these to the original task but usually without being able to state many valid conclusions. Communicates clearly the work undertaken but without giving reasons for the strategies used and/or explaining the assumptions made. Presents results in an orderly sequence. Uses an adequate range of mathematical language and symbols, including appropriate visual forms and, where appropriate, computer output. Response to questions is intelligible and audible although not as refined or concise as for Grade A. Uses some mathematical words relevant to the task and is generally familiar with the vocabulary of Level I. Can give reasons for choice of strategy although those involving successive decisions may not be explained in a logical order. Responds willingly and in some detail to questions. Uses discussion to clarify thinking and expression of ideas.</p>

A	<p>Shows excellent, clear understanding of the task. Where appropriate extends the task and/or creates sub-problems. Applies clear reasoning to plan strategies. Chooses efficient strategies. Uses appropriate concepts and methods and develops the methods as the work proceeds. Orders the information systematically and controls the variables. Uses efficient methods to simplify the task. Processes data very accurately. Discriminates between necessary and redundant information. Plans and schedules a range of relevant mathematical tasks. Applies a variety of skills, knowledge and procedures to a task. Recognises patterns. Makes and tests conjectures. Formulates general rules. Where appropriate, makes use of symbols when generalising. States results achieved and draws and states valid conclusions. Communicates clearly the work undertaken giving reasons for the strategies used and explaining some assumptions made. Selects the most appropriate methods for communicating results. Makes effective use of a range of mathematical language and notation, diagrams, charts and, where appropriate, computer output. The response to questions is clear, audible, and concise. Uses and responds to mathematical language relevant to the task and the examination level. Can explain steps in reasoning in a logical manner, including any assumptions made. Comments effectively on arguments put. Responds confidently in a variety of situations, initiates discussion, may ask further questions and sustains conversation.</p>
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Appendix 2**Coursework tasks and performance indicators**

from LEAG (1991)

Passola	272
Symmetry Groups	273
Topples	274
Pendulum	275
Inner Triangles	276
Areas Under Curves	277

PASSOLA—PASS II ROUND

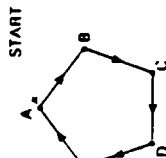
This task concerns a game called PASSOLA A

The idea of the game is that the ball is passed round a group of people

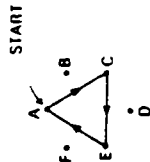
The rules of the game are

- (i) always pass the ball around in a clockwise direction,
- (ii) the *person number* is the number of people in the ring,
- (iii) the *step number* indicates the person the ball is passed to

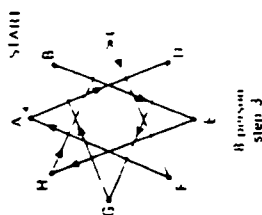
The diagrams below are given as illustrations to help with the rules



5 persons
step 1



6 persons
step 2



8 persons
step 3

1 Draw diagrams for each of the following

- (i) 7 persons, step 1,
- (ii) 7 persons, step 2,
- (iii) 7 persons, step 5,
- (iv) 8 persons, step 2,
- (v) 8 persons, step 6,
- (vi) 6 persons, step 3,
- (vii) 6 persons, step 4,
- (viii) 10 persons, step 1

(b) Write down any observations you can make about the various geometrical and/or number patterns which you can see

(c) Explain your observations

2 In some cases all of the people receive the ball, in other cases some do not

(a) State clearly the relationship between the PERSON NUMBER R and the STEP NUMBER S which determines whether or not all of the people receive the ball

(b) Try to explain why this happens

3 For a person number of 24, list the step numbers for which

- (a) all the people receive the ball,
- (b) the path of the ball is a square,
- (c) the path of the ball is a hexagon

4 Investigate further the relationship between the shape of the path of the ball and the step number, S

5 Generalise the results to parts 3 and 4

PASSOLA - PASS II ROUND

For the lower levels of G and F it is expected that the candidates would do the specific examples given. A level F candidate could also be expected to make some fairly unsophisticated observations. These might be nothing more than something such as:

"some people do not receive the ball if the person number and step number are both even"

For the middle grades of E and D candidates will be expected to tackle the investigation in some strategic manner, using well selected numbers rather than working at random. (Telling candidates of the need to work in such a way, or giving them a hint about doing so is not unfair and would not need to be recorded as help given.)

At grade D candidates will be expected to be able to give some examples where not all the people receive the ball, but not necessarily to generalise this result. Grade D candidates should show that they recognise results such as:

"100 person step 25" gives a square".

Grade C candidates will show similar expectations to those of grade D, however, they would be expected to communicate their results in a more formal manner. This might include statements such as:

"all the people do not receive the ball when the person and step number have a common factor".

At grade B candidates would be expected to go even further, perhaps making statements such as:

"if the person number is $3n$ and the step number is n then the path is a triangle".

Grade A candidates should obtain the results set out for grades C and B as above, but should also be able to offer some explanation as to why these results occur. The A and B candidates might also undertake some good work on the optional extensions.

Work on the optional extension should be used to enhance the grade awarded to a candidate.

OPTIONAL EXTENSION

Investigate the PASSOLA game further

You may wish to consider any of the following.

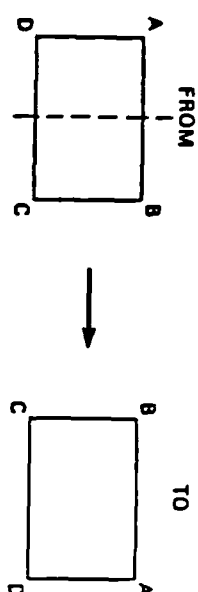
- (a) further investigations into the shapes generated,
- (b) a general rule for determining who gets the ball and when they get it,
- (c) the consequences of varying the step number, so that it is no longer fixed for each game
- (d) a further investigation of your own choice

SYMMETRY GROUPS

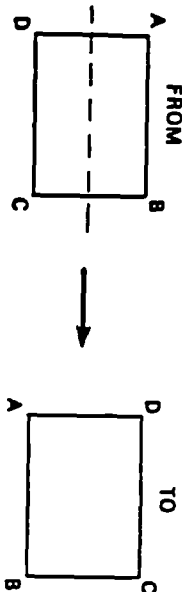
There are four transformations which can leave a rectangle in the same position although its corners have been moved.

These four transformations are,

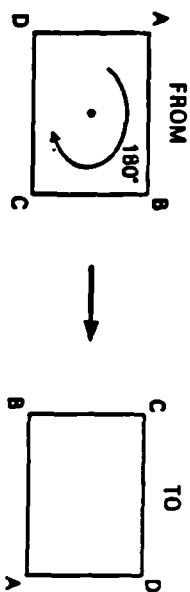
(i) a reflection in the vertical centre line



(ii) a reflection in the horizontal centre line



(iii) a rotation of 180 degrees about the centre



(iv) a rotation of 360 degrees about the centre (or doing nothing)



These four transformations make up what is known as the SYMMETRY GROUP of a rectangle.

Because there are 4 such transformations we say that the symmetry group of the rectangle has 4 elements.

- 1 Show, with labelled diagrams, that the symmetry group of a square has eight elements
- 2 Find all the elements of the symmetry group of
 - (a) an equilateral triangle,
 - (b) a regular pentagon,
 - (c) an isosceles triangle,

3 How many elements are there in the symmetry group of a regular hexagon?

- 4 Use your results for the square, equilateral triangle, regular pentagon, regular hexagon and, if necessary, any other regular figures of your own choosing, to find the GENERAL RESULT connecting the number of vertices of a regular polygon and the number of elements in its symmetry group. Explain this result.

- 5 Give, with explanations, the numbers of elements in the symmetry groups of
 - (a) a regular octagon,
 - (b) a circle,
 - (c) an irregular figure,
 - (d) an oval.

OPTIONAL EXTENSION

Give, with explanations, the number of elements in the symmetry group of some three dimensional solids

SYMMETRY GROUPS

Candidates could read around the task. This type of 'research' is to be encouraged - it is what professional mathematicians do. Credit should be awarded to candidates who tackle the task in this manner.

The task has been quite heavily structured. With this in mind it is possible to relate expectations for grades against the structure. The framework for so doing is as follows:

- | | |
|---------|---|
| grade G | showing all 8 elements for a square. |
| grade F | doing the square and any one in the part 2. |
| grade E | all of parts 1 and 2 with correct results. |
| grade D | some form of statement that the number of elements for the regular hexagon is 12 because it is 2×6 . |
| grade C | a clear and precise statement that the number of elements is $2 \times$ number of vertices for a regular polygon. Some formula expressing this, such as $n = 2v$ is to be particularly appreciated. |
| grade B | candidates should be able to explain why the general result works. |
| grade A | we would expect all that has gone before plus correct results - with reasons - to some of part 5. |

Again the optional extension can be used to enhance grades. Particular credit should be awarded to those who supply correct results, with reasons, for 3D solids.

TOPPLES

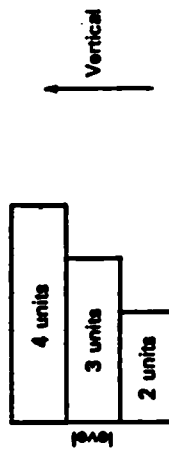
In this task you will be asked to balance some rods of different lengths on top of each other, until the pile topples.

The diagrams below are given as illustrations.



We start the pile with the 2 unit rod on the bottom and balance the three unit on top of it, being careful that the left hand edges are level.

Then we balance the 4 unit rod on top of the three unit rod.



We continue building the pile, progressing through the sequence of rods, until the pile topples.

You should find that this pile of rods topples when we get to the 5 unit rod.

So the pile that starts with the 2 unit rod at the base eventually topples when we get to the 5 unit rod.

Your task is to investigate the relationship between the length of the rod at the bottom of the pile and the rod which first makes the pile topple.

1. Starting with rods of different lengths at the base, build up your piles until each one topples. Make sure that the rods increase by one unit of length at a time.

(a) Record the length of the rod at the base and the length of the rod that makes the pile topple.

(b) Tabulate your results.

(c) Make any observations that you can.

(d) GENERALISE.

(e) Explain your result. (Well argued explanations based on intuition and insight will gain at least as much credit as those based on the principles of Physics.)

2. Imagine that you start with a rod of length 100 units and build up the pile using rods of lengths 101, 102, 103, ... units.

What will be the length of the rod that first makes the pile topple?

3. A pile topples when we place a rod of length 50 units on the top.

(a) What will be the length of the rod on the bottom of the pile?

(b) Explain your working.

OPTIONAL EXTENSION

Extend this investigation in any way of your own choosing.

TOPPLES

The generalisation for this task is well within the syllabus at intermediate and higher level; it is a simple linear function. We would therefore expect to see an algebraic (symbolic) representation of this generalisation from candidates at grade C and above. From the candidates at grades B and A we would expect to see use of this algebraic form in the two specific cases given and to offer, certainly at grade A, some explanation of why the pile topples.

For the award of a grade D, candidates should be expected to do much of that for a grade C but to lack an element of sophistication. They might, for instance, fail to generalise in an algebraic form but be able to state generalisation in words or through a specific example. They might even, at the lower level of grade D or top grade E, answer the specific examples by extending their table of results.

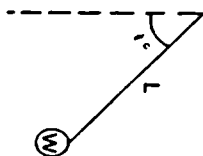
The candidates who score a grade B will certainly have tried to undertake the task in an ordered, strategic manner, looking at well selected specific cases. They will also obtain a good table of results.

At the two lower levels, we would expect to see some reasonable attempts at the investigation but not handled in any strategic fashion. The results at grade G are likely to be flimsy, few and not particularly accurate. For a grade F we would expect to see at least a couple, and preferably a trio, of correct results. The grade E and G candidates will not have been able to handle either of the specific examples.

As before, the optional extension should be used to enhance grades at all levels.

PENDULUM

The diagram shows a very easily constructed pendulum being held in its starting position



The pendulum consists of a piece of string of length L , and a bob of weight W . At the start the string makes an angle x° with the vertical.

When we let go of the bob, the pendulum will swing back and forth.

The PERIOD of the swing is the time it takes for the pendulum to swing from its starting position across to the other side and back.

Your task is to investigate what happens to the PERIOD of the swing by having different weights, starting angles and lengths.

- (a) Make a pendulum.
- (b) Investigate what happens to the period of the swing
 - (i) when you change the length of the string.
 - (ii) when you change the starting angle.
 - (iii) when you change the weight of the bob.

Please make sure that the length of the string is at least half a metre, that the starting angle is less than 30° degrees and that the bobs are of a small volume.

- (c) Record your results in the form of tables and graphs.

- (d) Comment on your results, offering any observations or generalisations

OPTIONAL EXTENSION

Investigate what happens to the period of the swing when the length of the string, starting angle and weight of the bob are all changed at the same time.

(Note: Even with a good stop watch, it might be difficult to record accurately the time of one period—that is one complete swing of the pendulum. Credit will be given for ways of overcoming this problem.)

Pendulum

Grade C

Pendulum made, with weight of bob and its length recorded accurately.
Starting angle set accurately.
Period of swing measured.

Grade F

As above.
Some changes made, as in b(i), (ii) and/or (iii).
Poor tabulation of results. No graphical communication.

Grade F

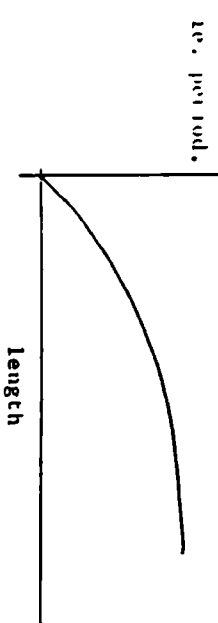
Systematic altering of length, angle and weight. Clear tabulation of results. Some attempt to overcome the problems highlighted in the note at end of task.

Grade B

More systematic altering of length, angle and weight. For instance fix angle and weight, change length in sensible units.
Offer some observations. For instance - (i) weight of bob is more or less irrelevant. (ii) as string gets longer so period of swing increases. Some graphical communication.

Grade C

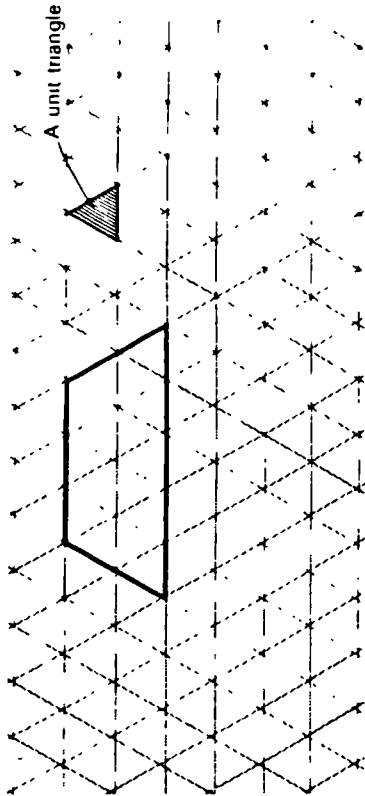
Look for all of above. Better graphical communication. Very systematic approach - such as change length in units of 5cm. Good graphical communication.



Offers some form of generalisation.
Uses attempts at extension to enhance grades - improve marks.
Look for systematic ways of changing the three parameters all at once.

INNER TRIANGLES

The diagram below shows a trapezium drawn on triangular lattice or isometric paper



The trapezium contains 16 of the unit triangles.

The dimensions of this trapezium are

- top length 3 units,
- bottom length 5 units,
- slant length 2 units?

1. How many unit triangles are there in a trapezium with dimensions

- (a) top length 2 units,
bottom length 4 units,
slant length 2 units?
- (b) top length 4 units,
bottom length 7 units,
slant length 3 units?

2. Give the dimensions of a trapezium containing

- (a) 8 unit triangles,
- (b) 32 unit triangles.

3. Investigate the relationship between the dimensions of a trapezium and the number of unit triangles it contains.

In your report you should

- show all your working,
- explain your strategies,
- make use of specific cases,
- generalise your results,
- prove or explain any generalisations.

OPTIONAL EXTENSION

Extend this investigation in any way you wish.

For the extension, the only constraints placed on you are that figures must be drawn on isometric paper and

Inner Triangles

Grade F

Part 1, obtaining answers of 12 and 33 to (a) and (b) respectively for a lower F. For a higher F, must obtain one of the answers to Part 2(a).

Grade B

Starts to approach the investigation in an ordered, systematic way. Look specifically for:

- (i) answers to 2(a) to define lower E.
- (ii) specific cases tried.

Grade D

The approach to the investigation should be more ordered. In particular look for cases where:

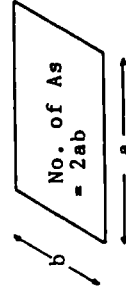
- (i) bottom length is kept constant, but slant height varied.
- (ii) distinction between parallelogram and other trapezia.
- (iii) sensible tabulation of results.
- (iv) explanation of methodology.

Grade C

Look for:

- (i) above at Grade D plus
- (ii) good explanation of technique used;
- (iii) clear identification of parallelogram and other trapezia.

The results are:



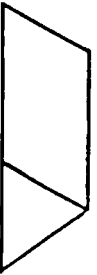
At Grade C we expect the first result expressed implicitly. Top C = second result given implicitly (by example.).

Grade B

Good approach to investigation, very well ordered and systematic. Look specifically for breakdown of



into



with recognition that top length - bottom length = slant height.

Obtaining result

$$2ab + b^2$$

is at the very top of Grade B.

This generalisation with very good write up = lower A.

Grade A

See above for lower A.

The quality of explanation of why the generalised result is as it is, defines the level of Grade A.

AREAS UNDER CURVES

This task concerns the areas of some figures

You will be asked to work out estimates of these areas and express each one as a fraction of the area of a rectangle which encloses each figure.

To help you with your estimates it is suggested that you may wish to consider using one of the following techniques

- (i) counting squares,
- (ii) cut and weigh, using an accurate balance,
- (iii) using thin strips which can be regarded as trapezia,
- (iv) a technique of your own choice.

1 Figure 1 shows an irregular figure enclosed by a rectangle measuring 3 cm by 2 cm

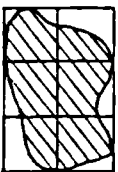


Fig. 1

- (a) Estimate the shaded area.
- (b) Express this area as a fraction of the area of the rectangle

2 Figure 2 shows a circle enclosed by a square.

This square is the smallest square in which this circle can be fully enclosed

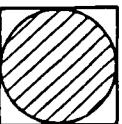


Fig. 2

- (a) Draw some circles of your own, choosing various diameters. Enclose each circle in its smallest square.
 - (b) Without using any formulae, estimate the area of each of your circles
- For each of your circles work out the fraction

$$\frac{\text{Area of Circle}}{\text{Area of Enclosing Square}}$$

The area of a circle of diameter d is given by

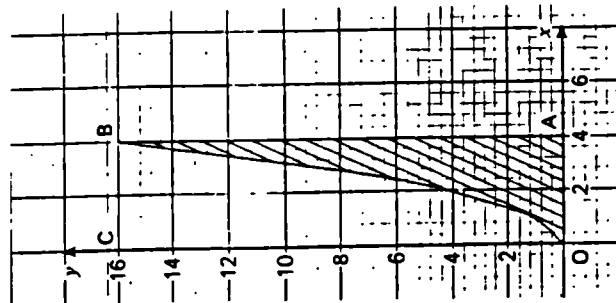
$$\text{Area} = \frac{\pi d^2}{4}$$

- (c) Using this formula, state what the exact value of the fraction in (b) above should be

3. In this part you will be looking at the areas under the curve

$$y = x^2.$$

This is drawn in Figure 3 for values of x from $x = 0$ to $x = 4$



The area of the shaded region is known as the

'Area under the curve from $x = 0$ to $x = 4$ '

Fig. 3

- (a) Obtain, by any means, an estimate for this area.

- (b) Express this area as the fraction

$$\frac{\text{Area under the Curve}}{\text{Area of the Enclosing Rectangle OACB}}$$

- (c) Show that this fraction is approximately the same whether we choose $x = 1$, $x = 2$, $x = 3$ etc., instead of $x = 4$.

(There are techniques beyond the GCSE syllabus which prove that these fractions are exactly the same.)

- 4 (a) Obtain an estimate for the fraction

$$\frac{\text{Area under the Curve}}{\text{Area of the Enclosing Rectangle}}$$

for the curves $y = x$, $y = x^3$, $y = x^4$.

- (b) From your results obtain the GENERALISED result for the fraction

$$\frac{\text{Area under the Curve}}{\text{Area of the Enclosing Rectangle}}$$

for $y = x^n$.

OPTIONAL EXTENSION

Investigate the fraction,

$$\frac{\text{Area under the Curve}}{\text{Area of the Enclosing Rectangle}}$$

when some of the graphs are combined

e.g.

$$y = 2x^3 \quad (\text{i.e. } y = x^3 + x^3)$$

$$y = x + x^2$$

$$y = x^3 - x$$

$$y = x^4 - 2x^2$$

etc

Areas Under Curves

Grade F

Doing 1.

Grade E

Lower E 2(a)
plus
Mid E 2(b)
plus
Upper E 2(c)

Grade D

Lower/Mid D 3(a) with result in region of 21/22 sq units.
Mid/Upper D Obtaining the fraction.

Look for accuracy of result to define lower/mid or mid/upper.

Note: Cut and weigh on electronic balance is the best.

Grade C

Being able to show, with reasonable level of accuracy, the results in 3(c).

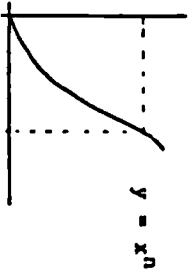
Grade B

Obtaining the fractions:

$y = x$ 1/2
 $y = x^3$ 1/4
 $y = x^4$ 1/5
along with $y = x^2$ 1/3

Grade A

Generalised result



Area under curve = $\frac{1}{n+1}$ Area of rectangle

for A

Appendix 3

Glossary of linguistic terms

Halliday's functional linguistics identifies three **Meta-Functions** that are performed by every text:

- the **Ideational** or **Experiential** function expresses "the categories of one's experience of the world" (Halliday, 1973: p.38) and one's interpretation of that experience.
- the **Interpersonal** function expresses social and personal relations between the author and others, "including all forms of the speaker's intrusion into the speech situation and the speech act" (p.41).
- the **Textual** function makes language "operationally relevant" in its context and "distinguishes a living message from a mere entry in a grammar or a dictionary" (p.42).

The **Transitivity System** consists of the processes represented in the text, the participants in the processes and the circumstances associated with them.

A **Process** is typically realised by the use of a verbal group while a **Participant** is realised by a nominal group.

Halliday (1985) identifies three main types of process:

- **Material processes** are processes of 'doing', although this may not necessarily be a concrete action. Every material process involves an **actor** and some also have a second participant or goal.
- **Mental processes** involve thinking, feeling and perceiving. They are distinguished from material processes by:
 - the necessity of involving a 'conscious' participant which would be referred to as *he* or *she* rather than *it*;
 - a second participant may be a 'fact' rather than a thing;
 - the usual (unmarked) present tense form is the simple present (e.g. 'She likes the gift') rather than the present in present (e.g. 'She is liking the gift').
- **Relational processes** are about being. They may be **attributive**, ascribing some attribute to the participant, or **identifying**, using one entity to identify another.

In addition to these three main types of process, Halliday also identifies other types which are related to these but distinct in some respects:

- **Behavioural processes**, which are processes of physiological or psychological behaviour such as breathing, dreaming, smiling, looking, listening. These share some characteristics of both material and mental processes.
- **Verbal processes** are about the symbolic exchange of meaning and involve a participant that may be anything which puts out a signal.
- **Existential processes** represent the existence or happening of a phenomenon.

A process may be transformed into a participant by means of **Nominalisation**, realising the process in the form of a nominal group, for example, *combination* (from the process *combine*) or *expectation* (from the process *expect*).

Lexicalisation is the naming of a concept and hence **Overlexicalisation** is the use of a variety of different names for the same (or closely related) concept.

Modality is “the speaker’s judgement of the probabilities, or the obligations, involved in what he is saying” (Halliday, 1985: p.75). It is realised through the use of:

- **modal auxiliary verbs**, such as *does, doesn’t, might, can, will, should*, etc.;
- **modal adjuncts**, such as *certainly, probably, usually*,
- or **adjective predicators**, for example, *I’m determined to . . .* or *you’re required to . . .*

The **Theme** of a clause is what its message is going to be about. In English, this is realised by being positioned at the beginning of the clause. The usual, unmarked theme of a declarative clause is its grammatical subject. Other elements may, however, also take this position and are thus foregrounded.

Coherence is a semantic rather than syntactic notion, referring to the extent to which a text stands as a unified whole.

Cohesion, on the other hand, refers to the linguistic resources by which coherence may be achieved. These include:

- **reference**, for example the use of pronouns to refer to participants that have been introduced in another part of the text;
- **ellipsis**, the omission of an element of the structure of the text, requiring the listener or reader to supply the missing part;
- **conjunction**, relating sections of text together by such terms as *namely, and, or, yet, then, so*;
- **lexical cohesion**, which may take the form of repetition of key words or semantically related expressions.

Deixis refers to orientation to the context in which the speech or writing event is located through the use of pronouns (*it, this, that*) or spatial or temporal deictics such as *here* or *now*.

Appendix 4

Extracts of an academic mathematics text

from R.H.Dye, 1991, 'Hexagons, Conics, A_5 and $PSL_2(K)$, *Journal of the London Mathematical Society* (2) 44: 270-286

1.1 When a group occurs geometrically there is a natural expectation that the geometry should account for its subgroups. Perhaps this is particularly so for the groups $PSL_2(K)$ and $PSL_3(K)$ that arise from those simplest of geometric objects, the projective line $PG(1, K)$ and the projective plane $PG(2, K)$ over a field K . Yet a simple graphic geometrical *raison d'être* for the existence, when it occurs, of the alternating group A_5 as a subgroup of $PSL_2(K)$ does not seem to be known. The initial aim of this paper is to discover a picturesque object that accounts for this occurrence of A_5 . We shall know that if $PG(1, K)$ is represented as a conic \mathcal{C} in $PG(2, K)$, then the pertinent object is a certain type of hexagon, distinguished by the special concurrencies of its edges. And there is a natural concurrent aim: obtain the action of $PSL_2(K)$ on these hexagons, and determine their geometry in relation to \mathcal{C} . There is, since A_5 is a subgroup of A_6 , a consequent aim: discover a configuration of these hexagons that accounts, when it occurs, for A_6 as a subgroup of $PSL_3(K)$. Such a configuration has been found, but to keep this paper within reasonable bounds the details will form a sequel. However, to paint the whole picture, we give a brief description in Section 1.6.

...

2.4 Our next step is the following.

THEOREM 2. *Suppose that H is a Clebsch hexagon in $PG(2, K)$. Then*

- (i) *each edge of H contains two Brianchon points;*
- (ii) *there are five triangles whose three sides contain, in pairs, the six vertices of H , each edge being the side of one triangle;*
- (iii) *there is a unique orthogonal polarity ρ with respect to which these triangles are self-polar;*
- (iv) *ρ corresponds to a conic \mathcal{C} unless K has characteristic 0 and the quadratic $x^2 + y^2 + z^2$ is anisotropic.*

Proof. By Theorem 1 we may assume that H is H^* . Moreover, (i) and (ii) were established in the fourth paragraph of the proof of Theorem 1. We saw in Section 2.2 that the edge joining $(1, j, 0)$ to $(j, 0, 1)$ was $jx - y - j^2z = 0$ and contained the Brianchon point $(0, -j^2, 1)$ which was also on the edge joining $(1, -j, 0)$ and $(-j, 0, 1)$. By (4), (6) the other Brianchon point of the former edge is $(1, -1, 1)$. By (4) this is also on the edge $jx^2 + jy - z = 0$ joining $(1, -j, 0)$ to $(0, 1, j)$ and the edge $x + j^2y + jz = 0$ joining $(-j, 0, 1)$ to $(0, 1, -j)$. Hence the other sides of the triangle Δ_1 of H with $jx - y - j^2z = 0$ for one side are the edges joining $(1, -j, 0)$ to $(0, 1, -j)$ and $(0, 1, j)$ to $(-j, 0, 1)$, namely $j^2x + jy + z = 0$ and $x - j^2y + jz = 0$. Now (4) gives the following display of the vertices of

$$\Delta_1: (1, -j^2, j), (-j, 1, j^2), (j^2, j, 1) \quad (13)$$

Operating by V we see that apart from Δ and Δ_1 the other triangles of H^* , with their vertices are

$$\begin{aligned} \Delta_2: (1, j^2, -j), (j, 1, j^2), (-j^2, j, 1); \\ \Delta_3: (1, j^2, j), (j, 1, -j^2), (j^2, -j, 1); \\ \Delta_4: (-1, j^2, j), (j, -1, j^2), (j^2, j, -1). \end{aligned} \quad (14)$$

Any orthogonal polarity for which Δ is self-polar must correspond to a quadratic form $Ax^2 + By^2 + Cz^2$ with $ABC \neq 0$. The demand that the first vertex of (13) is polar to the other two gives

$$Aj + Bj^2 - Cj^3 = Aj^2 - Bj^3 + Cj = 0 \quad (15)$$

...

Appendix 5

Case study student texts and analyses

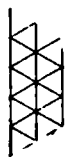
A5.1	Clive's 'Inner Triangles' Text	287
A5.2	Analysis of Clive's 'Inner Triangles' Text	292
A5.3	Steven's 'Inner Triangles' Text	297
A5.4	Analysis of Steven's 'Inner Triangles' Text	302
A5.5	Richard's 'Inner Triangles' Text	307
A5.6	Analysis of Richard's 'Inner Triangles' Text	312
A5.7	Steven's 'Topples' Text	315
A5.8	Analysis of Steven's 'Topples' Text	319
A5.9	Ellen's 'Topples' Text	327
A5.10	Analysis of Ellen's 'Topples' Text	333
A5.11	Sandra's 'Topples' Text	339
A5.12	Analysis of Sandra's 'Topples' Text	345

A5.1 Clive's 'Inner Triangles' Text

INNER TRIANGLES

I	II	III	Total
5	5	4	14

The problem that we were given was to see how many triangles there are in a trapezium.



— A unit triangle

A trapezium which contains 16 unit triangles. (Above)
The dimensions are :-
top length 3 units,
bottom length 5 units,
slant length 2 units.



Questions

1 How many unit triangles are there in

a trapezium with dimensions

a) top length 2 units,
bottom length 4 units,
slant length 2 units?

b) top length 4 units,
bottom length 7 units,
slant length 3 units?

2 Give the dimensions of a trapezium containing

a) 8 unit triangles

b) 32 unit triangles

3. Investigate the relationship between the dimensions of a trapezium and the number of unit triangles it contains

OPTIONAL EXTENSION

Extend this investigation in any way you wish

Answers

1

a) top length 2 units,
bottom length 4 units,
slant length 2 units
= 12 inner triangles

b) top length 4 units,
bottom length 7 units,
slant length 3 units
= 33 inner triangles

2 8 unit triangles
= top length of 1,
bottom length of 3,
slant length of 2

b) 32 unit triangles
= top length of 2,
bottom length of 6,
slant length of 4

3. Below the table shows the results of a quick conversion table if you have a trapezium with a slant of 1 and a top of 4 you look on the table and the answer is 3.

Slant

	1	2	3	4	5
1	3	8	15	24	35
2	5	12	21	32	45
3	7	16	27	40	55
4	9	20	33	48	65
5	11	24	39	56	75
Pattern Gap of	2	4	6	8	10

Gap of 2
between
each
answer

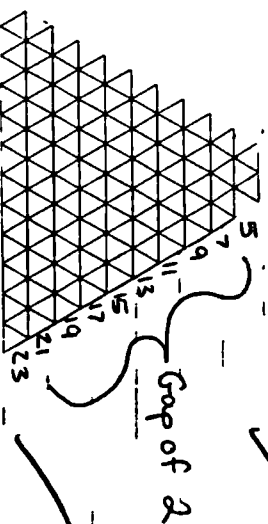
Base	Slant	Top	Total
2	1	1	3
4	2	2	12
6	3	3	27
8	4	4	48
10	5	5	75

Here is another
quick conversion
table

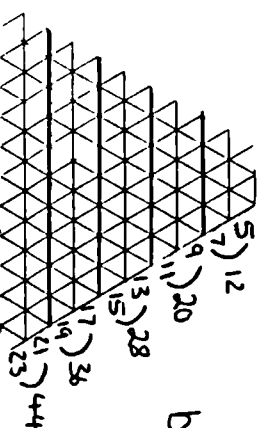
Below is a formula that our group work out, here it is

The top + The bottom x The slant

Also my formula is the one above but mine is below.



The numbers can be added together to get the next row of numbers. It can also tell you the answer from a 2 top, 2 slant and 4 bottom as well as a 2 top, 10 slant and a 12 bottom to get this add all the numbers together.



but this didn't
carry on

Extension

For my extension I am going to see how many triangles in a triangle and also how many hexagons are in a hexagon. My formula for the triangle is similar to the trapezium because a triangle is like a trapezium but without a top. Here it is

The slant length \times The bottom length

$$\frac{25}{2} = 25$$

Here is a conversion table for triangles.

	Slant				
	1	2	3	4	
1	1	0	0	0	
2	0	4	0	0	
3	0	0	9	0	
4	0	0	0	16	
b					
o					
+					
+					
o					
m					

1 = not possible

Hexagon

Like the trapeziums and the triangles I found a formula for hexagons quite quickly, here it is.

The number of triangles - 3 to give the number of hexagons inside it.

Here is my conversion table

Base	slant	bottom	\times	total
1	+1	+1	$\times 2$	6
2	+2	+2	$\times 4$	24
3	+3	+3	$\times 6$	54
4	+4	+4	$\times 8$	96
5	+5	+5	$\times 10$	150

Evaluation I found this investigation fairly easy and there were alot of patterns and formulas to find but once our group (Gareth, Sven, Joseph) and myself got under way there was no ~~stopping~~ us. ✓

A5.2 Analysis of Clive's 'Inner Triangles' Text

Clive, who was awarded a total of 14 marks out of 20 by his teacher for this piece of work, presented nine pages, including a title page containing only the title of the investigation and his own name. On the next page he states the problem in his own words and then presents the example given in the task statement. The next page, headed "Questions", is also copied directly from the task statement and is followed by a page headed "Answers" containing answers to questions 1 and 2 and two subsequent pages answering question 3. The next page is headed "Extension", followed by another page of extension work, and the final page, headed "Evaluation" contains a single sentence.

A5.2.1 Ideational aspects: Representations and algorithms

The participants and processes in Clive's text are summarised in Table A5.1 below. This table does not include the first part of the text in which the problem is stated because the questions were copied virtually word for word from the question paper. While this fact is clearly significant in itself as an indication of his attitude towards his task (see section A5.2.2 below), it is the author's original use of language which is of interest in analysing his picture of the nature of mathematics.

Table A5.1
Actors and Processes: Clive - Inner Triangles

	Human		Basic or Derived	Object Relational	Represent- ational	No actor (passive or infinite)
	general	specific				
Verbal					2	
Relational = sign	1		2 4	3	2	
Mental		4				1
Material	2	1	1			2
Behavioral	1					
Existential			2	6	3	

One of the most striking features of the summary of Clive's actors and processes in Table A5.1 is the large number of statements declaring the existence of relational and representational objects:

Here is another quick conversion table.

Below is a formula . . . here it is

. . . I found a formula . . . here it is.

. . . there were a lot of patterns and formulas . . .

The objects concerned are tables, formulae and patterns and these clearly play a significant part in Clive's text. Not only are they present in the text but their presence is declared, drawn to the reader's attention by the use of existential statements. The role of the specific human actors involved is to search for these formulae and patterns:

Below is a formula that our group work out . . .

. . . I found a formula for hexagons quite quickly . . .

while the general reader is advised to *look on the table* in order to find the answer. Moreover, the representational objects are not only tools to be used by the reader but may play an active role, using verbal processes to inform:

Below the table shows the results of a quick conversion table

Not only is it the table that shows the results, rather than the author, but the nominal *results of a quick conversion table* suggests that they are the results of the table itself, not of any human activity. Similarly:

It [the diagram] can also tell you the answer

It is significant that Clive uses *answer* here rather than *number of unit triangles*; the mathematical problem with which he is working is primarily about numbers and number patterns rather than about the ostensibly geometrical objects giving rise to them.

The origin of the tables in this text is obscured. They are merely declared to exist and on only one occasion (in the final part of the extension) is there any suggestion of human agency in the production of a table when the author claims ownership, declaring *Here is my conversion table*. The general lack of explicit human agency may indicate obliquely that most of the tables were constructed by other group members or by the group as a whole rather than by the author himself (and are, therefore, not to be claimed personally by the author). However, the image of *table* presented in the text is of a pre-existing basic object, rather than a deliberately constructed organising mechanism or problem solving tool. An idiosyncratic lexical item used by Clive is the expression *conversion table* or *quick conversion table* to describe each of his tables of results. His tables are clearly not what is usually referred to as conversion tables and this misused term may be negatively evaluated by a teacher/assessor.

Unlike tables, the formulae are *worked out* or *found* and ownership by the group or by the author himself is claimed each time a formula is introduced. The formulae are thus presented as the product of human activity. Since the overall purpose of the piece of coursework is to allow the teacher to assess the author, what the author has achieved must be made clear to the reader. Whereas the tables appear as autonomously existing entities, the formulae represent personal achievements.

Although the problem is about geometric objects, the diagrams in Clive's text also reinforce the idea that the problem is primarily about patterned sequences. For example, the first two diagrams, although in the form of trapezia, appear not to represent individual trapezia but rather whole sets of trapezia used to generate a number pattern. The commentary on the diagrams focuses the reader's attention on the numbers rather than on the geometrical figures:

The numbers can be added together to get the next row of numbers. It can also tell you the answer . . .

The referent of the word *it* in the extract above is not explicit but appears, perhaps because of a lack of appropriate technical vocabulary, to be "pointing" (Rowland, 1992) to the pattern displayed in the second diagram (derived from the first diagram by adding the numbers for each pair of rows). This extract also illustrates the suppression of human agency in this text by the use of the passive voice in the first sentence and by the the indeterminate *it* to perform the verbal process of *telling* the answer.

The infrequent use of the equals sign is another unusual feature of Clive's text. Apart from four occurrences signalling the answers to each of the parts of questions 1 and 2, it is used only twice: once to equate a diagram with the number of triangles inside it, and once to provide a key to notation used in a table, *O = not possible*. None of these uses plays a conventional identifying role.

The formulae, which play such a significant part in the text as the products of mathematical activity, are expressed as rules rather than as relations. Thus,

Below is a formula that our group work out, here it is.

The top + The bottom \times The slant

The result (and indeed the purpose) of using the formula remains implicit. A further example from the extension section of the text, however, shows that the purpose of a formula is to perform a calculation in order to achieve an answer.

. . . I found a formula for hexagons quite quickly, here it is.

The number of triangles \div 3

to give the number of hexagons inside it.

Here it is explicit that the formula is an algorithm rather than a relation between variables.

There are very few causal relationships expressed by Clive other than *getting* a numerical answer from a calculation or a table. This extract illustrates clearly the role of the human mathematician as mediator between the given information and the answer:

If you have a trapezium with a slant of 1 and a top of 1 you look on the table and the answer is 3.

There is, however, one interesting example which shows a different aspect of the nature of mathematics. This displays a recognition of the importance of structure:

My formula for the triangle is similar to the trapezium because a triangle is like a trapezium but without a top.

The world of patterns and formulae is seen to mirror the world of concrete objects; thus a similarity between triangles and trapezia is paralleled by a similarity between the formula for the number of units in a triangle and that for a trapezium. This statement is interesting given the general lack of reference back to the concrete in the rest of the text. Perhaps there is no need to make explicit connections elsewhere simply because it is so obvious to the author that a close relationship exists between formula and shape.

A5.2.2 Interpersonal aspects: expression of ownership and confidence

Examining the author's presence in the text, it appears that he considers it important to distinguish between the results that were achieved by the group in which he was working and those which he achieved individually. This is consistent with the assessment purposes of the task although some other writers are not so scrupulous about crediting their peers. For example, the first formula is introduced thus:

Below is a formula that our group work out

Ownership of this formula is then claimed by the author individually - he has not merely copied it from other members of the group - but he goes further to affirm that the rest of the work is his own alone:

Also my formula is the one above but mine is below.

Such statements of personal ownership of formulae and tables appear especially in the extension section of the text:

My formula for the triangle . . .

I found a formula . . .

Here is my conversion table.

In the context of assessment it is important to demonstrate this personal ownership because the reader is going to read evaluatively to assess not only the overall quality of the work but, more crucially, the achievement of the individual author.

The author's attitude towards the task itself appears to alter halfway through the text. At the beginning it is plain that the task is one that is imposed from without:

The problem we were given . . .

Not only was the problem *given* rather than freely chosen by the author and his group of co-workers, but the use of the passive voice, by obscuring agency, emphasises the impersonal nature of the task. Moreover, the presentation of the statement of the problem and of the specific questions is virtually identical to that given in the question paper; even the illustrative diagram, the layout and punctuation have been copied. This further strengthens the impression that the task has been imposed. When the author progresses to the extension, however, the representation of his relationship to the task alters. He both describes the task in his own words and uses the first person to claim ownership:

For my extension I am going to see how many triangles in a triangle and also how many hexagons are in a hexagon.

The informal, speech-like quality of this problem statement contrasts vividly with the formal statement of the original problem earlier in the text, copied from the question paper:

Investigate the relationship between the dimensions of a trapezium and the number of unit triangles it contains.

Clive's own language is not only less formal in its lexis (e.g. *see* rather than *investigate*) but is also less precise in its mathematical formulation. Unlike the precise technical expression of *the relationship between the dimensions of a trapezium and the number of unit triangles it contains*, the phrase *how many triangles in a triangle*, while quite clear within this particular context because of the work that has gone before, could be interpreted in different ways within the context of different tasks (e.g. if posed following solution of the "How many squares on a chess board" problem). The informal language might suggest an informal and sympathetic relationship between author and reader; the reader is assumed to share a common understanding of the problem and to be willing to interpret potentially ambiguous expressions in the way that was intended by the author. At the same time, the paraphrasing of the instructions given in the task statement may be taken as a sign that the author has understood the task and made it his own. On the other hand, a less sympathetic reader might interpret the lack of specialist language (in particular, the lack of mathematical symbolism throughout and the 'misuse' of the equals sign described above in section A5.2.1) as a sign of poor ability in mathematics.

No 'working out' is presented as evidence of the processes that the author went through to arrive at his formulae and tables. The reader is thus constructed as concerned with products only. However, these products include general methods for the reader to use to find specific answers. As well as using the algorithms presented as formulae, it is suggested that the reader might *look on the table* or use the pattern shown in a diagram because it *can also tell you the answer*. The reader is interested not only in the fact that the author has arrived at an answer but also in the mathematical power that that answer provides to a user. The diagrams, although presented as tools for solving the problem, are nevertheless 'public' tools, summarising the method of solution discovered by the author and making it available to his audience, rather than 'private' ones that merely record his own processes.

The author's level of confidence in his work is expressed in the modality of the text. On the whole, positive statements are made without any modification. Doubt enters in only when explaining the method for using an array of numbers. Although a positive possibility is expressed, the modification in itself suggests a degree of uncertainty:

The numbers can be added together to get the next row of numbers. It can also tell you the answer . . .

The source of the doubt is apparent when it is observed that the pattern on which these generalisations relies *didn't carry on*. It might be asked why the author has included this doubtful section at all. A number of interpretations are possible: he did not recognise that a pattern which does not *carry on* is of doubtful value; he felt that the reader would be able to make sense of it anyway; or he may have been following an instruction from his teacher to show everything he did, including dead ends.

Elsewhere, confidence is expressed through explicit statements about the work, for example:

I found a formula for hexagons quite quickly

I found this investigation fairly easy

although in each case the level of confidence is qualified. In the context of assessment within which this text is situated, such statements could be seen as double edged; on the one hand, the author may be seen as able to solve problems quickly and easily and hence be evaluated highly, while on the other hand, there is a danger that the author's extension might therefore be judged to be trivial because it was too easily completed. Hence the qualifications serve to hedge the author's bets in this situation. The final sentence of the text, however, is presented unambiguously:

. . . once our group . . . got underway there was no stopping us.

Hard work and perseverance are sure to be valued by the reader/assessor even if speed and finding the work easy are less certain of success.

A5.2.3 Textual aspects: A display of results

As described above (section A5.2.1) there are a large number of existential statements in this text, declaring the presence of tables and formulae. These frequently occur in clauses containing themes which direct the reader's attention to the relevant parts of the text. The following extract illustrates how these locational themes construct the text as a display of significant results:

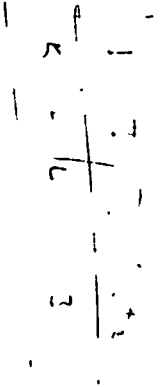
Below the table shows the results of a quick conversion table. If you have a trapezium with a slant of 1 and a top of 1 you look on the table and the answer is 3. [table 1] [table 2] Here is another quick conversion table. Below is a formula that our group work out, here it is. [formula] . . .

A similar pattern is to be found repeatedly throughout the text: a declaration of the presence of a representational or relational object, sometimes accompanied by an explanation of how to make use of it, together with the object itself.

As described at the beginning of this analysis, the overall structure of the text follows closely the way in which the task was originally posed with headings naming each of the discrete sections, defined by the questions given. This serves to position the text within the 'school mathematics' discourse as a response to a series of questions, particularly as the questions are presented word for word and the answers numbered. There are, however, two structural features which do not fit in with this convention: the title page, whose stylised lettering suggests an attempt at decoration, and the last section, entitled "Evaluation". Both of these serve to present the text as a single product rather than a sequence of unconnected answers and make it appear a 'project' rather than an exercise.

A5.3 Steven's 'Inner Triangles' Text

INNER TRIANGLES



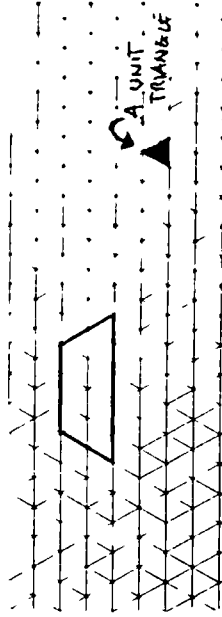
cii

CONTEND PAGE.

THE PROBLEM	2
QUESTION 1.....	3
QUESTION 2.....	4
QUESTION 3.....	5
PATTERNS.....	6
FORMULA.....	8

INNER TRIANGLES

The diagram below shows a trapezium drawn on triangular lattice as some of the paper.



The trapezium contains 16 of the unit triangles.

The dimensions of this triangle are:

top length 3 units

bottom length 2 units

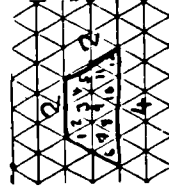
slant length 2 units

QUESTIONS

298

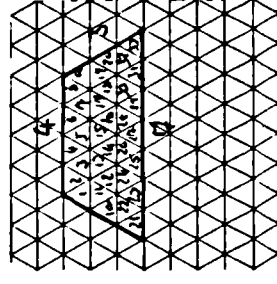
1 How many unit triangles are there in a trapezium with dimensions

- a) top length 2 units,
bottom length 4 units,
slant length 2 units.



= 12 units.

- b) top length 4 units,
bottom length 7 units,
slant length 3 units.



= 33 units.

Questions.

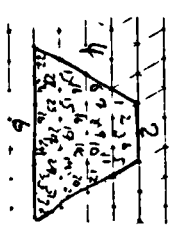
2. Give the dimensions of a trapezium containing:

a) 8 unit triangles



= top length 2,
bottom length 4,
slant length 2.

b) 32 unit triangles.



= top length 4,
bottom length 8,
slant length 4.

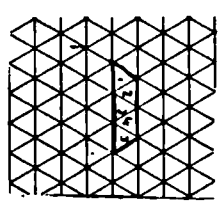
3. Investigate the relationship between the dimensions of a trapezium and the number of unit triangles.

PATTERNS

I have found that whenever you increase the top length or the slant length the number of unit triangles goes up by the same amount.

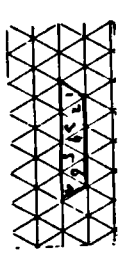
If you have a trapezium with:

top length 2,
bottom length 4,
slant length 1,
unit triangles = 5.



Then this will have 2 less units than a trapezium with:

top length 3,
bottom length 5,
slant length 1,
unit triangles = 5.



Thus happens when you adjust the top length. I have made a table up to show the results on a larger scale.

TABLE TO SHOW DIFFERENCE IN UNIT NO. WHEN TOP LENGTH IS INCREASED.

TOP	1	2	3	4	5
BOTTOM				.	.
SLANT	1	1	1	1	1
UNIT No.	3	5	7	9	11

As you can see the unit No. increases by two every time the top length increases by one.

This can be done by using any slant No. but if you change this you may find that the unit increases may be different. e.g.

TOP	1	2	3	4	5
BOTTOM
SLANT	2	2	2	2	2
UNIT No.	6	12	18	24	30

This time the unit increase is by 4 instead of 2.

On the next one when you remove the slant to three it increases to 6.

TOP	1	2	3	4	5
BOTTOM
SLANT	3	3	3	3	3
UNIT No.	15	21	27	33	39

As you can see the difference is

six.

Another interesting pattern is the way in which the unit No's increase when the top length stays the same and just the slant increases.

TOP	1	1	1	1
BOTTOM				
SLANT	1	2	3	4
UNIT No.	3	6	15	24

The first increase is by 5, then 3 to 8 and then from 8 to 15 is by 7 and finally 15 to 24 is increased by 9. This shows that it increases by the same amount as before but increases by 2. So it would go:

5, 7, 9, 11, ...

This pattern works whenever the top number is:

Formula

I have found a formula that works on all trapeziums.

$$y + x \times z = \text{Unit No.}$$

y = Top length

x = Bottom length

z = Slant length.

example.

Top length 4
Bottom length 6
Slant length 2

$$4 + 6 = 10$$

$$10 \times 2 = 20$$

There would be 20 unit lengths.

A5.4 Analysis of Steven's Inner Triangles' Text

Steven, awarded 13 marks out of a possible 20 by his teacher, presented a total of 11 sides of work. This starts with a decorative title page and a "contents page". The statement of the problem is then given, copied word for word from the given statement on the question paper (although there are some apparent copying errors: the word *trapezium* is twice mis-spelt as *trampezium* and once substituted by the word *triangle*). The next two pages, headed "QUESTIONS" contain the copied text of questions 1 and 2 together with answers to these questions. There then follow four pages of work on question 3, headed by the copied statement of the question and a heading "PATTENS" (sic). While most of these pages are filled, one, containing only a single table and one sentence, is written on the back of a sheet and appears to have been inserted as an afterthought; unlike the rest of the pages, it is not numbered. After the description of the patterns is a page headed "FORMULA" and the final page contains no more than the single word "Conclusions".

A5.4.1 Ideational aspects: patterns and variation

The participants and processes in Steven's text are summarised in Table A5.2 below. This table includes only the part of the text following the heading "PATTENS" as, prior to that, the bulk of the text has been copied from the original question paper.

Table A5.2
Actors and processes: Steven - Inner Triangles

	Human		Basic or Derived	Object		No actor (passive or non-finite)
	general	specific		Relational	Represent- ational	
Verbal					2	
Relational = sign	1		3 8	5		
Mental	3	2				
Material	4	1	5	3		3
Existential			1	1		

There is a high proportion of material processes in this text, mostly to do with changing, and in particular increasing, either lengths or numbers. Where this action is explicitly performed by a human agent, it is a general *you* rather than a specific person; in most cases, however, either the action is presented without an actor at all through the use of the passive, or it is the length or number itself which performs the action. In this passage from the section entitled "PATTENS", these instances of increasing and changing have been printed in bold type.

*I have found that whenever you **increase** the top length or the slant length the number **always goes up** by the same amount [e.g.] This happens when you **adjust** the top length. I have made a table up to show these results on a larger scale. **TABLE TO SHOW DIFFERENCE IN UNIT NO. WHEN TOP LENGTH IS INCREASED** [table 1]*

*As you can see the unit No. **increases** by two every time the top length **increases** by one. This can be done by using any slant No. but if you **change** this you may find that the unit **increases may be different**. e.g. [table 2] This time the unit **increase** is by 4 instead of 2. On the next one when you **increase** the slant to three it **increases** to 6. [table 3] As you can see the **difference is six**. Another interesting pattern is the way in which the unit No's **increase** when the top length stays the*


same and just the slant increases. [table 4] The first increase is by 5, from 3 to 8 and then from 8 to 15 is 7, and finally 15 to 24 is increased by 9. This shows that it increases by the same amount as before but increases by 2. So it would go: 5, 7, 9, 11, This pattern works whatever the top number is.

The lengths and numbers are presented as changing autonomously. Not only is human agency obscured through this but also through the author's use of the nominalisation *the unit increase*. The process of varying the values in the problem is not shown as something done by the author himself; rather, it shifts from being a process that may be carried out by any mathematician (*if you change this or when you increase the slant*), to a process performed by mathematical objects themselves (*the unit No. increases by two every time the top length increases by one*) or by some unspecified agent (*15 to 24 is increased by 9*), and finally to an object which may itself have properties and variations (*The first increase is by 5 . . . This shows that it increases by the same amount as before*). This nominalisation opens up the possibility of a higher complexity of generalisation, taking account of relationships between three variables rather than just two at a time and considering rates of change as well as individual changes.

The mathematical activity in this text is thus seen to be about variation in values. The variation is brought about through the autonomous existence of patterns of relations between numbers rather than through human activity. The human activity is to set the various patterns into action by adjusting the parameters and then to observe the results:

This can be done by using any slant No. but if you change this [i.e. the size of the slant No. which holds a different constant value in each of the first three tables] you may find that the unit increases may be different.

The author's own activity is explicitly mentioned only three times: at the beginning of each of the sections "PATTERNS" and "FORMULA" with the declaration *I have found . . .* and once to state *I have made a table up to show these results on a larger scale*. There is no attempt to recount the processes through which the results were "found". The only indication of any 'working out' is to be found implicitly in the small number of diagrams which illustrate the answers to questions 1 and 2 and the single example demonstrating the change in the number of unit triangles which accompanies a change in the top length of a trapezium. Each unit triangle in these diagrams of trapezia has been labelled with a number, suggesting that a process of counting has occurred:

e.g. 

This practical mathematical activity is, however, only in the background and is not referred to explicitly in words. Moreover, although it seems likely that a significant amount of drawing and counting must have taken place, only a very small number of specific cases have been given as examples. Such practical activity is, therefore, to be seen as less important than the description of patterns and the discovery of a formula.

Tables play an important part in this text, signalled by their prominent positioning on the page, large capital letters to name the variables, and the use of colour to distinguish between the values of different variables. Although the purpose of the first table is introduced as *to show these results on a larger scale*, all the tables, including this one, are subsequently presented as ways of organising data to show patterns. Thus the first table is headed:

TABLE TO SHOW DIFFERENCE IN UNIT NO. WHEN TOP LENGTH IS INCREASED

Each of the tables is integrated into the description of the patterns by drawing the reader's attention to relationships between the values in the tables rather than to the entries themselves, e.g.

As you can see the unit No. increases by two every time the top length increases by one.

The raw data is not interesting in itself but only in its interpretation.

Having seen that the focus of the main part of the text is on patterns and relationships, it is not surprising to find that the formula is apparently expressed in relational form as a predominately symbolic equation: $y + x \times z = \text{Unit No.}$ However, the author introduces it by stating *I have found a formula that works on all trapeziums*, thus suggesting that the formula's role is in fact a procedural one for working out the number of unit triangles. This interpretation is reinforced by the presentation of an example showing how the formula may be applied to find this number:

Top length	4	
Bottom length	6	$4 + 6 = 10$
Slant length	2	
		$10 \times 2 = 20$

There would be 20 unit triangles.

There is a discontinuity here between the earlier focus on patterns and relationships and the procedural focus of the page headed "FORMULA". A possible explanation of this could be the author's belief that the coursework genre within which he is working necessarily involves the statement and testing of a formula. Although such a formula does not follow naturally from the earlier development of his work, he has felt the need to include it because of the demands of the assessment context.

A5.4.2 Interpersonal aspects: an authoritative colleague

As was seen above, there is little explicit mention of the author's own activity in the text. It thus appears that the reader's concern ought to be with the mathematical content rather than with the author's processes. On the three occasions when the author does refer to himself, the effect is to claim ownership of the mathematics by stating, for example, *I have found that*, rather than to describe his action by using the alternative simple past tense *I found that*.

The reader is addressed as a colleague who is expected to take an active part in making sense of the mathematics. She is addressed directly in order to bring her attention to the significant features of the tables:

As you can see the unit no. increases by two every time the top length increases by one.

Similarly, the reader's attention is drawn to *another interesting pattern*, suggesting that she is expected to share the author's enthusiastic attitude towards his findings. The relationship between author and reader is thus constructed as one in which a general competence and interest in mathematics is shared but in which the author has expert knowledge about the particular problem considered in this text.

The results are presented with certainty, for example:

... whenever you increase the top length or the slant length the number always goes up by the same amount. e.g. If you have a trapezium ... Then this will have 2 less than a trapezium with ...

stressing not only the generality of the results but also the author's confidence in their validity. The only exception to this occurs when the author is making a prediction at a higher level of generality, going beyond the types of trapezia he had actually evaluated to suggest that similar patterns would continue:

This can be done by using any slant No. but if you change this you may find that the unit increases may be different.

Authority and confidence are further expressed through the vocabulary used to refer to mathematical objects; this may not, however, be seen by a teacher/assessor reader to be appropriate. Although the original words, copied from the given task, were used at the beginning of the text (with the apparent copying errors mentioned earlier), the author has clearly not felt constrained to use the given terminology but has used his own words in the rest of the text. The “unit triangles” named in the task, are referred to at various points as “unit triangles”, “unit no.”, “units”, “the number”. While this variety suggests an intimate relationship with the task, its informality is far from the usage of conventional mathematical discourse in which the precise and unambiguous naming of objects is seen as a crucial difference between mathematical and ‘ordinary’ language. There is a tension between the breadth of vocabulary expected of a fluent, successful writer in many other fields and the one-to-one relationship between label and object expected in mathematical writing.

A5.4.3 Textual aspects: logically ordered description

As described at the beginning of this analysis, the text is to a large extent structured by the questions posed in the given statement of the task; these questions have even been copied word for word and the answers signalled by the use of an equals sign, positioning it within a ‘school mathematics’ discourse. This structure is, however, contained within a framework that includes a coloured and decorated title page, a contents page and a page labelled “Conclusion”. This framework, together with the use of colour in tables and prominent lettering for headings, constructs the text as a ‘project’.

The extended section headed “PATTENS”, is initially presented as the answer to question 3 of the given task which instructs the student to “investigate”. Examining the thematic choices made by the author in this section, a high proportion of topical themes serve to construct the text as a largely descriptive report, focusing on the objects of the task: the lengths of the sides of trapezia, the number of unit triangles contained, and the increases in these quantities. In addition, however, there are parts of the text, illustrated by the following extract, in which thematic choices draw the reader’s attention to the logical progression of the facts presented.

This time the unit increase is by 4 instead of 2. On the next one when you increase the slant to three it increases to 6. [table] As you can see, the difference is six. Another interesting pattern is the way in which the unit Nos increase when the top length stays the same and just the slant increases. [table] The first increase is by 5, from 3 to 8 and then from 8 to 15 is 7 and finally 15 to 24 is increased by 9. This shows that it increases by the same amount as before but increases by two. So it would go: 5, 7, 9, 11 . . .

While the text is largely descriptive, the structure of relationships between the objects being described is also made clear to the reader; the various patterns described are part of a coherent overall system.

One of the sentences from the above extract presents an interesting example of the way in which a transformation can shift the focus:

Another interesting pattern is the way in which the unit Nos increase when the top length stays the same and just the slant increases.

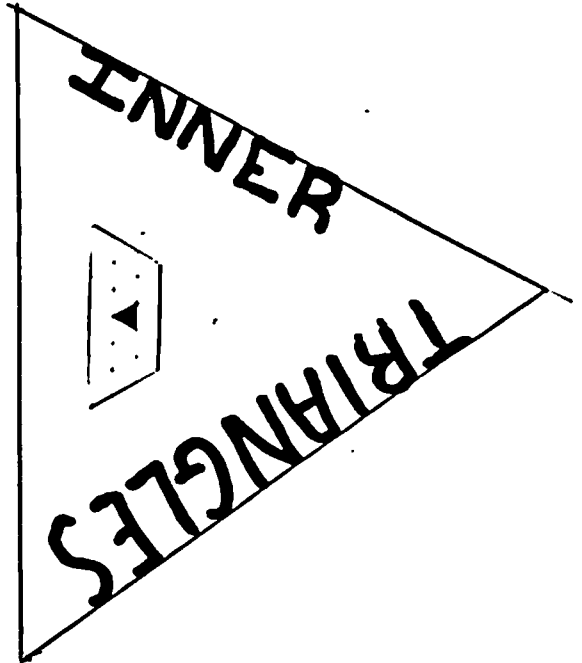
By presenting the sentence in this form rather than the unmarked alternative

The way in which the unit Nos increase when the top length stays the same and just the slant increases is another interesting pattern.

the reader’s attention is drawn primarily not to the topic of the next section of the text (in which top lengths are kept the same while the slant length is varied) but to the fact that it is another interesting pattern. The focus of the text is thus seen to be the system of patterns described rather than the details of the individual patterns.

The page headed "FORMULA", which follows the description of the patterns, has few cohesive links with the immediately preceding part of the text. Starting with the words *I have found a formula* it echoes the first words of the "PATTENS" section, while the way in which the formula itself is presented with a sequence of statements of identity using equals signs, each on a single line, is similar to the question-answer style of the "QUESTIONS" section. This discontinuity makes the formula appear an afterthought, included because it is part of the requirements of 'school mathematics' (like the answers to the specific questions at the beginning of the task) rather than part of the "interesting" exploration of patterns reported in the main body of the text. Similarly, the presence of the otherwise empty page headed "Conclusion" appears to be an (unsuccessful) attempt to conform to the requirements of the 'project' genre. The lack of cohesion and the mixture of genres suggest that the author is not completely in control of his writing and this is likely to be interpreted by a teacher/assessor as a lack of ability or a failure to complete the task adequately.

A5.5 Richard's 'Inner Triangles' Text



I	II	III	Total
6	5	3	14

34

1

Top length = 2
 Bottom length = 4
 Slant length = 2
 Unit triangles = 12 ✓

Top length = 4
 Bottom length = 7
 Slant length = 3
 Unit triangles = 33 ✓

Top length = 1
 Bottom length = 3
 Slant length = 2
 Unit triangles = 8 ✓

Top length = 7
 Bottom length = 9
 Slant length = 2
 Unit triangles = 32 ✓

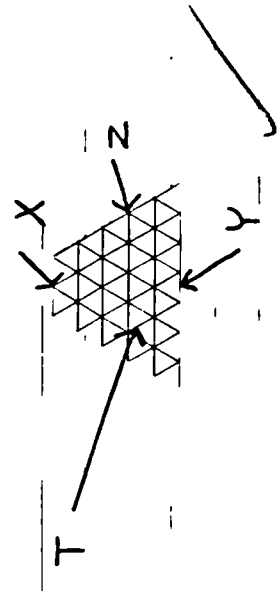
2

Working out.

Results

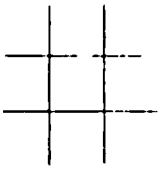
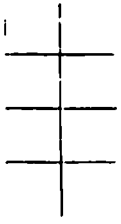
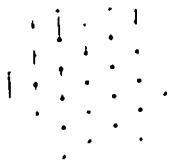
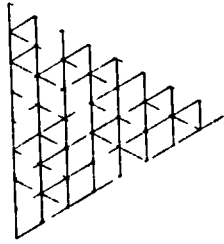
Top length	Bottom length	Slant length	Unit of triang
1.0m	3	2	8
1.0m	4	2	12
1.0m	5	2	16
1.0m	4	3	15
1.0m	5	3	21
1.0m	6	3	27
1.0m	5	4	20
1.0m	6	4	32
1.0m	7	4	40
1.0m	6	5	30
1.0m	7	5	45
1.0m	8	5	55

Formula



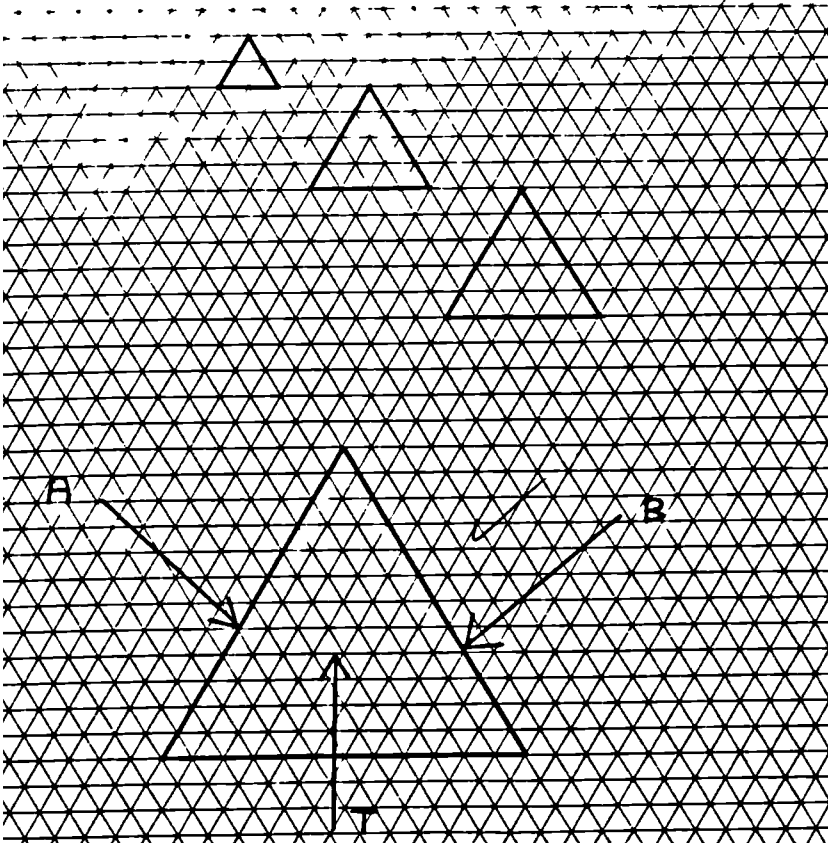
$$Z(X+Y) = T$$

EXTENSION



Working out

Triangles



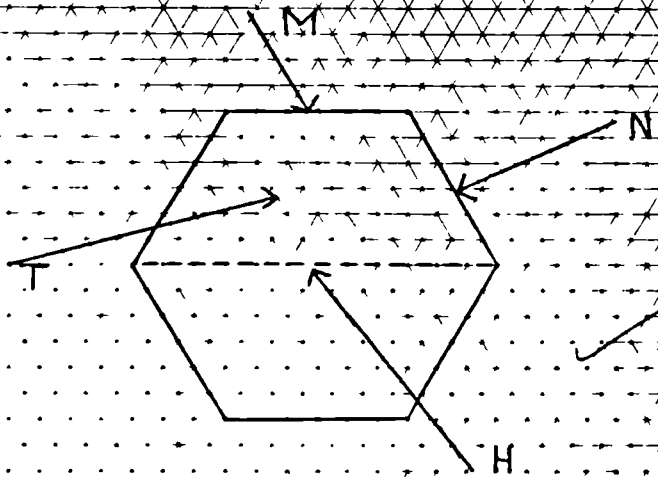
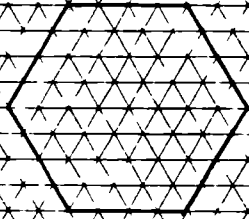
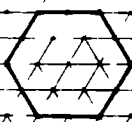
Length of side	Unit triangles
1	1
2	4
3	9
4	16
5	25

Formula

$$A \times B = T$$

310 Working out

Hexagons



Length of sides	Unit tr
1	6
2	24
3	54
4	96
5	150

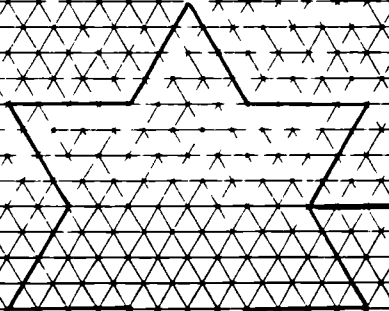
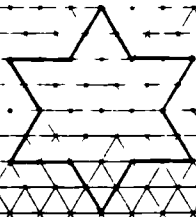
Formula

$$N(M+H) \times 2 = T$$

Eg

$$2(2+4) \times 2 =$$

Working out



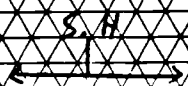
Perimeter	Slant Height	Unit tr
12	1	12
24	2	48
36	3	108
48	4	192

Stars

Formula

Do you mean num?

$$\text{Perimeter} \times S.H. = T$$



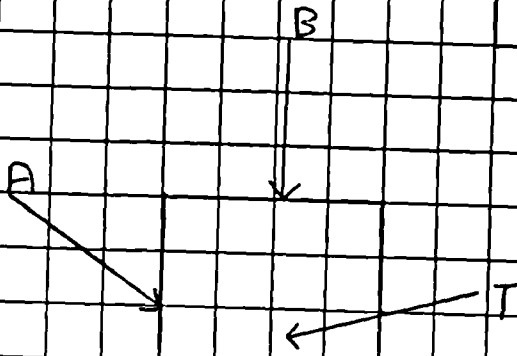
Squares

311

side A	side B	Unit triangles Squares
1	1	1
2	2	4
3	3	9
4	4	16
5	5	25
6	6	36

Formula

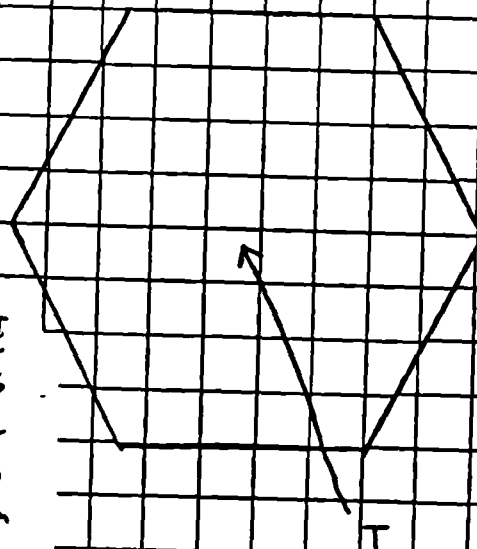
$$A \times B = T \quad \checkmark$$



Hexagons

side length	Unit squares
1 cm	
2 cm	7
3 cm	
4 cm	
5 cm	

Estimated



Formula

middle line + length of side

A5.6 Analysis of Richard's Inner Triangles Text

Richard was awarded 14 marks out of a possible 20 by his teacher. He submitted 18 pages altogether, including a title page on coloured card and 8 pages of rough work. The rough work, some of which is attached upside down, consists of diagrams, calculations, rehearsals of the tables and formulae appearing in the main body of the text, and doodles. Some of the pages have apparently had diagrams cut out from them. The presentation of this rough work, which contrasts sharply with the well-ordered and even decorative presentation of the first 10 pages, makes it clear that it was not originally intended for public consumption. Only the first 10 pages will be analysed here.

After the title page, there are three pages devoted to the initial part of the problem, containing respectively the numerical answers to the first two questions, diagrams with the heading 'Working out', and a table, diagram and formula. Another title page follows for the 'Extension', and there are then five pages each of which contains diagrams, a table and a formula pertaining to a different shape.

A5.6.1 Ideational aspects: abstract relationships between concrete objects

One of the most obvious characteristics of Richard's work is the almost complete absence of any indication of the author's involvement in the mathematical processes that gave rise to the content of the text. Examination of the actors and processes represented reveals that the only participants in the text are basic objects such as triangles and hexagons and measurements derived from them (e.g. *length of side* and *unit triangles*). No human actors are present; the author's agency in producing the mathematics is obscured by the use of nominalised headings such as *Working out*, *Results* and *Extension*. An implicit indication of his activity is, however, provided by a set of diagrams of four trapezia roughly drawn on isometric paper in which most of the small triangles inside each trapezium contains a faint dot. This suggests a pointing and counting process has occurred, giving rise to the numbers which label each trapezium. The order of presentation of each of the six pages containing diagrams, followed by a table, followed by a formula, may be read as suggesting a sequence of activity leading to the discovery or construction of the formula but here, as elsewhere, there is no verbal indication of the process. The formula itself is the final item on each page and is further highlighted by being labelled with an underlined heading "Formula". The focus of the text is clearly on the product of any mathematical activity rather than on the activity itself.

The only processes indicated explicitly are relational ones, using the equals sign. This is used in two types of statement. On the first page after the title page there are four sets of statements of the form:

Top length = 2
Bottom length = 4
Slant length = 2
Unit triangle = 12

These are labelled to indicate that they are answers to the first two questions given on the question paper. The equals sign is used to ascribe a property to each of the variables listed. Although the two questions asked for the values of different variables (question 1 asking for the number of unit triangles while question two asked for the other dimensions), Richard's presentation does not distinguish between what was given in the question and what is new information. This again serves to obscure the activity giving rise to the new information; these appear merely as lists of associated facts rather than as the result of any action.

Elsewhere in the text, the equals sign appears in a number of formulae relating the dimensions of various shapes to the number of unit shapes contained within them. These are mostly presented as relations between symbols, e.g.

$$N(M+H) \times 2 = T$$

the reference of the symbols being indicated by labels on diagrams accompanying the formula. In some cases words are used alongside the symbols, e.g.

$$\text{Perimeter} \times S.H = T$$

It appears that words are used here and in other formulae in those cases where it is more difficult to show the reference of the variable on a diagram. There is thus a very close relationship maintained between the variable name and its concrete referent, suggesting that the participants in these relationships are the lengths themselves rather than abstract symbols.

While these formulae appear to be relational statements, there are aspects which suggest that they may have a procedural function. The order of each statement may be read as a generalised 'calculation gives result' format and in one case, the hexagon, the formula is followed by a numerical example, demonstrating a specific calculation and result.

There are no explicit expressions of causal relationships within the text.

A5.6.2 Interpersonal aspects: a formal mathematical text

As has been seen above, Richard has obscured his own presence in the text. Indeed, the total absence of any explicit reference either to the author's actions and attitudes or to his reader contribute to the impersonal formality of the text. The largely non-verbal nature of the text also contributes to this formality and the few words that do appear form a restricted, technical vocabulary consisting only of nominal terms such as *length of side* and *unit triangle* which are used repeatedly and consistently.

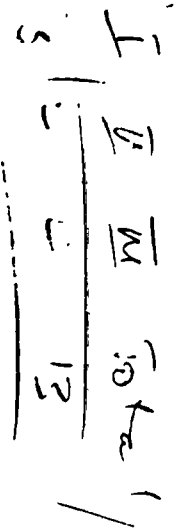
The author's relationship to his reader is thus constructed as a distant and formal one. The use of question numbers alongside the answers at the start of the text and the highlighting of formulae mentioned in the previous section, by emphasising 'answers', suggest that the reader is an assessor. At the same time, the specialised vocabulary used and the generally non-verbal style of communication through diagrams, tables and symbols suggest that the reader is expected to share Richard's knowledge of the task and to be able to understand what has been done and the meaning of the results without recourse to any further form of explanation. The reader is thus seen to be an 'expert', fluent in specialist mathematical forms of communication. She must also be familiar with the task as there is no statement of the task included within the text.

A5.6.3 Textual aspects: a school maths text

The presence of title pages at the beginning and in front of the extension work, together with the careful presentation indicate that this text is a complete piece of work being presented for assessment. However, the internal structure of the text locates it firmly within 'school maths'. As well as the use of question numbers to label the early parts of the text, the repetitive structure of the various parts of the extension is similar to the sequences of similar exercises which form a major part of everyday writing in the mathematics classroom. Each of the five pages of the extension contains a small number of diagrams on the left hand side and a table with a formula underneath it on the right hand side. There is no explicit cohesion between these components; in order to make sense of the page, the reader must be familiar with the school genre of 'investigation' which expects the sequence data-pattern-generalisation to be present.

A5.7 Steven's 'Topples' Text

Topples



USE EXAMINATION

INSTRUCTIONS:

TOPPLES

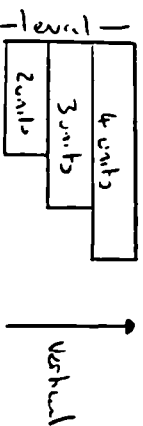
In this task I have been asked to balance some rods of different lengths on top of each other, until it will topple.

The diagrams demonstrate the principle.



I start the pole with one rod of a two unit length on top of a three unit rod on top keeping on side level so it produces an inverted step pattern.

Then I balanced a four unit rod on top of the three unit rod.

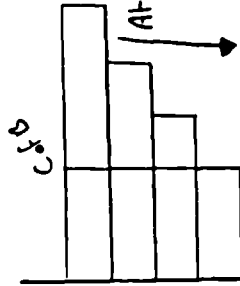


I then continue building the pole, putting a block on top of the 4-unit rod, increasing by one each time until the pole topples. I found that the pole toppled at five.

My task is to investigate the relationship between the width of the rod at the bottom and the extra which first makes the pole topple.

e) Explain your result.

The reason that the pile topples could be because the weight over the starting pile becomes too much and gets pulled down by gravity.



At five there is too much weight
At No 4 it will still be O.K.

There is too much weight on the right hand side so the pile topples over, there is nothing to support the weight.

1. Starting with rods of different lengths at the base, build up your piles until each one topples. Make sure that the rods increase by one length at a time.
- 2) Record the length of the rod at the base and the length of the rod that makes the pile topple.

✓ done

b) Tabulate your results.

Start rod	Topple rod
1	3
2	5
3	8
4	10
5	13
6	15
7	18
8	20
9	23
10	25

Difference



d) Make any observations that you can.

There were definite patterns that I can pick up. The topple pole goes up in three and then two and then three and so on. This is just in the results I have found, not be continuous all the way through.

1) GENERALISE. With such a cube pattern a formula should be easy to find.

Formula

I have found a formula which can be used with all the results I have and which it will be able to predict other results without having to make piles to find a result.

My formula is:

$$(A+A) + \left(\frac{A}{2}\right) = b$$

A = Starting block No.
b = topple block.

e.g.

Start No. 1

$$(1+1) = 2 \quad \left(\frac{1}{2}\right) = 0.5$$

$$2 + 0.5 = 2.5. \quad (\text{you then round this up})$$

ans. 3 ✓

Start No. 10

$$(10+10) = 20 \quad \left(\frac{10}{2}\right) = 5$$

$$20 + 5 = 25 \text{ ans}$$

2. Imagine that you start with a red length of 100 m and build up the pile using rods of lengths 101, 10103, ... units.

What will be the length of the red that in the pile topple?

$$(100+100) = 200 \quad \left(\frac{100}{2}\right) = 50$$

$$200 + 50 = 250$$

250 would be the one at which it would

An alternative way to do this would be to take 1 result of a pile starting at 10 and multiply it by 10.

e.g.

$$(10+10) = 20 \quad \left(\frac{10}{2}\right) = 5 \quad \checkmark$$

$$20 + 5 = 25$$

$$25 \times 10 = 250.$$

or you could even take the basic result 1 with rounding it up and you could multiply it by 100.

e.g.

$$(1+1) = 2 \quad \left(\frac{1}{2}\right) = 0.5$$

$$2 + 0.5 = 2.5$$

$$2.5 \times 100 = 250.$$

3 A pile bopples when we place a rod of length 50 units on top.

a) What will be the length of the rod in the bottom of the pile?

I estimate that it will be 20 units at the bottom. I say this using the same method that I used in Q2. e.g. I have taken a look at my early results and seen that a 2-unit starter has a bopple rod length of 5 units so if I multiply them by 10 I get a starting rod of 20 and a bopple rod of 50.

Or alternately I have found another formula or equation which can be used to find the result. First I tried it of an answer I knew.

e.g. a block bopples at 25 what is the starting block.

$$\left(\frac{a}{2}\right) - \left(\frac{a}{10}\right) = b \quad \begin{array}{c} \downarrow \quad \downarrow \quad \downarrow \\ 12.5 - 2.5 \quad 10 \end{array} \quad \left\{ \left(\frac{25}{2}\right) - \left(\frac{25}{10}\right) = 10 \right.$$

a) = bopple block length
b = starting block length.

So now I will try this with the result in hand.

$$\left(\frac{a}{2}\right) - \left(\frac{a}{10}\right) = b \quad \begin{array}{c} \downarrow \quad \downarrow \quad \downarrow \\ \left(\frac{50}{2}\right) - \left(\frac{50}{10}\right) = b \end{array} \quad \begin{array}{c} \downarrow \\ 25 - 5 = 20 \end{array} \quad b = 20$$

A5.8 Analysis of Steven's 'Topples' Text

Steven was awarded 13 marks out of a possible 20 for his work on the 'Topples' task. He presented 7 pages (all written on one side only), including a coloured title page. A paraphrase of the description of the problem situation from the question paper is then given, followed by two pages on which each of the specific questions 1(a) - 1(e) is stated and responses given. The next page, headed "Formula", contains a formula and examples using the formula. The final two pages contain the text of questions 2 and 3 respectively and Steven's responses to these questions. The response to each of these questions completely fills a page, only leaving room at the very foot of the final page for the words "THE END".

A5.8.1 Ideational aspects: from practical activity to number patterns to calculation

The participants and processes in Steven's text are summarised in Table A5.3 below. Those parts of the text which have been copied verbatim from the question paper have been omitted, but the paraphrase of the description of the problem situation is included in the analysis and the significance of the changes introduced by Steven is discussed below. There is a noticeable predominance of material processes here, many with a human actor, which largely refer to the manipulation of the physical apparatus and the carrying out of calculations. The text is not, however, homogeneous, but can be divided into a number of separate parts distinguished not only by their surface subject matter but also by the picture of mathematics and mathematical activity presented.

Table A5.3
Actors and Processes: Steven - Topples

	Human		Basic or Derived	Object		No Actor (Passive or non-finite)
	General	Specific		Relational	Represent- ational	
Verbal		2			1	1
Relational (= sign)		3	5 (26)	7		
Mental		7				2
Material (---> sign)	6	17	12 (5)			7
Existential			2	2		

The first page after the title page contains a paraphrase of the question paper's description of the practical task of building up a pile of rods until it topples. It is in this section that the majority of the material processes with specific human actors are to be found, describing what "I" do to build a pile. The fact that the statement of the task has been paraphrased rather than simply copied is likely to be interpreted by a teacher/assessor as a sign that the pupil has understood the problem. From the author's point of view, this section may have been written in order to fulfil this assessment requirement. Nevertheless, it is of interest to consider the nature of the changes that have been made and their significance in creating a picture of the nature of the task. For a full analysis of the question paper itself, see chapter 8.

<u>Question paper text</u>	<u>Steven's text</u>	<u>Changes</u>
In this task you will be asked to balance some rods of different lengths on top of each other, until the pile topples.	<i>In this task I have been asked to balance some rods of different lengths on top of each other, until the pile topples.</i>	you → I future → past tense
The diagrams below are given as examples.	<i>The diagrams demonstrate the principle</i>	passive → active diagrams as actors examples → principle
[diagram]	[diagram]	no change
We start the pile with the 2 unit rod on the bottom and balance the three unit one on top of it, being careful that the left hand edges are even.	<i>I start the pile with one rod of a two unit length and place a three unit rod on top keeping one side level so it produces an inverted step pattern.</i>	We → I balance → place "being careful" - omitted pattern elaborated in terms of appearance rather than relationships
Then we balance the 4 unit rod on top of the three unit rod.	<i>Then I balanced a four unit rod on top of the three unit rod.</i>	We → I present → past tense
[diagram]	[diagram]	no change
We continue building the pile, progressing through the sequence of rods until the pile topples.	<i>I then continue building the pile, putting a block on top only increasing by one each time until the pile topples.</i>	We → I "then" inserted "progressing through the sequence" interpreted
You should find that this pile of rods topples when we get to the 5 unit rod.	<i>I found that the pile topples at five.</i>	You → I normative future › past "when we get to ..." → "at ..." "5 unit rod" → "five"
So the pile that starts with the two unit rod at the base eventually topples when we get to the 5 unit rod.		Summary omitted
Your task is to investigate the relationship between the length of the rod at the bottom of the pile and the rod which first makes the pile topple.	<i>My task is to investigate the relationship between the length of the rod at the bottom and the rod which first makes the pile topple.</i>	Your → My

The changes from second person and inclusive *we* to first person singular play the interpersonal function of indicating that the author recognises that the task is addressed to himself. This passage serves a dual function of demonstrating that the author has understood the given statement of the task and that he has followed the implicit instructions contained within it to build a particular pile. This and other interpersonal aspects related to changes in the wording will be discussed in section A5.8.2. In addition, however, the changes from using *we* to using *I* remove generality from the text. Rather than being a description of a procedure that might be carried out by any member of the community and that contains a suggestion that it might also be generalisable to other sizes of rods, it is now a description of a particular procedure being carried out by the author himself. This particularity is also reflected in the changes from present or future tense to past tense as well as the insertion of the word "then"; it is a narrative account of what Steven actually did with a specific set of rods (although these changes of tense are not consistent throughout the passage).

The change of wording from "when we get to the five unit rod" to "at five" suggests a move away from considering this to be a problem about physical manipulation of rods to considering it a problem about numbers. Human participation in the process of building the pile of rods has disappeared as has the process itself. The "five" is treated as an object in its own right rather than an attribute of a physical object. This shift from a physical to a numerical problem is continued in the next section. Throughout the rest of the text, there is no further involvement of human participants as manipulators of physical objects. Thus the practical aspects of the task, although initially emphasised in the question paper, do not appear as of fundamental importance to either author or reader.

A further distinction of roles is to be found in the section headed "Formula". While the author uses the first person singular at the beginning of the section to make the claim that he has discovered a formula, the actor in his final sentence, which gives information about how to apply the formula, is a general *you*. This shift from the specific to the general is associated with the different types of processes being represented. The author himself is individually responsible for the mental activities of finding a formula and predicting results, while 'rounding up' is a material action which forms part of a generalised procedural solution to the problem.

The next section of the text consists of exact copies of each part of question 1 as stated on the question paper, followed by the author's response. The practical nature of the activity that was reflected in the previous section is absent here. When asked to make observations, Steven focuses on patterns in the numbers generated rather than on the physical properties of the piles of rods.

There is one definite pattern that I can pick up. The topple pile goes up in three and then two and then three and so on.

The "definite pattern" is emphasised by being linked by an arrow pointing to a column of numbers headed "Difference" that had been generated from a table of results. These numbers are also pointed to by arrows from the table. Their significance is thus brought to the reader's attention as central to the subject matter of this page. The pile of rods is no longer present as a physical object which must be kept even and may balance or topple. Instead it has become an abstract quantity, "the topple pile", which "goes up" by a number. It is not clear what this quantity relates to in the physical situation (the length of the top rod on the pile or the number of rods in the pile?) but this is no longer important within a task which has been transformed into a problem about patterns of numbers. While the author is still present in the text, his role is now to "pick up" an existing pattern rather than to create anything new.

The shift of focus from the practical activity to abstract patterns is paralleled by a shift from an experimental interpretation of the results to an abstract, non-empirical interpretation. Having identified the "definite pattern", Steven initially qualifies his observation:

This is just in the results I have it may not be continuous all the way through.

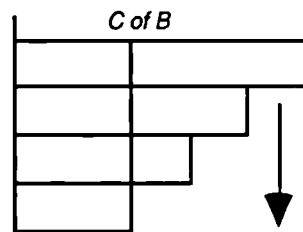
The modality here suggests uncertainty about the generalisability of the results and a recognition of the empirical nature of the emerging pattern. In response to the next instruction (GENERALISE), however, Steven moves away from this position. He responds at a meta-level:

With such a definite pattern a formula should be easy to find.

This is a generalisation, not so much about the present problem as about mathematics in general. Mathematics consists of patterns and formulae and where a pattern is found, a formula will also exist. While there is still some uncertainty expressed in the modal "should", this could be interpreted as uncertainty either about the continuation of the pattern in this case (and hence the existence or complexity of the formula) or about the author's own ability to find the formula. The statement, by using the non-finite form "easy to find", obscures any individual human agency. Easiness is presented as an absolute property of the mathematical situation and is not relative to the personal characteristics of the individual mathematician. In the assessment context of the production of this text it is thus important for the author that he should find the formula as he will otherwise be judged to have failed to do something that he himself has defined as easy.

The task asks for an explanation of "your result". The reasoning that Steven presents in his response to this request, however, does not relate either to his own physical actions as described in the first section or to the number patterns and formulae that he claims to have found. It is the behaviour of the piles of rods which he explains in terms independent of human action:

The reason that the pile topples could be because the weight over the starting pile becomes too much and gets pulled down by Gravity.



*At five there is too much weight.
At No. 4 it will still be O.K.*

There is too much weight on the right hand side so the pile topples over, there is nothing to support it.

There is no indication of how the weight becomes too much. The fact that the author himself placed the extra weight on the pile is obscured, contributing to the generality of the statement (in contrast to the removal of generality found in the statement of the problem discussed above). The schematic nature of the diagram, with no attempt to provide perspective or realism, is consistent with this impersonal, abstract impression. This section appears unconnected to the rest of the text, not only because of the lack of reference outside itself apart from the question number in the margin, but also because of the different types of participants and processes contained within it.

Interestingly, the formula itself is not presented as an answer to one of the given questions, but on a separate page headed "Formula" (without a question number), indicating its high status as something which should exist in its own right rather than merely as part of "generalising". The symbolic formula itself is announced, positioned centrally on the page and presented in large, clear letters, inside a box:

My formula is:

$$(A + A) + \left(\frac{A}{2}\right) = b$$

This again emphasises the high status of the formula and suggests it is an answer or conclusion for the whole task; the only other use of such a box in this text is to surround the words "THE END!!" at the bottom of the final page. The presentation also echoes the style of some mathematics text books which use such signals to draw attention to important passages (Shuard & Rothery, 1984).

The validity of the formula is established:

I have found a formula which can be used with all the results I have and with it I will be able to predict other results without having to make piles to find a result.

The earlier suggestion that the pattern might not continue has been abandoned and the fact that the formula fits all the previous results is enough to guarantee that it will be successful in all future cases. Although two examples are presented, these appear as demonstrations of the procedures needed to apply the formula without any attempt to relate them back either to the previous experimental results or to any physical interpretation¹. This section is concluded with the statement:

You will have to round up some numbers that come to .5 to get your number.

The aim of using the formula is to find a result or a number rather than to find out when a pile of rods will topple. It is interesting to note that, although the original formula presented in the box is given in symbolic and relational terms, the variation required in the formula to ensure that all answers are whole numbers is expressed procedurally without using algebraic symbolism. The symbolism required to express this relationally was probably not available to Steven; there is, however, an inconsistency here and on the last two pages which suggests that the relational form of the formula given in the box is more a reflection of Steven's understanding of the algebra requirements of the coursework genre than of his way of thinking about the relationships between the problem variables. Having stated the formula, Steven proceeds to exemplify its application, presenting this as a fragmented procedure which transforms the picture of the subject matter of the page from the statement of a relationship between variables into the presentation of a method for arriving at particular numerical answers.

start No. 10

$$(10 + 10) = 20 \quad \left(\frac{10}{2}\right) = 5$$

$20 + 5 = 25 \leftarrow \text{ans}$

The equals signs, while technically validly equating quantities, may be read as material "operators" (Kieran, 1981): 20 and 5 are products of material processes which are subsequently operated on once more to produce a final answer. This reading is reinforced by the labelling of the outcome of the string of operations as "ans". Similar fragmented strings of operations are given in response to question 2:

¹The problem that was originally posed to the students demands this move away from the practical towards the abstract. The instruction to "Generalise" suggests that it is possible and valid to do so. It would be difficult for a student to challenge this by seriously calling into doubt the continuation of the observed pattern in the results. The nature of the physical situation, moreover, makes it extremely difficult to take an empirical approach to validating any formula. The equipment that is most likely to be available consists of rods which are no more than twelve units long. In most cases, the student's original experimental results include all possible constructions and this makes it impossible to confirm in practice any of the "predictions" made by using the formula.

$$(1+1) = 2 \quad \left(\frac{1}{2}\right) = 0.5$$

$$2 + 0.5 = 2.5$$

$$2.5 \times 100 = 250$$

On the final page, however, the response to question 3 is given in a mixed style with both relational and procedural aspects:

Or alternately I have found another formula or equation which can be used to find the result . . .

$$\left(\frac{a}{2}\right) - \left(\frac{a}{10}\right) = b \quad \left| \quad \begin{array}{ccc} \left(\frac{50}{2}\right) - \left(\frac{50}{10}\right) = b & & \\ \downarrow & \downarrow & \\ 25 - 5 = 20 & & b = 20 \end{array} \right.$$

Here, each equation maintains the whole relationship between the variables a and b , while the arrows draw attention to the partial procedures which need to be carried out in order to evaluate b for the given value of a . The arrows here take on the role that was played by equals signs in the previous example. The introduction to this section also draws attention to the procedural purpose of the algebraic expressions. This mixing of relational and procedural elements is Steven's response to the demands of a coursework task which explicitly requires specific numeric answers while the discourse within which it is situated simultaneously expects and values algebraic generalisation. His apparent lack of consistency, however, may be interpreted in the assessment context as a lack of control of algebraic notation and failure to comply with conventions.

The first words of this section draw attention both to the importance of "the result" in Steven's text and to the arbitrary nature of the method of achieving the result. At various points, three different methods are used: the original formula $(A + A) + \left(\frac{A}{2}\right) = b$, scaling up from known results by multiplying by 10 or 100, and the second formula $\left(\frac{a}{2}\right) - \left(\frac{a}{10}\right) = b$. While the second formula expresses the inverse relationship between the variables, I would argue that it should not be called the inverse of the first formula as there is no relationship between the two expressed in the text, in particular, there is no suggestion that one is derived from the other but each is introduced using the words "I have found . . .". The methods are introduced as independent alternatives without any consideration of why they might give similar results:

An alternative way to do this would be . . .

Or you could even . . .

Or alternately . . .

Although, when working with numbers, Steven appears to expect patterns and relationships, formulae are not presented as part of a coherent system. This may also reflect the limitations of a procedural view of algebra; it is only by seeing a formula as an object in its own right that it is possible to consider sets of formulae and hence to look for patterns and relationships between them.

A5.8.2 Interpersonal aspects: authority and confidence

As was seen in the previous section, Steven has 'translated' the statement of the problem into the first person singular, making the problem appear to be his own and providing evidence to the reader/examiner that he has fulfilled the practical component of the task. It is clearly important from the assessment point of view that the author should be seen to have fulfilled the requirements of the task. The way in which he structures the rest of his text around the specific questions asked, heading each

response with a verbatim copy of the question, enables him to demonstrate that he has done so. At one point, one of the questions asked him to do something that he had already completed elsewhere in the text; he nevertheless provided a further response in the form of a tick and the word *DONE*. One of the roles ascribed to the reader of this text thus appears to be that of examiner, checking that all the questions have been answered.

Steven's use of the first person throughout most of the text makes similar claims to have complied with the more general requirements of 'investigation'. Thus, for example, he states at the beginning of the section headed "Formula":

I have found a formula which can be used with all the results I have and with it I will be able to predict other results without having to make piles to find a result.

Finding a formula and making predictions were not specifically mentioned in the task set but are likely to have been stressed by Steven's teacher as important components of any investigation. The tenses he has chosen to use here suggest that he is providing a commentary and demonstrating his knowledge of the conventions of doing investigations rather than providing a narrative of the processes he actually went through. Similarly, in the final section of the text,

I estimate that it will be 20 units at the bottom. I say this using the same method that I used in Q.2. eg. I have taken a look at my earlier results and seen that a 2 unit starter has a topple rod length of 5 units so if I multiply them by 10 I get a starting rod of 20 and a topple rod of 50.

the use of the present rather than the past tense raises the status of the description of the processes from being merely a narrative of what the author did to being a more general statement of the way things are and of a method that could be replicated in other cases or by other participants. Again the mentions of estimating and using earlier results bring to the reader's attention processes that are valued within the investigation discourse.

The tenses used and the modality in the examples above, as at most other points in the text, suggest confidence and a sense of authority in relation to the mathematics and the reader. At one point, presenting his third alternative method of calculation, Steven celebrates his achievement, stating that:

you could even take the basic result of 1 without rounding it up and you could multiply it by 100.

The modifying phrase *could even* does not suggest doubt about whether the method is correct or whether the reader might wish to follow the procedure but rather emphasises the degree of choice that the author's skill in devising formulae has made available to the interested reader. Again, at the end of the "Formula" section, he states:

You will have to round up some numbers that come to .5 to get your number.

There is not only certainty about the correctness of the methods described but also confidence and authority in instructing the reader. There is an expectation that the reader will be actively interested and involved in carrying out the procedures described and that she will be willing to be instructed. This appears to contrast with the reader's role as examiner suggested by the question-answer structure mentioned above, although the two roles are not necessarily incompatible.

The vocabulary used to refer to the rods and piles of rods that form the subject matter of the task also suggests that Steven is confident in his use of language and in his sense of ownership of the task. He develops and uses the terms *topple block length* and *starting block length* consistently in formal contexts, particularly when defining the variables in his formulae and interpreting his results. Elsewhere in the text, however, he uses a wide variety of terms, including *start rod*, *start no.*, *a pile starting with 10*,

20 units at the bottom, a 2 unit starter, a starting rod, starting block, starting block number. This multiplicity suggests a fluency and familiarity with the task but simultaneously gives an impression of informality and a lack of conformity with conventions of mathematical language use. This may not be approved by a teacher reading as an assessor looking for 'appropriate' forms of communication.

A5.8.3 Textual aspects: variation within a coherent whole

As has been seen already, Steven's text does not form a homogeneous whole but may be separated into a number of sections with contrasting styles. This is apparent when the thematic structure is examined. The first section, which paraphrases the problem, has mainly personal and topical themes, forming a personal narrative; a similar structure is found at the start of the answer to question 3, suggesting that the response to this question (which uses an inverse relationship) requires a fresh start to what is almost a new problem. In contrast, the response to question 1, which contains most of the practical and exploratory work on the problem, has a sequence of existential and topical themes which appear to construct it as a descriptive report:

There is one definite pattern that I can pick up. The topple pile goes up in three and then two and then three and so on. This is just in the results I have. It may not be continuous all the way through.

For example, in the first sentence of the above extract the declaration in the thematic position focuses the reader's attention on the existence of the pattern rather than on the fact that it was 'picked up'. Finally, the last section of the text appears to contain an attempt to present an argument, using themes which suggest a train of reasoning:

*I have taken a look at my early results and seen that a 2 unit starter has a topple rod length of 5 units so if I multiply them by 10 I get a starting rod of 20 and a topple rod of 50.
Or alternately I have found another formula . . . First I tried it on an answer I knew . . . So now I will try this with the result in hand . . .*

The reasoning, however, presents a justification for the chain of actions carried out by the author rather than a logical explanation of the results achieved.

The text as a whole contains a mix of features which locate it both within the question-and-answer mode of traditional school mathematics and within a 'project' genre. On the one hand, the text is largely structured by the numbered questions given on the question paper; each question is copied from the paper and immediately followed by a response. On the other hand, a number of features also occur which are not typical of school mathematics. Firstly, there is a title page with some attempt at coloured decoration. The text is framed by an initial statement of the "Project Title . . . Topples . ." and, at the foot of the final page, "The End!!", suggesting that the text should be read as a coherent whole rather than as a series of answers to discrete questions. Although some parts of the text, in particular the responses to the various sections of question 1, have few links with other sections, the final two pages in particular refer several times to results and formulae occurring earlier in the text, for example:

*I say this using the same method that I used in Q.2.
eg. I have taken a look at my earlier results . . .*

Such explicit links strengthen the impression that this is unitary text.

TOPPLES!

INTRODUCTION

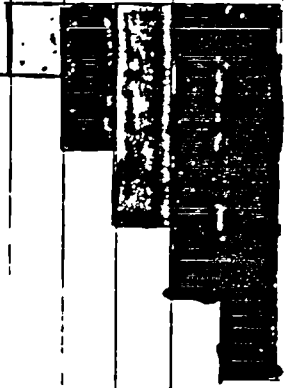
The idea is to see when the blocks topple e.g.



The 1 unit Rod topples at 2 units.

NOTE: The Rods have to be vertical all times.

Verticals →



I	II	III	Total
6	5	4	15



The 2 unit Rod toppler at 5 units.
This goes on. I will show my Results
in a table.

Results

Unit Number of Rod	Toppler at (units)
1	2
2	5
3	8
4	10
5	13
6	15
7	18
8	20
9	23

C.

OBSERVATIONS

There is a pattern where the number of unit toppler at ie, The number 1 unit toppler at 2, the number 2 unit toppler at 5. ~~The~~ difference = 3

Unit number of Rod	Toppler at (units)	Difference
1		
2	2	3
5		

The number 3 unit toppler at 8, the number 4 unit toppler at 10. The Difference = 2

Unit number of Rod	Toppler at (units)	Difference
3	8	2
4	10	

~~The~~ number 5 unit toppler at 13 and number 6 unit toppler at 15. Difference = 2. etc.

FORMULAR

I thought I had a formula but it only worked for even numbers.

Only for even numbers.

X unit of Rod by 2.

$\frac{1}{2}$ unit of Rod and add together

E.g.

2 in the unit of Rod. Times it by 2 = 4

$$2 \times 2 = 4$$

half of 2 = 1 $\therefore 1 + 4 = 5$

2 unit Rod toples at 5.

E.g.

$$4 \times 2 = 8$$

half of 4 in 2 $\therefore 2 + 8 = 10$

4 toples at 10.

d.

Generating

THE REAL FORMULAR - works for odd & even numbers

The unit at the bottom + itself.

Half unit at the bottom, if it doesn't come to a whole number round it up to the nearest higher number.

E.g.

$$4 + 4 = 8$$

half of 4 in 2 $\therefore 8 + 2 = 10$

With 4 as the bottom unit, it will toples at 10.

E.g.

$$7 + 7 = 14$$

half of 7 is 3.5 round it to 4 $\therefore 4 + 14 = 18$

With 7 as the bottom unit, it will toples at 18.

e. The smaller the unit at the bottom the more likely the load will fall quicker. ie a one unit rod can't even balance a 2 unit rod, yet a 3 unit rod can balance 8 units.

2. If I start with a rod of length 100 units and build up to the pile using rods of length 101, 102 & 103 etc, the length of the rod that will make the pile topple by using my formula is,

$$\text{Base} = 100 \times 2 = 200$$

$$\text{half of } 100 = 50 \therefore 200 + 50 = 250$$

With 100 length unit at the bottom the pile will topple at 250 units.

3. A pile topples when we place a rod of 50 units on top.

The length at the bottom of the pile would be 20.

$$20 + 20 = 40$$

$$\text{half of } 20 = 10 \therefore 10 + 40 = 50$$

50 makes the 20 unit rod at the bottom topple.

2. EXTENSION from question 2.

Formula:

$$\text{Base} + \text{itself} = \text{Ans}$$

$\frac{1}{2}$ base and + Ans

✓ If I started with Rod length 70.

The length of rod that would make the pile topple would be 185 unit rod.

Base

Answer

$$70 + 70 =$$

$$140$$

$$+ 70 = 35$$

$$35 + 140 = 185$$

Should be 175

Extension work

This time I decided to have the unit go up in 2's. E.g. I started with a 1 unit straight away put a 3 unit rod on etc.

Unit number of Rod	Topple at?	Difference
1	3	→ 1
2	4	→ 3
3	7	→ 3
4	10	→ 3
5	13	→ 3
6	16	→ 3
7	19	→ 3
8	20	→ 1
9	23	→ 3
10	26	→ 3
11	29	→ 3

If you take a look at the differences on the table it shows it goes up in 3's but when the number of units is 8 a topple at 20 it changes to 1. The pattern stops the to my surprise starts up again

Unfortunately I was unable to find a
 way of telling when the rods were going to topple
 on top of what length of rod.

Pattern

no. of 2's	unit number of Rod	Topple at (unit)
1	$1 + 2 = 3$	3
1	$2 + 2 = 4$	4
2	$3 + 2 + 2 = 7$	7
3	$4 + 2 + 2 + 2 = 10$	10
4	$5 + 2 + 2 + 2 + 2 = 13$	13
5	$6 + 2 + 2 + 2 + 2 + 2 = 16$	16
6	$7 + 2 + 2 + 2 + 2 + 2 + 2 = 19$	19
6	$8 + 2 + 2 + 2 + 2 + 2 + 2 = 20$	20
6+1	$9 + 2 + 2 + 2 + 2 + 2 + 2 + 1 = 23$	23
8	$10 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 26$	26
9	$11 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 29$	29

⊕ Apart from where the pattern goes wrong
 originally the pattern is to ~~2~~ as shown
 above.

A5.10 Analysis of Ellen's 'Topples' text

Ellen was awarded 15 marks out of a possible 20 for her work on the Topples task. She presented 19 pages altogether, of which the last 8 are written in pencil and clipped together to indicate that they are rough working. Some of the rough working appears to be a rehearsal of part of the content of the first 11 pages, while the rest consists mainly of calculations which may have been used while trying to derive a formula to fit the data. It is only the first 11 pages that will be analysed here as they are the part that has been written for public reading; this is also the part that was used in the interviews with teachers. It may be assumed that the rough working was done by the pupil initially as private writing, its submission being required by the examination as 'evidence' but not forming a coherent part of the text.

After a coloured title page, the second page contains a paraphrase of the statement of the problem and the third page a table of results. The next page, headed *Observations*, contains two tables showing difference patterns and written comments on these differences. This is followed by a page with the heading *FORMULAR* (sic) and two under the heading *Generalising*. Each section is labelled with the relevant question number, relating it to the statement of the task provided by the examination board. The next page contains answers to questions 2 and 3 and this is followed by a single page *EXTENSION from question 2..* The last two pages are headed *Extension Work*.

A5.10.1 Ideational aspects: observing an autonomous material world

The participants and processes in Ellen's text (excluding the rough working) are summarised in Table A5.4 below.

Table A5.4
Actors and Processes: Ellen - Topples

	Human		Basic or Derived	Object		No Actor (Passive or non-finite)
	General	Specific		Relational	Represent- ational	
Verbal		1			1	1
Relational (= sign)		3	11 (25)	2		
Mental		2				4
Behav-ioural		1				
Material	7	6	27	6		4
Existential				2		

The table shows a preponderance of material processes involving both human and inanimate actors. It is, however, worth looking at the detail within this broad category. There is in particular a clear distinction between the kinds of material processes performed by the different types of actors. Of the 27 cases noted in which a basic or derived object is the actor in a material process, nearly all involve a rod toppling or balancing. When referring to her own activity, Ellen once mentions building or placing rods on piles:

I started with a 1 unit rod and straight away put a 3 unit rod on etc.

This is, however, the only case in which she clearly indicates her own physical actions. The other instances of material activity by a specific human actor appearing in the table (in all these cases indicated by the use of the first person singular) occur where the words given in the task have been closely paraphrased; they do not, therefore, contribute significantly to the picture of the author's mathematical activity constructed by the text. The material activity by general human actors, moreover, is almost exclusively confined to performing calculations. The subject matter of mathematics portrayed here is a world of material objects which behave in ways which are not on the whole dependent on human manipulation. Thus, when asked to explain her results, Ellen responds with an answer which makes no mention of the process of building piles:

The smaller the unit at the bottom the more likely the load will fall quicker. ie a one unit rod can't even balance a 2 unit rod, yet a 3 unit rod can balance 8 units.

Even in her statement of the original problem, it may be seen in comparing her introduction with that provided by the examination board that she has obscured all human involvement in the practical activity of building the piles of rods. For example, the warning about the importance of the way in which the rods are placed on top of one another is changed from a description of "careful" human behaviour to a statement that "The Rods have to be verticle (sic)". A property of the pile of rods substitutes for the human act of building the pile. Even the author's own task is changed from "to investigate the relationship . . ." to "to see when the blocks topple"; she has the role of a passive observer of the physical world rather than an active participant.

Question paper text

Ellen's text

In this task you will be asked to balance some rods of different lengths on top of each other, until the pile topples.

The idea is to see when the blocks topple e.g.

The diagrams below are given as examples.

[diagram showing pile starting with 1 rod]

[diagram]

We start the pile with the 2 unit rod on the bottom and balance the three unit one on top of it, being careful that the left hand edges are even.

The 1 unit Rod topples at 2 units.

NOTE: *The Rods have to be verticle (sic) at all times.*

[diagram illustrating a "verticle" pile of rods]

Then we balance the 4 unit rod on top of the three unit rod.

[diagram showing pile starting with 2 rods]

[diagram]

We continue building the pile, progressing through the sequence of rods, until the pile topples.

You should find that this pile of rods topples when we get to the 5 unit rod.

So the pile that starts with the two unit rod at the base eventually topples when we get to the 5 unit rod. *The 2 unit Rod topples at 5 units.*

Your task is to investigate the relationship between the length of the rod at the bottom of the pile and the rod which first makes the pile topple.

Most of the remainder of the human activity in the text is related to the author's own problem solving processes at the level of decision making rather than carrying out tasks. Thus, at the beginning of the section with the formulae, she states:

I thought I had a formular (sic)

and she introduces one part of the extension work, explaining the problem in terms of her own thought processes:

This time I decided to have the units go up in 2's.

The role of human beings is thus presented as one of observing and deciding what to observe, calculating and thinking rather than engaging directly with the material world.

There is great emphasis throughout the text on the role played by the rods; in many cases the word *Rod* is even capitalised. The author uses a number of different, apparently interchangeable terms to refer to the rods; these include *rod*, *number of units*, *number n unit*, *unit number of rod(s)*, *unit of rod*, *n unit rod*, *unit*, *length of rod*. The individual object, its length, the pile that starts or finishes with a given rod, and the number of rods in a pile are not always clearly distinguished. This over-lexicalisation, while indicating the significance of the physical objects for the author, may not be consistent with generic requirements for the use of precise mathematical vocabulary. In particular the term *unit* is used in an unconventional way.

Objects such as tables, patterns and formulae also play an important role. There are several tables at various points throughout the text. The first merely displays a set of results but the rest all include some indication of patterns to be seen within the set of results, labelled with *difference* or *pattern*. At one point, the reader's attention is explicitly drawn to the patterns indicated within the table:

If you take a look at the differences on the table it shows it goes up in 3's

but in other cases the tables are integrated more implicitly merely by juxtaposing them with verbal descriptions of the patterns within them, for example:

The number 3 unit topples at 8, the number 4 unit topples at 10. The Difference = 2

<i>unit number of rods</i>	<i>Topples at (units)</i>	<i>Difference</i>
3	8	2
4	10	

The table is not presented as a self-sufficient means of communication but as one of a number of alternative ways of presenting the same information. This may be interpreted as a sign of awareness of the needs of various readers or as a sign that the author is aware that the coursework genre requires her to include tables in her text.

The question paper asked the students to make observations and generalisations. In response to this, Ellen has sought to find patterns and formulae. While she comments on patterns in the numbers she has recorded as results in her table, and while her rough working appears to show that she spent considerable time working with numbers in order to construct her formulae, she relates both patterns and formulae closely to the physical situation from which they arise. Thus her first formula is expressed as follows:

× unit of Rod by 2.
 $\frac{1}{2}$ *unit of Rod and add together*
E.g.
2 is the unit of Rod. Times it by 2 = 4
 $2 \times 2 = 4$
half of 2 is 1 ∴ 1 + 4 = 5
2 unit Rod topples at 5

The results of the human activity of performing calculations parallel the results observed in the physical world; each time a calculation is performed it is accompanied by a sentence interpreting the result in terms of rods toppling.

As may be seen in the example above, the formulae are all presented as procedures, the use of the imperative suggesting that they are to be carried out by some general reader. At one point the author attempts a more conventional impersonal expression of the formula:

Base + itself = Ans
 $\frac{1}{2}$ *base and + Ans*

Although the first line of this formula looks at first sight like a relational statement, I would suggest that, when both lines are read together, the equals sign may be seen to be acting as an operator, producing the value *Ans* which is to be used in the next stage of the procedure. The use of the word *and* in the second line suggests that $\frac{1}{2}$ like + must be read as an instruction to the reader rather than as part of an algebraic expression. The formula serves as a description of human activity rather than a statement of a mathematical relationship.

Although the symbol *l* is used several times within examples of the use of the formulae, it appears to mean something like 'now take the previous results and use them in the next step of the calculation' rather than being used as a logical connective. An example of such use may be seen above. The main causal relationships expressed in the text are between material states and material actions, e.g.

With 4 as the bottom unit, it will topple at 10.

or between material objects and material actions, e.g.

If I started with Rod length 70. The length of the rod that would make the pile topple would be a 185 unit rod.

In the latter example there is a suggestion that the author is herself involved in the causal relationship, but her role as an agent is secondary to that of the "185 unit rod". The punctuation again reinforces the parallel but separate roles of human activity and the physical world.

A5.10.2 Interpersonal aspects: formal beginning, informal extension

In considering the modality and presence of personal pronouns, Ellen's text may be seen to be divided into two very different sections. As has been seen already, the first section, which deals with the statement of the problem and the answers to the specific questions posed on the question paper, is almost entirely impersonal and is presented as a sequence of facts and algorithms with few expressions of doubt or opinion. Human involvement is only implied by the use of imperatives in the specification of procedures. The one exception to this is during the development of the generalisation in which Ellen presents a picture of her development of "THE REAL FORMULA" through a stage at which she had a formula which only fit part of the data:

I thought I had a formula but it only worked for even numbers

One of the pieces of advice provided to students is to show all their work, including errors and false trails. Given the assessment context and the traditional school mathematics emphasis on correctness, this is a piece of advice that many students may find difficult to accept. In this case, however, the partial formula was not actually wrong and the fact that Ellen went on to achieve a complete formula means that she is taking no risks by presenting the original incomplete one. In this first part of the text, she appears to be presenting a formal piece of work to an assessor.

When she moves on to "Extension Work", however, her relationship to her subject matter and to her audience becomes both more personal and more tentative. It is interesting to examine the whole of this section:

*This time I decided to have the units go up in 2's. E.g. I started with a 1 unit rod & straight away put a 3 unit rod on etc, [table]
If you take a look at the differences on the table it shows it goes up in 3's but when the number of units is 8 and topples at 20 it changes to 1. The pattern stops then to my surprise starts up again.
Unfortunately I was unable to find a way of telling when the rods were going to topple on top of what length of rod.
Pattern [table]
Apart from where the pattern goes wrong originally the pattern is to + 2 as shown above.*

The author starts by claiming responsibility for posing the problem herself in this extension; she is personally involved with deciding on and carrying out the task. This sense of involvement is continued in her advice to her reader on how to read the table. The reader too is expected to be interested and involved in trying to make sense of the data. As well as making a claim to ownership of this part of the task, however, Ellen is also laying herself open to criticism because of her inability to find a formula to fit her data. The repeated comments on the lack of consistency in her results and her failure to find a formula suggest that she sees her reader as a trusted colleague who will share and appreciate her surprise rather than condemning her. The contrast between the informality and personal nature of this section and the formality of the earlier part of the text raises some questions about the role and nature of the 'extension' in coursework and its relation to the original problem. In this case, rather than being 'more of the same', the extension seems to be an opportunity to take risks and to demonstrate that the approved processes have been attempted even if no 'correct' result has been achieved.

A5.10.3 Textual aspects: a descriptive project

The first half of Ellen's text has almost exclusively topical themes, presenting a descriptive report of the setting of the problem and of the patterns that she has observed, e.g.

The number 3 unit topples at 8, the number 4 unit topples at 10. The difference = 2.

This is followed by sequences of imperative statements, detailing the procedures to be followed. The later parts of the text, however, are dominated by contextual themes, in particular ones which draw attention to the conditions which make the following parts of the statements true. Thus, when asked for the lengths of the rods that will make piles of given starting lengths topple, she responds:

With 100 length unit at the bottom the pile will topple at 250 units

and

If I started with Rod length 70, the length of rod that would make the pile topple would be a 185 unit rod.

The effect of ordering the statements in this way is to order the information content from what is given in the question to the answer. When answering a question which asks for the length of the base given that a rod of length 50 will make the pile topple, however, this information order is maintained by inverting the wording as well as the relationship:

50 makes the 20 unit rod at the bottom topple

In this case, without careful reading of the context, the statement might appear to be the answer to the question 'Which rod makes the pile with 20 units at the bottom topple?'. This, together with the fact that Ellen shows no evidence of having used an inverted formula to achieve this result, may influence a reader-assessor to judge her grasp of the inverse relationship to be limited.

Although all the questions originally posed are answered and are labelled in the margin by a question number, they are not copied but are (as was shown in section A5.10.1) heavily paraphrased and interpreted flexibly with some changes in the order of presentation. Moreover, the headings used for different sections of the text are not directly related to the wording of the question paper. For example, the headings include *Introduction*, *Formula*, *The Real Formula*, *Extension work*, *Pattern*. The heading *Extension from Question 2*, labels a section that is positioned after the completion of question 3. While Ellen's text is clearly referenced to the question paper, it stands as an independent coherent whole rather than a collection of answers. This, together with features such as an elaborate title page, and the use of colour to underline the headings, suggests that it is a 'project' rather than a 'school maths' exercise.

the 2 white rod at the base eventually topples when we goe
to the 5 white rod.

340

COLOUR LIST

Light Blue = A rod of length 2

Pink = A rod of length 3

Red = A rod of length 4

Yellow = A rod of length 5

Mauve = A rod of length 6

Crimson = A rod of length 7

Grey = A rod of length 8

Blue = A rod of length 9

Orange = A rod of length 10

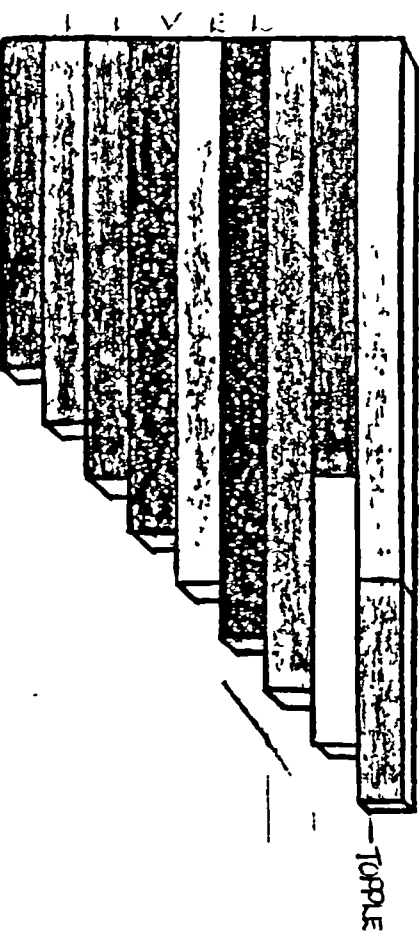
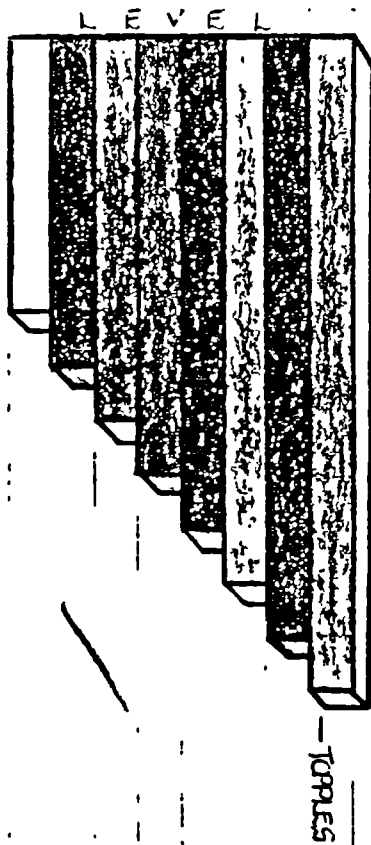
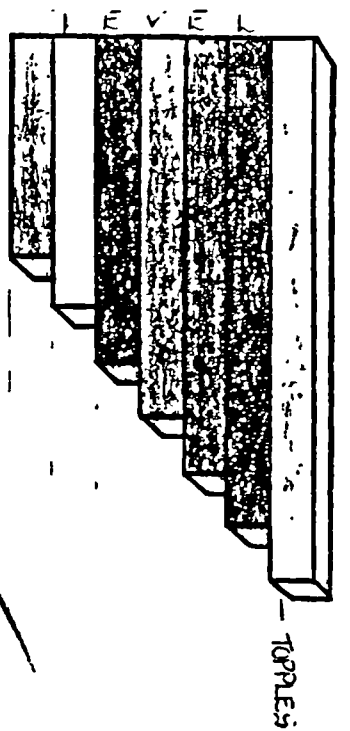
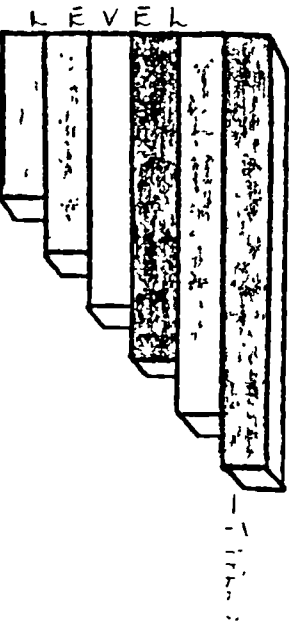
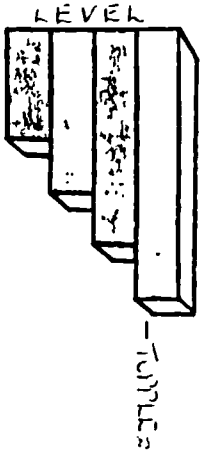
Black = A rod of length 11

Purple = A rod of length 12

QUESTION 1

Showing with rods of different lengths at the base, build up your piles until each one has 10 rods. Make sure the rods increase by 1 unit of length at a time.

a)



b) Tabulate Results

LENGTH OF ROD	LENGTH OF ROD THAT FIRST MAKES THE FILE TOPPLE
1	2
2	5
3	8
4	10
5	12
6	14

c) How I worked them out

Length of Rod Length of rod that first makes the file topple

$$\rightarrow 1 = \text{Twice its length plus 2 extra's}$$

$$\rightarrow 2 = \text{Twice its length plus 2 extra's}$$

$$= \text{Twice its length plus 2 extra's}$$

$$1 + 1 = 2 + 2$$

$$= \text{Twice its length plus 2 extra's}$$

$$2 + 2 = 4 + 2 = 6$$

d) Predictions

$$= \text{Twice its length plus 2 extra's}$$

$$6 + 2 = 12 + 2 = 14$$

$$= \text{Twice its length plus 2 extra's}$$

$$7 + 2 = 14 + 2 = 16$$

$$= \text{Twice its length plus 2 extra's}$$

$$9 + 2 = 18 + 2 = 20$$

QUESTION 2

Imagine that you start with a rod of length 100 units and build up the pile using rods of lengths 10, 102, 103, ... units. What will be the length of the rod that first makes the pile happy?

LENGTH OF ROD	LENGTH OF ROD THAT FIRST MAKES THE PILE HAPPY
100	202
101	204
102	206
103	208
104	210
105	212
106	214

To work this out, you can use the fact that the sum of the first n natural numbers is $\frac{n(n+1)}{2}$. So, for example, the sum of the first 10 natural numbers is $\frac{10(10+1)}{2} = 55$. This means that the sum of the first 10 rods is 55 units long.

QUESTION 3

A pile happens when we place a rod of length 50 units on the top.

a) What will be the length of the rod on the bottom of the pile? Explain your working.

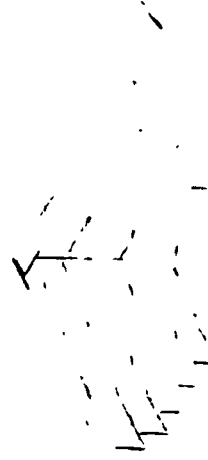
If a pile happens when we place a rod of length 50 units on the top, the length of the rod on the bottom pile will be 24. The formula is opposite of that when you're trying to work out the length of the top rod.

$$50 - 2 = 48 \div 2 = 24$$

OPTIONAL

EXTENSION

a) Here is a sketch of an optional extension stage 1 through of.



You can try and build a pile by joining some cylinders and see when they topple, in the same way as the original experiment and compare them together.

1 tried this optional extension and these are my results.

Length of Rod = 2	Topple on 3
Length of Rod = 3	Topple on 6
Length of Rod = 4	Topple on 7
Length of Rod = 5	Topple on 11
Length of Rod = 6	Topple on 12
Length of Rod = 7	Topple on 15 (on 16)

b) Here is another sketch of another optional extension stage 1 through of.



You can try building up the pile using 2 more units than what we originally did, 1 unit more each time.

Here are my results.

Length of Rod = 2	Topple on 4
Length of Rod = 3	Topple on 7
Length of Rod = 4	Topple on 10
Length of Rod = 5	Topple on 11
Length of Rod = 6	Topple on 16
Length of Rod = 7	Topple on 19
Length of Rod = 8	Topple on 20
Length of Rod = 9	Topple on 25
Length of Rod = 10	Topple on 26

So really, if you think about it, you can make up many optional extensions from this initial design.

A5.12 Analysis of Sandra's 'Topples' text

Sandra was awarded 15 marks out of a possible 20 for her work on the Topples task. She presented 24 pages, 12 of which are stapled separately and appear to have the status of rough working. In fact, the rough working consists entirely of a rehearsal of material contained in the first 12 pages, the major difference being in the extent of colour used in its presentation. It is only the first 12 pages which will be analysed here.

The text starts with a coloured title page and a verbatim copy of the description of the task that was given in the question paper provided by the examination board. The rest of the text is also structured by the given questions, each question being started on a separate page with a large coloured heading. In each case the answers are numbered and the question is copied word for word. The only exception to this structure is the omission of any response to question 1(e) which asked the student to explain the results achieved in the original experimental part of the problem.

A5.12.1 Ideational Aspects: concrete examples and general procedures

The participants and processes in Sandra's text (excluding the rough working) are summarised in Table A5.5 below. Those parts of the text which are copied verbatim from the question paper have not been included, although the significance of the fact that they have been used is discussed.

Table A5.5
Actors and Processes: Sandra - Topples

	Human		Object			No Actor (Passive or non-finite)
	General	Specific	Basic or Derived	Relational	Represent- ational	
Verbal						
Relational (= sign)			2 (31)	1 (7)	(11)	
Mental	7	4				3
Behavioural	1					
Material	9		33			
Existential			2		2	

In interpreting the figures in the above table, it must be taken into account that a very high proportion of occurrences of the equals sign (46/49) and of material processes by basic objects (21/33) occur in blocks of repetitions of just four basic statements. For example, one block consists of 15 statements of the form:

Length of rod = 2, Topples on 3
Length of rod = 3, Topples on 7
etc.

The fact that the author has chosen to present her work in this way rather than, say, making greater use of tables, is itself significant. The results presented thus are not just abstract numbers but are explicitly related to their origins in the material world. The repetitive nature of the task of collecting and analysing the data is also demonstrated. At the same time, however, by stating each instance separately in this

way, the importance of each individual observation or calculation is emphasised at the expense of overall observations, patterns and generalisations.

The emphasis on concrete examples is even stronger at the beginning of the text where the author demonstrates her participation in the practical activity prescribed by the task by presenting a set of six diagrams showing piles of rods. The diagrams are in a naturalistic mode, displaying not only the lengths of the rods (the relevant variable in this task) but also their colour, their three dimensional nature, and the fact that as the piles became larger, two rods were sometimes put together to make up the required length at the top of the pile. The prominence of these diagrams and the evident time and care that was put into their construction strengthens the apparent importance of concrete activity and the material world.

The repetitive form of presentation also, however, distorts the picture of the balance of different sorts of processes and participants in the text as a whole. If, instead of counting each individual occurrence, each block of repetitions were counted as a single occurrence, the summary would be as shown in Table A5.6.

Table A5.6
Adjusted Actors and Processes: Sandra - Topples

	Human		Object			No Actor (Passive or non-finite)
	General	Specific	Basic or Derived	Relational	Representational	
Verbal						
Relational (= sign)			2 (5)	1 (1)	(1)	
Mental	7	4				3
Behavioural	1					
Material	9		13			
Existential			2		2	

This adjusted table highlights the relatively large number of mental and material processes ascribed to a general human actor. While the author herself is shown only to be carrying out the tasks prescribed by the set questions (working out answers, thinking of an extension), she makes extensive use of the general *you* and *we* as actors in more specific activities, including calculations, building piles of rods, observing, comparing results and thinking. Mathematics is thus shown to be a human activity and, at the same time, this activity is generalised. Thus the description of the first extension

You can try and build a pile by joining them together and see when they topple, in the same way as the original experiment and compare them together.

provides a procedure that could be followed by any mathematician rather than a specific account of what Sandra herself did.

The author's own activity is signalled in inconsistent ways, at some times more explicitly than at others. In labelling the various sections of question 1, for example, she first copies the imperative instructions from the question paper:

b) Tabulate Results

then indicates her own personal involvement in the activity:

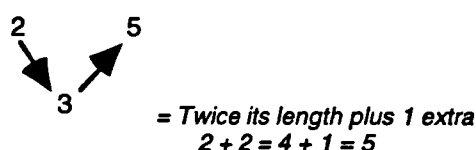
c) How I worked them out

and finally uses a nominalisation:

d) Predictions

which obscures her own role as the agent making the predictions. Such inconsistencies in the style of the text suggest a lack of control over linguistic resources that may be interpreted by a reader as a sign of immaturity in the writer.

Like the description of the extension problem given above, the generalisations are expressed primarily in procedural terms. Sandra's original exploration of patterns in the data is expressed relationally as a list of individual statements of the relationship between the length of the rod at the bottom of the pile and the length of the rod that causes the pile to topple, e.g.



While the verbal part of this statement appears to be relational, it is immediately operationalised into a procedure. Indeed, this set of statements is listed under the heading "How I worked them out", emphasising the procedural aspect again. When providing a statement for rods of any length, the generalisation is not only entirely procedural but also involves a human actor:

To work this out, you double the length of the rod then add 2 and that number that you get, is the length of the rod that first makes the pile topple.

Although the author makes no use of algebraic notation at any point in the text, she does use the term *formula* to refer to her procedures. The context in which it is used suggests that the purpose of a formula is entirely procedural:

The formula is opposite of that when you're trying to work out the length of the top rod.

$$50 - 2 = 48 + 2 = 24$$

In these extracts, the equals signs appear to be used as logical connectives between stages in the calculations rather than serving to signal identities. This is consistent with the generally procedural view of mathematics within the text as a whole. Elsewhere, equals signs are also used to ascribe labels or values. Very little symbolism is used in this text and, where there are symbols, they tend to be used in informal and unconventional ways.

The final sentence in the text provides some insight into Sandra's view of the purpose of extensions in investigative work. Having posed two further problems and presented some data related to each of them, she concludes:

So really, if you think about it, you can make up many optional extensions from this maths assignment.

Her achievement is to have 'made up' the problems rather than to have proposed any solutions to them.

A further consequence of the extensive use of lists in this text is the impression that the subject matter being considered is a collection of isolated (although similar) events rather than a coherent system. There are, in fact, very few causal relationships expressed; on the whole, these present material objects as causes of material actions, for example a column in a table of results is labelled:

Length of rod that first makes the pile topple.

In one case, the human action of placing a rod is the cause of a material event and this state of affairs is seen to be causally related to the length of one of the rods:

If a pile topples when we place a rod of length 50 units on the top, the length of the rod on the bottom pile will be 24.

This unusually complex sentence expresses more sophisticated causal relationships than may be found elsewhere in the text. It echoes the words of the original statement of question 3 (given immediately above it in Sandra's text):

*A pile topples when we place a rod of length 50 units on the top.
a) What will be the length of the rod on the bottom of the pile?*

This may be interpreted by a reader to indicate that the author is mimicking the question rather than constructing her own meanings. While consistently using the vocabulary provided by the question paper allows Sandra to be credited with making correct and precise use of mathematical language, it simultaneously lays her open to the charge of not 'really understanding'.

A5.12.2 Interpersonal aspects: a formal text

As has been seen above, the author's own activity is largely absent from this text. The exceptions to this do not describe what she has done but demonstrate that she has fulfilled the requirements of the task. For example, she claims ownership of the 'extension' work:

*Here is another sketch of another optional extension that I thought of. . . .
Here are my results.*

This is a very formal and generally impersonal text with no intrusion of opinion or emotion. One of the main aspects that contributes to its formality is the extent of repetition that has already been remarked upon. This repetition is not only in the form of lists but may also be seen in the close (and in many cases exact) similarities between the wording of different sections of the text and between the wording of questions and the wording of the accompanying answers. Such repetition suggests a concern with 'correct' and precise use of language and hence a formal relationship with the reader. It may, however, also give the impression of a lack of fluency with the language. For example, Sandra uses the phrase *Length of rod that first makes the pile topple* (which was used in the question paper) not only when writing in full sentences but also as a label for a variable and as a heading for a column of a table where it would be more usual to find a more abbreviated phrase.

Another major contributor to the formality of the text is its presentation which is both meticulous and decorative, making extensive use of colour. Again, there are repetitive aspects of this; there is great consistency in the size, lettering and colour of the page headings, for example, and the tables are all constructed to be the same size and with their columns aligned at the same oblique angle. The colour, although it also makes the text attractive to look at, is used in a way that suggests that its intention is more functional. Keys to the colours used in the diagrams are included, implying that the colour should help the reader to make sense of the diagrams. All these features serve to construct this as a formal and public text, addressed to an audience which has a very distant relationship with the author - possibly distant both in space and in status.

A5.12.3 Textual aspects: question-and-answer

As has been noted, a major feature of Sandra's text is the presence of blocks of repeated statements, including calculations. This, together with the use of question numbers and the copied questions, locates the text within the traditional school mathematics genre of question and answer. In the part of the text containing the 'extension', because the problems are not specified by the question paper,

Sandra has had to construct her own wordings. The interpersonal and existential themes used in this section suggest a descriptive report of the nature of the activity.

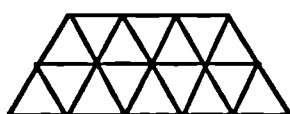
The presentation of this text is striking in its use of colour, elaborate headings and diagrams and careful handwriting. While this might suggest that it is a 'project', the structure of the text is nevertheless firmly in the question-and-answer mode. The large coloured headings announce the question number rather than the function of each section within an overall coherent project.

Appendix 6

Extracts of generalisations from 'Inner Triangles' texts

A6.1 Extract No.1

Once Suzanne and I had completed tasks 1 and 2, we set out to discover if there was any connection between the triangular area and the lengths of the sides. First we lettered the sides:-



a = slant line
 b = base line
 c = top line
 d = area

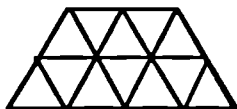
In a very short time we had discovered a relationship between the lengths of the sides and the area (triangular). We were able to put this into a formula:-

$$ab + ac = d$$

This simplified becomes:-

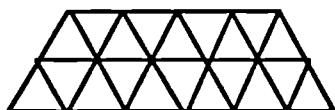
$$a(b + c) = d.$$

To make sure this worked, we checked it out:



$a = 2$
 $b = 4$
 $c = 2$

$$2(4 + 2) = 12$$



$a = 2$
 $b = 6$
 $c = 4$

$$2(6 + 4) = 20$$

This formula worked with all trapeziums

A6.2 Analysis of extract No.1

Human activity is important in this text in fulfilling the key roles of looking for, finding and validating a connection, relationship and formula. The arrival at a formula that "works" is the culmination of this activity. It is important that it is stated that the formula "works" for all trapezia as this validates the work as a whole even though there is no evidence presented that this is actually true; the author is claiming completeness for her work. The discovery of the relationship is not explained; lettering the sides of the trapezium is the only intermediate step that is provided. This contributes to the importance of algebra in the text. Not only is the discovery of an algebraic formula the purpose of the work but conventional rituals are fulfilled during this process: the naming of the variables is stressed both by announcing that it was done and by displaying it prominently and underlining it. The formula is also "simplified" although without any indication of why this might be appropriate; the impression given is that "simplifying" simply is the thing to do with a formula.

As well as stating the actions taken by the author and her friend, the value of their activity is asserted. They did it "in a very short time" and were capable of performing a potentially difficult task: "We were able to put this into a formula".

When checking the formula the identification of each example is represented both by a diagram and by the statement of the values of the variables. Taking into account the general lack of detail about the processes gone through earlier, this attention to detail suggests a fulfilment of conventions in order to satisfy the demands of the teacher/examiner to present examples in a standard form including: diagram, translation of diagram information into algebraic notation, use of this information to solve the problem.

This text is a "story" about what the author did. The past tense is used throughout; the only exception being the statement that the formula "becomes" a different form when simplified. This single use of the present tense emphasises a distinction between the activity of the human discoverers of mathematics, which is situated in a particular past time and is the dominant focus of this text and the mathematics itself which is timeless. The story aspect is reinforced by the use of temporal themes ("Once . . .", "First . . .", "In a very short time . . .") which link together the different stages in the problem solving process. There is a single example of a clause with a reasoning related theme: "To make sure this worked . . ." but this reason is at the level of the human problem solving process rather than at the level of logical connections within mathematics.

A6.3 Extract No.2

If you add together both the top length and the bottom length and times it by the slant length, you will end up with the number of unit triangles in that trapezium.

You can write this as $S(T + B)$

<i>Top Length</i>	<i>Bottom Length</i>	<i>Slant Length</i>	<i>Area inside trapezium</i>
1	2	1	3
2	4	2	12
3	6	3	27
4	8	4	48
5	10	5	75
6	12	6	108

A6.4 Analysis of extract No.2

Mathematical activity is presented as arithmetical procedures and writing in algebraic notation which is performed by a general "you". The author does not have an explicit role in this text.

The generalisation is procedural: "you will end up with . . .". In the algebraic notation, the "what you end up with" is omitted. It could be said that the formula is presented in functional form as a procedure with an implicit output.

The style is speech-like in its redundancy; the words “together” and “both” are not necessary to understanding and implementing the procedure as they are implicit in the instruction to “add”:

you add together both the top length and the bottom length

The assertion “you will end up with . . .”, while apparently expressing certainty, nevertheless allows more possibility for uncertainty than if the verb had been in the simple present (i.e. “you end up with”). The unmodified form would have suggested greater, timeless generality. Similarly, the modified “You can write this . . .” expresses the author’s tentativeness in making the statement.

The table is not integrated into the rest of the text and its role is not clear. The data in it is apparently arranged systematically but it is not apparent whether this is intended to confirm the truth of the formula or to demonstrate its use or merely to show that the author knows that it is important to include a table. There is no attempt to verify the formula by reference back to the concrete.

A6.5 Extract No.3

I put the results in a table and from here I can find a formula.

I noticed:

If you add the top length and the bottom length, then multiply by the slant length, you get the number of unit triangles.

For example:

$$3 + 5 = 8$$

and

$$2 + 4 = 6$$

$$8 \times 2 = \underline{16}$$

$$6 \times 2 = \underline{12}$$

This, therefore is the formula:

(TOP LENGTH + BOTTOM LENGTH) \times SLANT LENGTH = No. OF TRIANGLES

A6.6 Analysis of extract No.3

Mathematics is a human activity. The 1st person role is an organisational one: to put results in a table, to find a formula, to notice things. The 2nd person role is to carry out arithmetic procedures.

What was “noticed” is of a procedural nature. This is reinforced by the way in which the two examples are laid out, showing the procedure in two steps. The procedure is then translated into “the formula”. The form of this formula is, however, ambiguous or transitional; while it appears on the surface to be in a relational form, it could be interpreted as procedural by a direct translation:

+ \rightarrow you add, \times \rightarrow you multiply, = \rightarrow you get

The use of capital letters for “TOP LENGTH” etc. in the formula suggests that these terms are acting as variable names rather than as a physical attribute. Again the statement of the formula appears transitional between consideration of physical attributes and abstract algebraic notation.

The importance of the formula is indicated by the box around it, the use of capital letters and the existential declaration

This, therefore is the formula

Causality resides both in the table, which brings about the possibility of finding a formula, and in the transition from the general procedure to the formula. In both cases it is manipulation of symbolic objects which brings about the result.

A distinction is made between specific events, denoted by the use of the 1st person and the past tense, and general results, denoted by the 2nd person and the present tense.

What is the role of the two examples? They show how to apply the given procedure to a set of numbers. There is no direct link made to actual concrete objects that these numbers might be supposed to refer to. Not only has the problem become one about numbers rather than about trapezia but the reader either has to take the validity of the examples on trust or do some active work to reconstruct the concrete referent.

Reasoning is important in the second part of the text, playing a significant role in the thematic position of each of the three sentences:

If you . . . you get . . .

◊

For example: . . .

This, therefore, is . . .

This sequence is apparently in the form: hypothesis, test, theorem (although the 'theorem' is not 'proved' except possibly by the two examples whose role appears to be to demonstrate the application of the procedure rather than truth of the formula which follows).

Appendix 7

Steven's additional coursework texts

A7.1	'Passola - Pass it Round'	357
A7.2	'Symmetry Groups'	362
A7.3	'Pendulum'	366

A7.1 Steven - 'Passola - Pass it Round'

PASSOLA-
PASS IT ON

I	II	III	Total
5	4	3	12

Passola - Pass it Round.Problem:

This task concerns a game called PASSOLA.

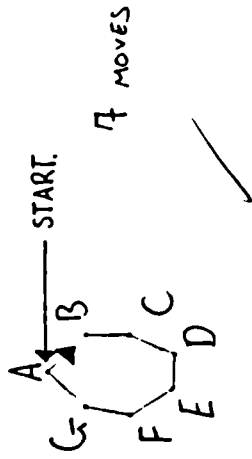
The idea of the game is that the ball is passed around a group of people.

The rules of the game are:

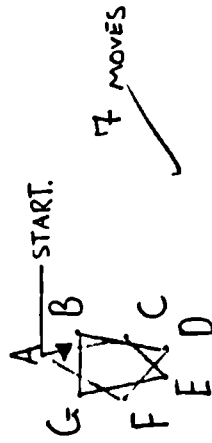
- i) always pass the ball around in a clockwise direction.
- ii) the person number is the number of people in the ring.
- iii) the stop number indicates the person the ball is passed to.

1 Draw Diagrams for each of the following.

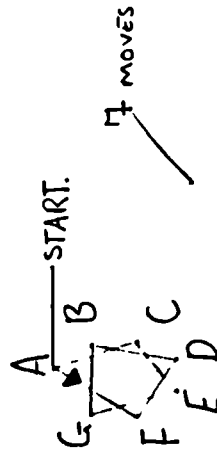
i) 7 person step 1,



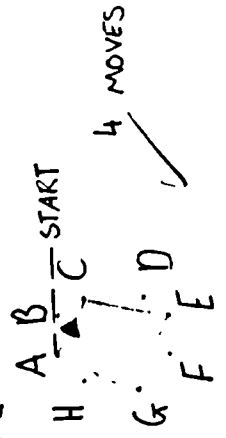
ii) 7 person step 2,



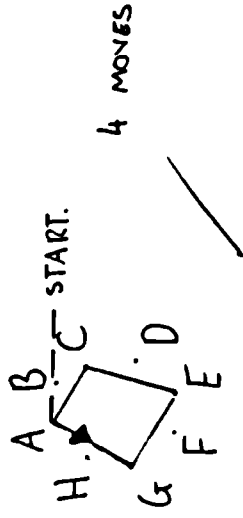
iii) 7 person step 5,



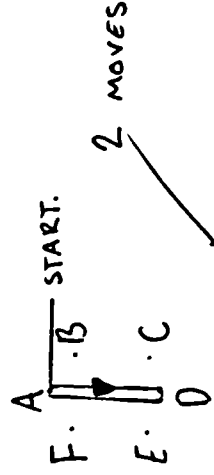
iv) 8 person step 2



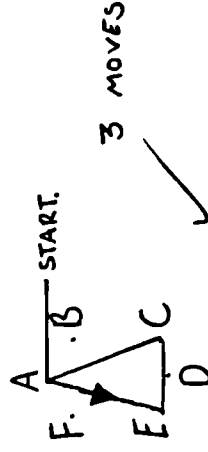
v) 8 person step 6,



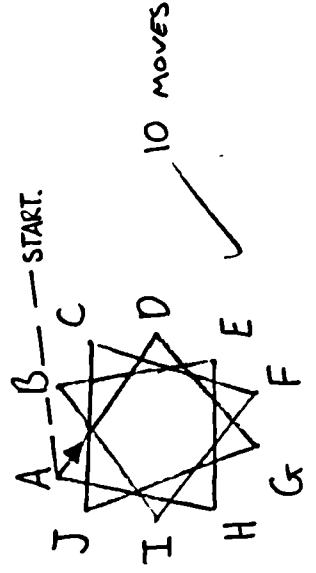
vi) 6 person step 3



vii) 6 person step 4,



viii) 10 person step 3,



b) Write down any observations you can make about geometrical and/or number patterns which you can see.

- i) All shapes are symmetrical with a few exceptions.
- ii) When the step number is half of the person number then only two moves are taken.
- iii) When the step number is half of the person number then it will only take two moves. This only works on even numbers.
- iv) When the step number is a third of the person number then you have a triangle.
- v) When the person number is even and the step number is odd then the moves taken will be the same as the person number.
- vi) Also when person number is odd then the moves taken will be the same as the person number.

You can see these patterns demonstrated on the grid on page: 8. Just match the colours.

2

In Some cases all the people receive the ball in other cases some do not.

a) state clearly the relationship between the person number and the step number which determines whether or not all of the people receive the ball.

i) If the person number is odd then the moves taken will be the same as the person number, so everybody will receive the ball irrespective of the step number.

ii) If the person number is even and the step number is odd then all the people will receive the ball.

b) i) this happens, because the ball is missing one (that has already been passed to) by one or more, this keeps on going until it gets back to the start.

ii) this happens, because the step number is odd it will not go into the person number which is even, so it has to go around a few times before it gets back except for a step number of one but this still has the same number.

3. For a person number of 24, list the step numbers for which:
a) all the people receive the ball

b) the path of the ball is a square,

c) the path of the ball is a hexagon,

4 Investigate further the relationship between the shape of the path of the ball and step number.

If you look at the grid on page 8, you will see that if you take any point on there and look at the person and step number write them down and double them ^{eg.}

$$\begin{array}{l} \text{Person number} = 3 \times 2 = 6 \\ \text{Step number} = 1 \times 2 = 2 \\ \text{moves taken } \underline{3} \rightarrow \underline{3} \end{array}$$

Then the moves needed be the same. This works on everyone.

STEP NUMBER

	1	2	3	4	5	6	7	8	9	10	→
1	0	2	3	4	5	6	7	8	9	10	
2	0	0	3	2	5	3	7	4	9	5	
3	0	2	0	4	5	2	7	8	3	10	
4	0	0	3	0	5	3	7	2	9	5	
5	0	2	0	4	0	6	7	8	9	10	
6	0	0	3	4	5	0	7	4	3	5	
7	0	2	0	2	5	6	7	8	9	10	
8	0	0	3	4	5	3	7	0	9	5	
9	0	2	0	4	5	2	7	8	0	10	
10	0	0	3	2	3	7	4				

The colours match with others in other pages. The grid demonstrates my theorem.

Most of this grid was made up without having to draw diagrams, just using what I had learnt.

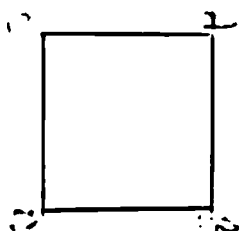
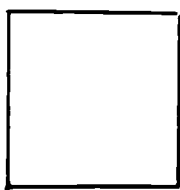
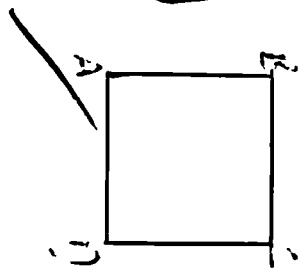
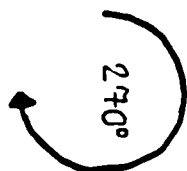
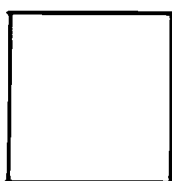
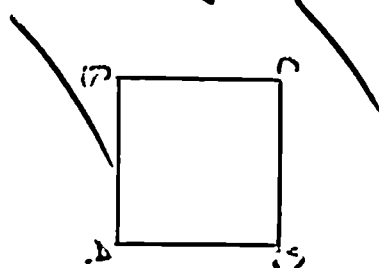
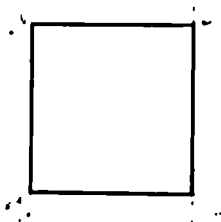
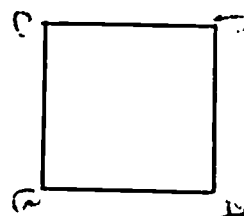
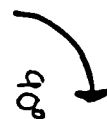
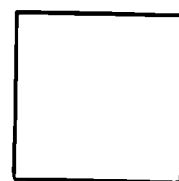
The number in the middle are how how many passes the ball has taken.

A7.2 Steven - 'Symmetry Groups'

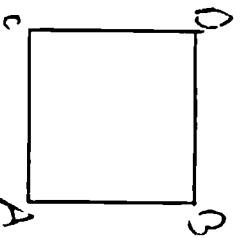
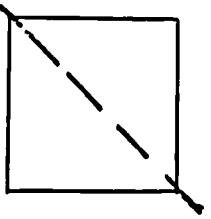
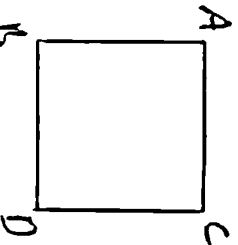
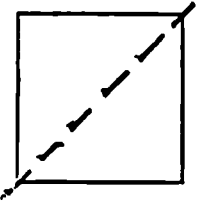
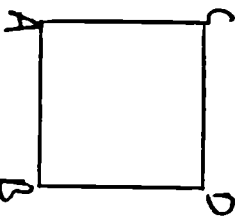
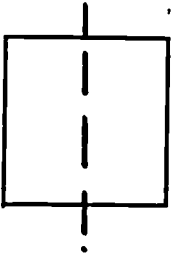
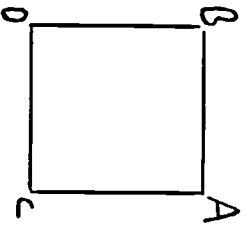
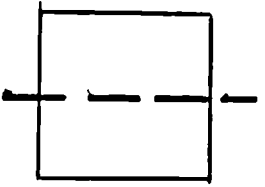
SYMMETRY
GROUPS

I	II	III	IV
5	5	5	5

ROTATIONS

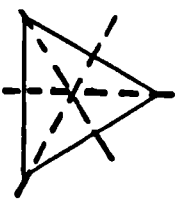


REFLECTIONS



2. FIND ALL THE ELEMENTS OF THE SYMMETRY GROUP OF:

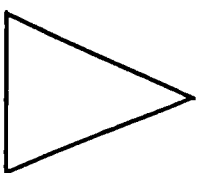
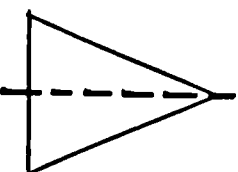
- AN EQUILATERAL TRIANGLE,
- A REGULAR PENTAGON,
- AN ISOSCELES TRIANGLE.



∞ 120°
 ∞ 240°
 ∞ 360°

AN EQUILATERAL TRIANGLE HAS SIX ELEMENTS

A REGULAR PENTAGON HAS TEN ELEMENTS.



∞ 360°

AN ISOSCELES TRIANGLE HAS TWO ELEMENTS.

All regular shapes have twice as many elements as they do sides.

FORMULA.

I have found a formula for regular shapes it is

interesting. - it seems to work!

$$(x \times 10) \div 35 - (x + y) = \text{Elements}$$

SIDES	$x \times 10$	$\div 35$	$- x + y$	Elements
3	30	9.54	5.54	6
4	40	12.73	7.73	8
5	50	15.91	9.91	10
6	60	19.09	12.09	12
7	70	22.28	14.28	14
21	210	66.84	41.84	42

X = SIDES
Y = NUMBER ON TABLE ?

FROM-TO	NUMBER OF Y.
2 TO 9	+1
10 TO 19	+2
20 TO 29	+3

To obtain the number of elements you have to round off the number to the nearest INTEGER

3) HOW MANY ELEMENTS ARE THERE IN THE 6th SYMMETRY GROUP OF A REGULAR HEXAGON.

$$(10 \times 6) \div 35 - (6 + 1) = 12$$

There are twelve elements in the symmetry group of a regular hexagon.

5. GIVE, WITH EXPLANATIONS, THE NUMBERS OF ELEMENTS IN THE SYMMETRY GROUPS OF.

- a) A Regular Octagon,
- b) A Circle,
- c) An Irregular figure.

a) SIXTEEN ELEMENTS.

b) INFINITE ELEMENTS.

c) IT WILL VARY FROM EACH SHAPE.

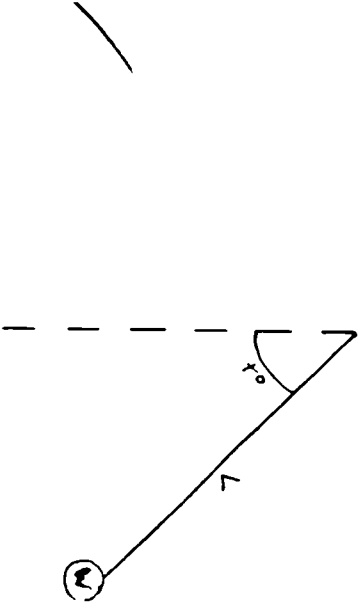
A7.3 Steven - 'Pendulum'

LONDON EAST AFRICAN CLUB

G.C.S.E. EXAMINATION

LIVELY AND I

The diagram shows a very easily constructed pendulum held in its starting position.

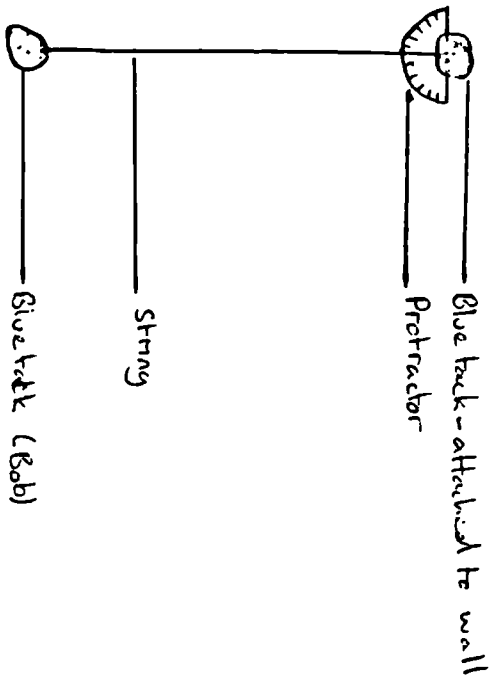


The pendulum consists of a piece of string length L , and a bob of weight W .
 At the start the string makes an angle x° with the vertical.
 We let go of the bob the pendulum as it goes back and forth.
 The PERIOD of the swing, the time it takes for the pendulum to swing from its starting position across to the other side and back.

My Task is to INVESTIGATE
 WHAT HAPPENS TO THE PERIOD
 OF THE SWING BY HAVING
 DIFFERENT WEIGHTS, STARTING
 ANGLES AND LENGTHS.

a) MAKE A PENDULUM.

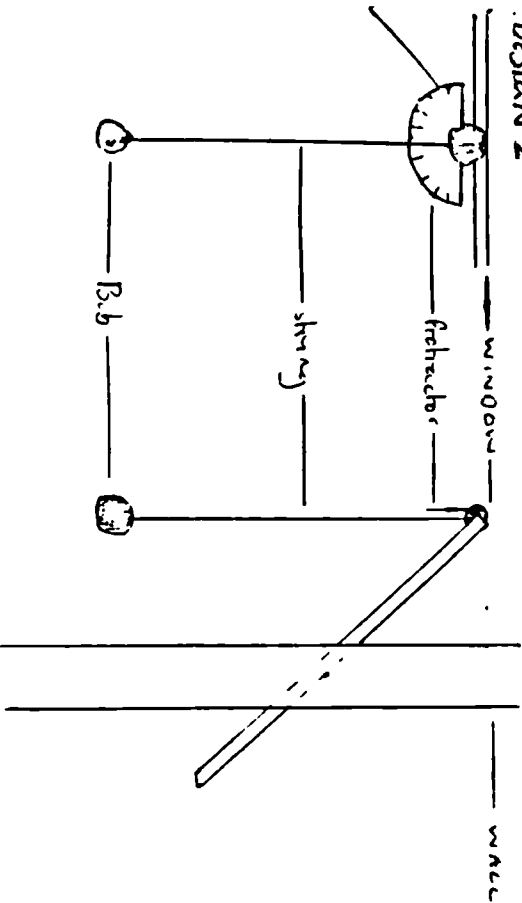
DESIGN 1.



Design worked very well there was no obstructions on the swing. I decided Design two would be the best to use

This design started out to be very promising, but when the experiment was started, I found the bob got caught on the wall, and this could have affected my results. I decided against the positioning of pendulum but the design was very good.

DESIGN 2



"GRAPHS"

1. INVESTIGATE WHAT HAPPENS TO THE PERIOD OF THE SWING:

- when you change the length of the string, ✓
- when you change the starting angle, ✓
- when you change the weight of the bob. ✓

2. RECORD YOUR RESULTS IN THE FORM OF TABLES & GRAPHS.

(i)

LENGTH (m)	WEIGHT	ANGLE (°)	Time (s)
1 m	N	30°	1.75
.75 m	N	30°	1.25
.50 m	N	30°	0.98
.25 m	N	30°	0.78

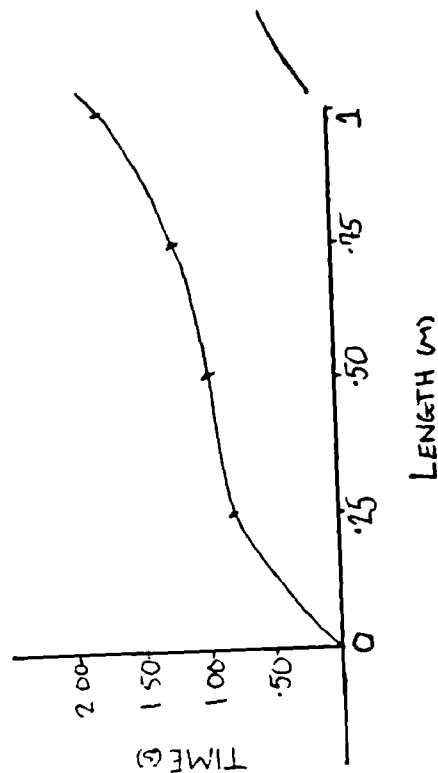
(ii)

LENGTH (m)	WEIGHT	ANGLE (°)	Time (s)
.50 m	N	40°	1.00
.50 m	N	30°	0.98
.50 m	N	20°	0.97
.50 m	N	10°	0.90

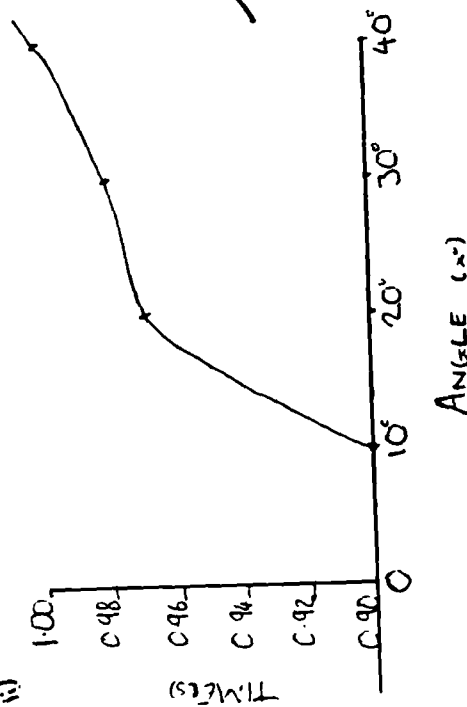
(iii)

LENGTH (m)	WEIGHT	ANGLE (°)	Time (s)
.50 m	N	30°	1.03
.50 m	N	30°	1.00
.50 m	D	30°	1.01
.50 m	D	30°	0.90

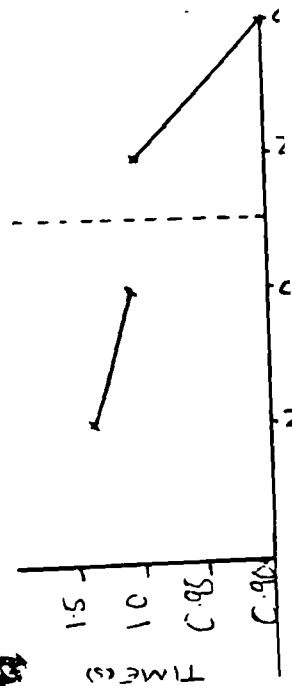
(i)



(ii)



(iii)



COMMENT ON YOUR RESULTS, OFFERING ANY OBSERVATIONS OR GENERALISATIONS.

PROBLEMS

The period of the swing is proportional to the length of the string.

I had a swing time as long as 1.50s.
10m probably has a swing time as long as 5m.

The reason why the swing period is longer when the string is longer is because the bob has further to go, and is therefore going to take more time to swing back and forth.

When the angle is changed from 10° up to 40° the period of swing changes, it is a very small change but it is there.

This could be because when the bob is dropped from a higher height there is a faster speed and therefore it takes nearly as long.

When I doubled the weight of the bob I found that the period of the swing got smaller. I believe this is because when you double the weight it may fall faster but on the upswing it will rise up as high because of the greater mass, it will also add swing as far on the way back so the time will be the same.

IMMEDIATE REACTIONS

When you have a swing you can never get an exact time because of the time it takes to stop and start the stopwatch. This could leave a large difference between times.

SOLUTIONS!!

When I did my timing I decided that you could not trust one result, so I took three results for each time and then found the average time, this was very well and all the results fell into place.

One small thing that helped was to have somebody else say ready, get, go and then drop it on go, this small detail helped get the time more correct.

OPTIONAL EXTENSION

INVESTIGATE WHAT HAPPENS TO THE PERIOD OF THE SWING WHEN THE LENGTH OF THE STRING, STARTING ANGLE AND WEIGHT OF THE BOB, ARE ALL CHANGED AT THE SAME TIME.

WEIGHT: 0
LENGTH: 1M

ANGLE (°)	TIME (s)
40°	1.87
30°	1.75
20°	1.64
10°	1.55

WEIGHT: 0
LENGTH: 1M

ANGLE (°)	TIME (s)
40°	1.72
30°	1.40
20°	1.37
10°	1.35

WEIGHT: N
LENGTH: 75

ANGLE (°)	TIME (s)
40°	1.36
30°	1.25
20°	1.14
10°	1.12

WEIGHT: 0
LENGTH: 75

ANGLE (°)	TIME (s)
40°	1.35
30°	1.34
20°	1.29
10°	1.35

* BAD TIMING.

WEIGHT: N
LENGTH: 50

ANGLE (°)	TIME (s)
40°	1.00
30°	0.98
20°	0.97
10°	0.9

WEIGHT: 0
LENGTH: 50

ANGLE (°)	TIME (s)
40°	1.19
30°	1.09
20°	0.98
10°	0.83

OBSERVATIONS

When the angle and length is increased there is a large rise in the amount of time it takes for a swing. see $W = N \times L = 1$.
When a lot of things are changed at once there is an increase all round as they all combine to get a longer time.

Appendix 8 **Teacher background questionnaire**

Name

School

Male / Female

Full-time / Part-time

How many years have you been teaching?

How many years have you been at this school?

Do you hold a post of responsibility in the department or school? Please specify.

.....

Do you teach any subject(s) other than mathematics? Please state the subject(s) and approximate proportion(s) of your timetable.

subject	time
---------------	------	-------

.....	
-------	--	-------

.....	
-------	--	-------

When did you first enter pupils for GCSE with coursework?

What levels have you entered pupils at?

.....

Which examination board(s) ?.....

What INSET have you had related to doing or assessing coursework? Please describe the content (briefly), how long it was and who it was provided by.

.....

.....

.....

.....

Appendix 9

Summary of teacher background information

Task	School	Teacher	M/F	Years teaching	Years in present school	Post of responsibility	GCSE coursework experience	Exam boards	INSET experience
Inner Triangles									
	J	Amy	F	>15	>5	Head of Dept.	2 years all levels	SEG	1 day SEG
	J	George	M	no response	3	none	2 years all levels	SEG	none
	N	Joan	F	15	1	Head of Dept.	5 years all levels	LEAG, SMILE, SEG	"a tremendous amount" most recently: 1 day SEG
	R	Andy	M	18	10	Head of Faculty	5 years all levels	LEAG	within the school
	R	Dan	M	24	16	Head of Dept.	5 years all levels	LEAG	1 day LEAG
	R	Fiona	F	10	7	2nd in Dept	5 years all levels	LEAG	none
Topples									
	G	Harry	M	9	6	(pastoral)	5 years Foundation Intermediate	LEAG	within the school + 1 day
	G	Charles	M	10	3	2nd in Dept	5 years all levels	LEAG	GAIM
	F	Grant	M	6	2	2nd in Dept	4 years all levels	SEG, MEG, LEAG	1 day LEA 1 day SEG
	H	Jenny	F	10	9	none	5 years all levels	MEG (SMP), LEAG	1 day LEAG
	H	Carol	F	no response	7	Head of Dept	5 years all levels	MEG (SMP), LEAG	various by LEA, LEAG and in school

Appendix 10

Teacher interview schedule

Several days in advance of the interview the teachers were asked to complete the background questionnaire and were given the coursework task with the request to "read the enclosed coursework task and think about what you would look for when assessing pupils' work produced in response to it."

Stage 1

Remind the teacher of the task.

What are you going to look for when assessing responses to this task?

Ask further questions only to clarify or to prompt for more. (I do not want the teachers to focus on my agenda but to make their own explicit.)

Stage 2

Give the teacher the three pieces of coursework.

I'd like you to look at and assess these three pieces of pupils' coursework. Please talk aloud while you are doing it so that I will know how you are making your judgements.

Ask further questions:

- a) to clarify
- b) to remind the teachers to talk about what they are doing
- c) after the basic assessment has been done to ensure that I know:
 - (i) a rank order for the three pieces
 - (ii) positive and negative comments on each piece of work

Stage 3

These are some extracts from some other pupils' work on the same task. Which of them do you think has expressed themselves best?

Why?

What advice would you give to each of these pupils to improve their work?

Appendix 11

Comparison of teachers reading the same texts

(Extracts from interview transcripts)

A11.1 Joan and Fiona reading Richard's 'Inner Triangles' text

A11.1.1 First reading of Richard's text (Joan: 129-165; Fiona: 32-102)

Comments on each group of pages have been juxtaposed to facilitate the comparison between them. This grouping parallels the structure of the analysis in chapter 11. There are no omissions from the interview transcripts of the first reading.

pages 1-3: Title: answers to questions 1 and 2: 'Working out' in the form of diagrams related to questions 1 and 2 drawn on isometric paper.

Joan

Now was Richard in the same group, cos he [Clive] has written his group members down ... no. Ok now he's done his working out as diagrams rather than - and as well as written answers ...

Fiona

I mean the first thing that I would notice from looking at a piece of coursework is that he has actually answered question one and question two here and it's laid out. So it shows that he's actually you know answering the questions that's been asked rather than going off on his own tangent to begin with and I think those are obviously correct those answers. So he's got question one done correctly. He obviously understands the problem for, that's been given. He's also got number two all done correctly which was more difficult. So you know, if we were grading he's certainly moving up to a grade F at this point. Um . . and then he's, and he's shown his working out which is good so that's been put on here. So he's had only, he's obviously not copied these from somebody else .. assume that that's his own working out and that you can see even from this how he's been counting them and stuff. So question one and two it's sort of . . the first thing from that is he's understood the problem, he's able to answer questions one and two without any major difficulties.

page 4: Results table, diagram and formulaJoan

Gosh . . . he's leapt into a formula . . . so let's just see if it works so he's given the top the bottom, the slant height and number of triangles, top plus bottom times the slant height . . . let's try one of these [referring to the data in Richard's table] . . 3 and 5 is 8 . . it seems to . . yeah, so that seems to work just as a quick check. And he's explained it quite nicely with the diagram. What he hasn't done is given any sort of lead up as to how he arrived at it. Which he probably did orally in the group but, you know, it would be nice to have it written down. . The presentation's nice on both of them by the way. .

Fiona

F He's gone on next to give results and a formula. Let's see what else he's done, working out here . . . one, three, two . . . Ok, well he's obviously been systematic here in his working out cos he's kept the slant height two to begin with then he's done it as three, then as four, then as five. So he's obviously got it ordered. Um let's just see how he's come up with these though. . . That's his working out for that bit. He's got a formula . . Oh, so he's got an extension. So it seems there's no justification and no working out at all on this is there. So that's a major draw back here because unless he's got it at the end, if he's included rough work and it's there then .. this is all extension though. His extension seems ok but this . . This is a major problem because he's got these results but unless one is there in the class and you're a teacher you don't know whether this is his results or somebody else's. He hasn't shown any diagrams of where these results have come from. He hasn't done any drawings as far as I can see. He's come up with a formula which is z equals. z must be the slant height. Is equal to x plus y equals t . I assume that's right, I don't know.

I Yes, I think that's right

F That is right is it? Ok . . but again even that's not, I mean he's given, one thing that I think they have to do is when they give a formula they should explain it using quite a few examples and show how it works. The thing that I always look for and I say to the kids is: you write it up as if you're writing it for somebody who's never seen this problem, who's never done that but would be able to understand it if they were to read it. And from this, somebody . . I don't think it's clear enough for somebody to use it and then work out, I mean he hasn't done even one example of how it works. So I think he's got a major problem here, he hasn't shown how he's got it. If you were in the class, obviously we can award marks when they've done something in class but it's not written down. If you're the teacher in the class and you knew that he's just omitted it by mistake um but also he hasn't included his rough work and

that's, when this happens, often you find what you need in the rough work and that can, you know. So none of his results are justified there at all. I think he could have problems with that.

pages 5-6: Title (Extension): Triangles - diagrams, table, formula

Joan

And he's done the same sort of thing for the triangles, he hasn't done the bit of justifying about the top being cut off [a reference to Clive's work]. .

Fiona

Then there's his extension. The extension was 'Extend in any way you wish . . the only constraint is that figures must be drawn on isometric paper' Right, ok. So he's done triangles and here now you see he's actually got the drawings as well to go with it. At least he's done some of them. So he's found that it's a times b. So that's quite good. equals t. Right.

pages 7-10: Four similar pages each with diagrams, table, formula for Hexagons, Stars, Squares and Hexagons (the last two on squared rather than isometric paper)

Joan

he's gone on to hexagons . . Phew, that seems to be ok . . I wonder . . the fact that he's drawn that dotted line [across the middle of the hexagon] across the middle makes me think he was looking at it in terms of two trapeziums but he hasn't said that here, yet anyway unless it's further on, so that seems like a very sensible idea . . but it would have been a good idea perhaps if he'd written in there . . Ah the same sort of thing here . . now this time he's looked at the perimeter, which seems to be jumping about. With the star I thought maybe he'd try and put it into triangles as he's already done a triangle one but . . Now squares . . no real overall conclusion . . . so perhaps . . a table at the end showing all the different shapes he'd looked at . . with some general conclusions . . which would be, if he had done that, . .

Fiona

And then hexagons. [The stars and squares are passed over; the next comment refers to the hexagons drawn on squared paper.] But he hasn't done his answers there, so that's not very good. He's put down some kind of a formula here but he hasn't got any answers or anything to go with it. Right, his extension is much better than the actual investigation itself I would have thought. Even then it still lacks detail.

Summing up immediately after completing the first reading

Having read the piece of work, both Joan and Fiona (without further prompting) provided an overall evaluation:

Joan

. Looking for a bit more [...] and prove your [...] . . [looking at National Curriculum document] . . That's a difficult choice actually because on the face of it it's definitely better than Clive's because it's got the symbolic formulas written in there but it's not as good as Clive's because he hasn't justified it as he's gone along. . I probably would give him a 7 though, all the same. I'm sure he'd have put an 8, he'd have got a level 8 rather, if he'd have put the things together at the end . . The problem we always have at level 8 is this thing about counter examples which I

think is really difficult. But if he'd made, you see, he could have put the hypothesis in there by saying "I think an octagon . ." well you couldn't, could you do an octagon? No . . "I think such and such a shape would have . . this formula" so that would have probably taken him up to level 8 so ok that's a gut reaction level 7, maybe a bit generous.

Fiona

It would have been nice for him to have shown how he's got these answers, you know just to make sure that we know he's not, he's counted it up and done it properly. His algebra's quite good. The algebraic notation is you know using brackets is quite, quite advanced. But it's lacking det.. the main problem I think with him is that it lacks detail. He hasn't justified what he's given us. He hasn't explained anything. He hasn't really made use of specific cases. A lot of the things that he's actually 'show all your working' he's almost ignored all the things

they've asked him to do in question three, and gone ahead and got formulae by whatever method I don't know. Is that ok?

A11.1.2 Subsequent comments

After reading all three pupils' texts, the teachers were asked to rank them and to justify their ranking. In undertaking this process, the key features used in order to arrive at an assessment are picked out by each teacher, often contrasting strengths and weaknesses both within and between pupils' texts. The extracts that follow are those passages during which Richard's text was discussed.

Joan

Extract A (218-230)

- J but it's very difficult to choose between Richard and Steven
 I Can you say, justify why?
 J I mean that's, it's difficult. Richard's got more in the way of the use of symbols and he's extended it quite a lot more than Richard, sorry, than Steven. So Richard seems to have extended it more than Steven. On the other hand I really really thought that Steven was going to suddenly come up with an overall generalisation. He just didn't quite get there so I have a feeling that if he were allowed to redraft this with a few comments I suspect that in the end Steven might come up with something better than Richard but on the face of what is there on the paper Richard has actually developed it much further so I think you would have to say that's the best one because he's just done more - of different things, not just more of the same.
 I In spite of the fact that he hasn't justified what he's done?
 J Mm. That's the big draw back with that one. . . They're actually three very different pieces of work, aren't they? and good in their own rights in different ways. I mean if somebody had actually put those three together that would have been an excellent p ece of work . .

Extract B (267-276)

Now, Richard's really is very good as um . . I would think that somewhere or other Richard's got hiding loads and loads of rough work and he has just given in the best bits and he hasn't . . the downfall of it is that he hasn't really explained his steps as he's gone along . . . It looks like the finished product. It doesn't show us how he got there. The finished product's very good. . . This is where it would have been a good idea to have more than one of us here cos we could have argued about what we said was the best one, which is always much better. You know, even with, as I said to you before, the levelling guidelines, we still come up with some inconsistencies obviously. Cos it is a little bit of a personal interpretation

Fiona

Extract A (264-288)

F You see Richard has got this, the way he has written it out a correct formula is good, but his actual I mean, he's spent no time on the investigation, I mean there's you know, one sheet of paper and that's the whole investigation done so there's no evidence at all. His extension was probably the most detailed of the whole of all of them. Then I did, that was the one I just done, Clive was second, so. He's answered the questions hasn't he. Did Richard not even answer the questions?

I Yes I think he did. Have I got them out of order somehow?

F I was going to say, I thought they all had started that didn't they. I've mixed it all up haven't I? [the order of the papers]. That's better. Right, they've all answered the questions, and they've done it correctly. And they're all about on a par at that stage. . Then and then he has done the working out. He hasn't done the working out for this bit. I think Steven did. . . So even on the first question. Richard has answered that and not shown, has he shown the working out, I've forgotten now. Yes he has, but he hasn't [Clive]. But then if we go onto the next bits, . . Steven has answered question two very well as well cos he again has shown how he worked it out. And so has Richard who's shown his working there for question two. And Steven hasn't . . . Right. I think because, I mean obviously he's done a nice extension, found a nice formula, but um Richard would really have to put a lot more effort into the actual investigation I think. He's put no effort, you know, he's done some work and he's got a table of results but where is the evidence of that. So he's certainly got a problem I would think that would have to be addressed.

Extract B (294-299)

I think Richard must be at the in at the would come third out of the three. I mean, having said that, his extension's very good but I don't think one can give, you know an extension will only get, you know, bring them up a slight bit, consider that he's missed all of this um it wouldn't bring him up high enough. It would give him a few extra marks, so probably Steven followed by Clive followed by Richard

A11.2 Jenny and Charles reading Steven's 'Topples' text

A11.2.1 First reading of Steven's text (Jenny: 19-74; Charles: 50-107)

Comments on each group of pages have been juxtaposed to facilitate the comparison between them. This grouping parallels the structure of the analysis in chapter 11. There are no omissions from the interview transcripts of the first reading.

pages 1-2: Title: statement of the problem

Jenny

. . . I hate it when kids write out the problem which is what he's done isn't it. Is he a he? Yes it says Steven Ellis. I thought there must be some reason why I thought it was a he. . . He doesn't say when it topples

Charles

[no comment]

page 3 (top): table of results

Jenny

. . . I assume that's right. He's got a, he's got, do two three two three two three come in? Well they could do I suppose

Charles

. . . . I didn't do one, that was silly wasn't it . . .
. ok . . Well obviously you've got a reasonable set of results. They're systematically laid out. I'm not sure how correct the results are but based on that there seems to be a nice pattern that is found - two three two three two three and so on. .

page 3 (bottom): description of pattern and comment on generalisation

Jenny

. . . Well I like his enthusiasm that he thinks with such a definite pattern a formula should be easy to find [laughs]

Charles

. . . Well there doesn't seem to be . . well read on a bit, I don't know Ok so the generalisation doesn't . . We don't seem to have . .

page 4: explanation in words and diagram of physical reasons for the results

Jenny

. . . He's doing physics this one

Charles

[no comment]

page 5: formula and examplesJenny

- J . . . [sighs] Can I untag it, sorry, cos I need to look at his table to see whether his formula works for his table . . one ought to check. There's one plus one plus a half of one [...] It doesn't work for the first one anyway . . . maybe [..] it works for the odd for some of them and not for the others so it works for even numbers and not for odd ones . . . it's not me being stupid is it?
- I If you see, he does actually comment on that
- J Ah and then you round up oh yes I hadn't got to that bit, right. Yeah ok well that's fair enough.

Charles

- C oh we do have a formula. Let's look at this first 'I found a formula' . . .
- I What are you actually looking at?
- C I'm just checking to see if they're using this formula correctly first. So we've got a formula and obviously I'm plugging some numbers in to see if it fits in with their . . . So I'm looking to see if they understand the formula. I'm not sure where they've got it from. So I'd have liked to have seen something to show how they arrived at this formula. So it may have been something that they got from someone else and then fair enough they may be able to understand the formula and use it but did they know how to arrive at the formula is something I would be looking for. Um . . I think we've left out a plus sign here - oh I see we've done two separate bits and added them together and then you round it up to three. Oh actually, right, ten plus ten is twenty . . which fits in . . ok . . that seems to work doesn't it.
- I So what sort of thing do you think he might have done in order to show how he arrived at the formula?
- C Well. I think to arrive at a formula like that you're not going to arrive at it straight off the top of your head first time. I think there'd be some attempts perhaps to look at other sets of numbers that might fit in with these. Perhaps looking at, I mean he's got square numbers coming hasn't he? He's got two A . . Oh, right, I see why it works now [laughs]. . . Perhaps I'd have liked to have seen perhaps some more tables of values with some calculations along the way. So I tried this and it didn't quite work, when I tried that I noticed this. Often what I get from kids is oh well I just plugged away on my calculator and suddenly it was there. And I'm highly suspicious of it, you know. But then there are some kids who will set down and show how they got to it and I'm much more happy about that, that I've got something in front of me. Perhaps I'm wrong, I mean, because often kids can do these things in their head or they can suss it out without having to write it all down. But I like to see some evidence on the paper [laughs] that's something I can judge isn't it. It's very difficult to judge what's going on in somebody's head. Um .. so perhaps this . . yeah .. It's a funny way to express it, um . .

page 6: answer to question 2 showing three alternative ways of calculating the result for 100 units

Jenny

J Sorry I've put it back in the wrong place, oh no that's right. . . . He's taken [..] starting at ten and multiplied by ten [sighs] I don't know what he means by that

I Which bit is that?

J An alternative way to do this would be to take the result of a pie starting at ten and multiply it by ten . . oh I suppose yes, yes I do know what he means, right

Charles

C right . . . so now he's gone on
[reading] . . . Hmm, this bit's quite . . .
[reading] . . . ok . .

I So what do you make of that bit then?

C . . I'm still trying to figure out the answer for myself in my head at the moment.
Um ok so I mean he's found the rule and he's quite successfully used it from what I can see to make predictions about what's going to happen for things that he obviously can't set up. So that shows that he understands the formula which he's come up with quite well, I think.
There's also found some sort of linearity in the results whereby he can just multiply up numbers which again shows quite a good understanding of the problem I think.

page 7: answer to question 3 showing two alternative methods, including a second formula

Jenny

J [sighs] . . . this is formula, A over two minus A over ten equals B. B is the topple number. You start with that . . where's his formula gone [looking back at original formula several pages back] Have you got a bit of paper and a pencil, I need to rearrange his formula and see whether he knows what he's doing because I can't do it in my head. . . So he's got two A plus half A equals B, so that's two and a half A equals B. So B should be, so A should be equal to two B over five. He's a bit long winded this kid isn't he? . . . [sigh] Is a half minus a tenth two fifths - probably [laughs] A half is.. a half is five tenths minus two tenths is three tenths . . I'm not at my best today. Um a half is five tenths so then a tenth is one . . . It doesn't seem to me to work maybe I'm stupid though . .

I I think it

J It does. What's a half minus a tenth?

I Five tenths

J Minus one tenth is

I four tenths

J Four tenths . . I've got there right sorry . . And that's the end.

Charles

Um . . and then again he's managed to use the formula working backwards to get this final answer as well from that. So he seems to have a good grasp at the end of the day of the formula he's come up with and how it works in solving the problem.

Summing up immediately after completing the first reading

Having read the piece of work, Both Jenny and Charles (without further prompting) provided an overall evaluation.

Jenny

- J Well now I've got to think what to do with him. Well he can yes he's algebra's better than mine anyway
- I He did have time
- J [laughs] More than sort of five minutes or whatever. Now I've got to think of all these awful statements which I loath . . Now I'm inclined to look at this and think well I think this kid ought to be getting a C-ish cos he's alright really. So that's my starting point and I will now look and see if I can remember what horrible things he's got to get. He's got to get a sevensish to get a C, is that right?
- I Yeah that's right
- J So let's look. Follow new lines of enquiry. What do they mean by that? Well yeah I suppose he's followed new lines of enquiry, has he? I don't know, I mean, what's new? Examined and comm.. well he's examined and commented constructively on generalisations or solutions, he's definitely got that. Let's go back to 6a. Well he can certainly have 6a I think. I'm sure he's posed his own questions, I'm sure there's some questions there. I suppose that's critical examination of mathematical presentation [sighs]. Well this one here this boxes one which Christine [..] as long as he's got a formula we reckoned that was alright for following new lines of inquiry. I'd give him 7a, 7b. Perhaps we ought to look at 8ab . . Mm I don't think, it's stupid, I do loath the National Curriculum. Have you noticed I loath the National Curriculum? [laughs]
- I You're not alone
- J . . Understand the role of counter-example . . no I think I'd, I mean I don't like spending long doing this: 7a 7b
- I You started from the basis of saying you thought it was about a C, so what is it that makes you say it's about a C?
- J Because he's manipulated algebra fairly efficiently. Which is bugger all to do with Using and Applying [laughs]. But then what is Using and Applying?
- I Ok
- J Alright, happy with that?

Charles

- C Um . . so . . yeah, I mean on the face of it it seems like quite a nice piece of work. It seems to hold together as a piece of work. Doesn't seem to be flying off on tangents and so on. We've got a formula. I didn't think it's expressed very well. I think it could be expressed better if I was looking at the algebra but saying that, he's obviously used the formula and obviously understands the formula and gets reasonable results from it. Um . .
- I How would you have preferred it to be expressed?
- C The formula? Well I think for the level of work that this person's doing I'd have thought they might have written it down as two and a half A or something, no I suppose two and a half A's a bit tricky though isn't it. Two A plus . . no I see your point there, it's not as straight forward as you think is it [laughs]. Two A. . I'd like to have seen two A plus a half of A or two and a half of A or something like that. I think I don't like the A plus A. I mean there's nothing wrong with it. There's nothing wrong with it and I don't think I'd discredit pupils for it but . . I had to look at it for a while to figure out what it was: A plus A plus A over two. It . . . I mean as a whole piece of work it hangs together very well I think. Apart from as I say it seems to come out of thin air a bit, apart from that everything else comes together quite well. I haven't really read this bit . . something to do with [?] the weight . . . That's an explanation . . . Um . . perhaps some idea of where the two and a half came from. . .

A11.2.2 Subsequent comments

Jenny (201-211)

Right, what's his best bit. His worst bit is that he didn't attempt to extend it when I think he probably could have done. Lazy little bugger. I think he's got enough mathematical ability probably, um, that he ought to have been able to try and do something a little bit more, I don't know. I mean he's got, I mean the fact that he's got this centre about thing there . . makes me think that he's got extra knowledge which he could have used if he could be bothered to think about other things and he hasn't. Um, . . well a good thing is that he has attempted to generalise by using um letters and has been almost consistent about them hasn't he. He's used an A and a B there and he's use an A and a B at the end and they're the same A and B, except that one's a capital and the other's small. I think, aren't they? Or maybe they aren't, maybe he's switched them . . I think that's . . ok

Charles

Extract A (187-190)

Well I don't know cos I think these two are very similar. I think probably Ellen understands it a bit better than Steven does but the Steven's got his little bit of algebra in there which . . I mean you're always looking for your algebra aren't you so [laughs]

Extract B (204-208)

Then again, not knowing the kids, and not having seen anything like that, you don't know how much they've picked up from other people and things like that. I suppose if I had to do it just on that I'd probably put Steven probably slightly above Ellen at the end of the day and then, I feel almost embarrassed saying, you know, about the results but you know, I'd probably put them in that order. Steven, Ellen and then Sandra.

Extract C (211-212)

I can't really complain there, that's fine. Steven can try to improve his algebra I think there. The way he sets the actual formula out

Appendix 12

Harry reading Sandra's naturalistic diagrams

Extract of interview transcript (Harry: 198-229)

- H Now one of her illustrations here shows I think that she's used, she's actually using split rods. We haven't actually tried that but I'm sure that would have an influence on the result.
- I Probably the rods only go up as far as twelve don't they
- H Yeah [laughs] I wonder what she makes of that. One that - I'd be interested to see if the results show anything different. Um, . . . well she's got something different in the results, she hasn't as yet said anything about it. No she's actually analysing - she's got, she's mentioned twice the length plus one and twice its length plus two. Um I'll go through it and see if she says what that means. . . . She's actually making - she is making predictions based on an increase in the length by a number of fixed units and it's an increase of two every time, double it's length times it then plus two as opposed to plus half of the total length. Which is going to make - which is going to mean basically her answers are wrong cos the formula is incorrect to start with. She's actually adding rather than multiplying.
- I Does that actually matter that she's got the er a different formula from what the others got?
- H From what I can see, it looks wrong.
- I Yeah but she actually came up with different answers there as well [the practical results] in her original data
- H Mmm. . . . I think this is just going to go back to what was said about the blocks being split. So I think that's had an effect there. Um . . . when it's five at the bottom . . . yeah she's got twelve um I'm not convinced that would make it topple. But in her illustration, is that is that I can't see which one.
- I This is the one with five on the bottom. That isn't split.
- H Right. So I'm not sure that that's - I'm not sure that it would topple then. But if she's convinced that it did topple - again we said about the accuracy of the actual modelling of it the setting up of it. If it toppled it may be something to do with it but she's convinced that it toppled. Then we have to accept that. Um but it's the six one the next one I think where the problem may be. Because she feels that she's gone, she's extended it to one next level and she's spotted that it's still two extra um that I think that's what's made her convinced that that is the correct solution. Um, but it would be this, and I would I would - post investigation again I would try to see if I can get any - go down to CDT or whatever and get some blocks of wood and chop them up and see you know if it does make a difference. I would get actually longer lengths um to try and eradicate this. So that's what's done it I think. That's what's caused the damage.

Appendix 13

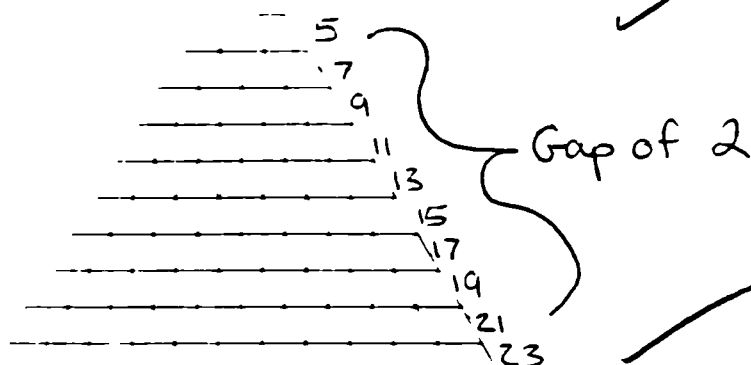
Extracts of 'coherent' sections of student texts

A13.1	from Clive's 'Inner Triangles' text	388
A13.2	from Richard's 'Inner Triangles' text	389

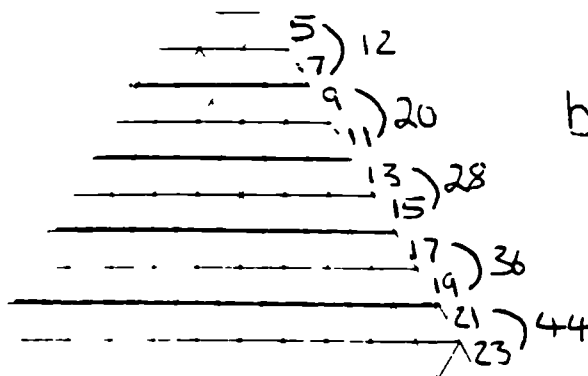
Below is a formula that our group work cut, here it is.

The top + The bottom \times The slant

Also my formula is the one above but mine is below.



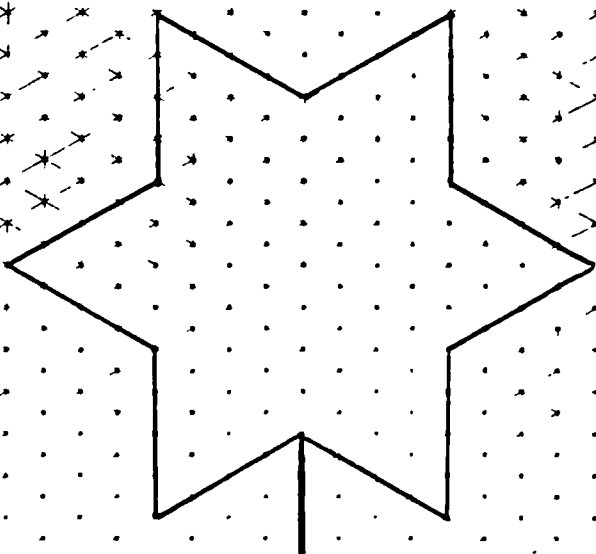
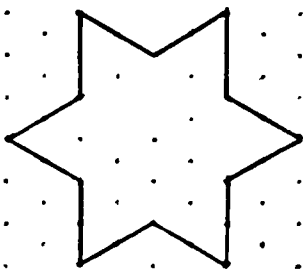
The numbers can be added together to get the next row of numbers. It can also tell you the answer from a 2 top, 2 slant and 4 bot as well as a 2 top, 10 slant and a 12 bottom to get this add all the numbers together.



but this didn't
carry on

Working out

Stars



Perimeter	Slant Height	Unit Triangles
12 24 36 48	1 2 3 4	12 48 108 192

Formula

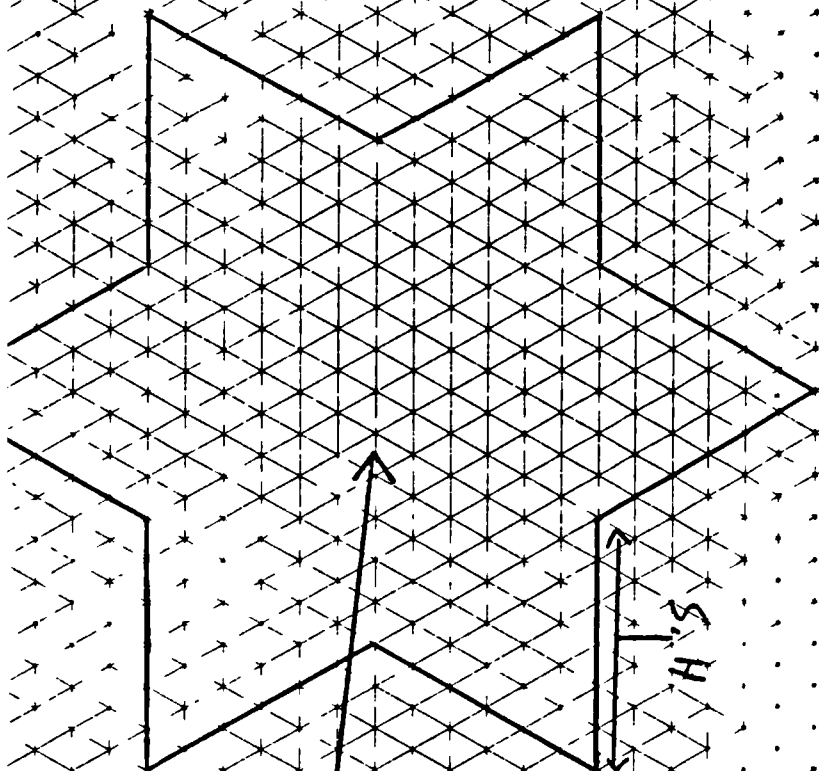
Perimeter \times S.H. = T

Do you mean multiply by

1

same

A13.2 from Richard's 'Inner Triangles' text



o

Appendix 14

Expression of generalisation

(Analysis of student text extracts)

A14.1 Algebra in the 'Inner Triangles' texts

Each of the three complete 'Inner Triangles' texts read in the interviews contained a generalised expression for the number of unit triangles within a trapezium given the dimensions of the trapezium. Apart from the omission of brackets, all these generalisations are correct. Clive and Richard both also included generalisations for other figures. However, since these largely shared the characteristics of the students' original generalisations, they will not be considered separately here.

Clive:

$$\textit{The top} + \textit{The bottom} \times \textit{The slant}$$

- Words are used rather than symbols for the variable names, although these names are succinct and capitalised, suggesting that, in spite of the names, they are abstract quantities rather than concrete objects.
- No dependent variable is indicated and hence no relation is explicit, suggesting that this describes a procedure.
- The brackets that would make this correct are missing.

Steven:

$$y + x \times z = \textit{Unit No}$$

- Abstract symbols are used as names for the independent variables. These were defined by a verbal key. The dependent variable is named in words, suggesting that it has a different status (perhaps that of 'answer') in the relationship.
- The generalisation is expressed as a relation between variables. The different status of the dependent variable noted above, however, provides a suggestion of a procedural aspect which is reinforced by the accompanying comment that the formula "works on all trapeziums".
- The brackets that would make this correct are missing.

Richard:

$$Z(X + Y) = T$$

- The variables are entirely symbolic. They were defined by a labelled diagram. The dependent variable T is named by what may be the initial letter of its referent (triangles), suggesting a different status. This suggestion is, however, far weaker than it was in Steven's above.
- The generalisation is expressed as a relation between variables. The lack of any examples demonstrating the use of the formula means that there is no indication of a procedural element.
- Brackets are used correctly.

A14.2 Algebra in the 'Topples' texts

The three 'Topples' texts also each contained a generalised expression for the length of the rod which makes a pile of rods topple, given the length of the bottom rod in the pile. In addition, Sandra and Steven included an expression of the inverse. All these generalisations fit the data collected and reported by the student. Sandra's data was different from that collected by Steven and Ellen; hence her generalisation is not equivalent to theirs. The ways in which teachers responded to these differences between the texts is discussed in chapter 14.

Steven:

$$(A + A) + \left(\frac{A}{2}\right) = b$$

- The variable names are entirely symbolic. They were defined by a verbal key.
- The generalisation is expressed as a relation between variables. The examples provided, however, emphasise the procedures needed to evaluate b.
- The brackets are technically correct but conventionally unnecessary.

Steven's 'inverse' has similar characteristics to those described above:

$$\left(\frac{a}{2}\right) + \left(\frac{a}{10}\right) = b$$

It should be noted that there is no explicit suggestion within the text that the two generalisations are related to one another. In particular, there is no evidence that this second generalisation was achieved by manipulation from the first. Although a and b are again used as variable names, their reference is reversed.

Ellen:

*The unit at the bottom + itself.
Half unit at the bottom, if it doesn't come to a whole number round it up to the nearest highest number*

- Words are used rather than symbols for variable names.
- The generalisation is eventually presented as a procedure to be carried out by a human agent (addressed in the imperative). The use of the addition sign and the lack of a consistently imperative formulation suggests, however, that there has been an attempt to express the generalisation in a more abstract form.

Sandra:

Sandra initially described her pattern by a repetition of the words "Twice its length plus 2 extras" next to a numerical exemplification for each of her pieces of data. She used colour to indicate which number played which role and used a key to the colours which identified the two variables as "Length of Rod" and "Length of rod that first makes the pile topple". The repetition and the consistent use of the colours suggests a degree of generalisation in this section of the text, although it is not made explicit at this point. On a later page she provided an explicit generalisation:

To work this out, you double the length of the rod then add 2 and that number that you get, is the length of the rod that first makes the pile topple.

- Words are used as variable names. There is no symbolic element.

- The generalisation is presented unambiguously as a procedure to be carried out by a human agent.

In response to question 3 which asked for the length of the rod on the bottom of the pile given the length of the rod that makes the pile topple, Sandra provided not only the specific length requested but also, by her use of the word *formula*, an indication that she considered her solution to be generalisable:

The formula is opposite of that when you're trying to work out the length of the top rod
 $50 - 2 = 48 + 2 = 24$

- The variables are only stated implicitly within a calculation that may be read as serving not only as a solution to a particular question but also as a generic example.
- The generalisation is procedural.

A14.3 'Inner Triangles' extracts

The three extracts from further 'Inner Triangles' texts consist of the section of each text containing the expression of the generalisation for the number of unit triangles in a trapezium. The extracts were chosen to display a range of contrasting features.

No.1:

In a very short time we had discovered a relationship between the lengths of the sides and the area (triangular). We were able to put this into a formula:-

$$ab + ac = d$$

(....)

This simplified becomes:-

$$a(b + c) = d.$$

(....)

- The variable names are entirely symbolic. They were defined by a verbal key.
- The generalisation is explicitly relational. The author describes it as a relationship and, although she followed this with some examples, the stated purpose of the examples was to "check out" the validity of the formula rather than to show how it works.
- The formula is manipulated correctly into a 'simplified' form.

No.2:

If you add the top length and the bottom length, then multiply by the slant length, you get the number of unit triangles.
 (....)

This, therefore is the formula:

$$(TOP LENGTH + BOTTOM LENGTH) \times SLANT LENGTH = No. OF TRIANGLES$$

- The variable names are verbal but the use of capital letters in the formula makes them appear more abstract.
- The generalisation is presented both as a verbal description of a procedure to be carried out by a human agent and in a more abstract form as a relation between variables. The procedure is translated into "the formula". The form of this formula is, however, ambiguous; while it appears on the surface to be in a relational form, it could be interpreted as procedural if it is read as a direct translation from the original verbal description: + → you add; × → you multiply; = → you get

No.3:

If you add together both the top length and the bottom length and times it by the slant length, you will end up with the number of unit triangles in that trapezium.

*You can write this as $S(T + B)$
(. . . .)*

- The variables are named first in words and then in symbols. The symbols are the initial letters of the verbal descriptions, maintaining the connection to the problem context.
- The generalisation is given first as a procedure with a human agent. The symbolic form contains no dependent variable and hence does not express a relationship. The preamble "You can write this as" strongly suggests that it is merely a translation of the procedure.

Appendix 15

The discourse of 'Investigation'

The types of task that are currently labelled as 'investigations' and the activities which are called 'investigating' have been present in some mathematics classrooms in the United Kingdom for many years and are, in particular, to be found described in the pages of publications of the Association of Teachers of Mathematics (ATM) at least as far back as the 1960's¹. The purpose of this chapter, however, is not to provide a historical review of the presence of these activities within the mathematics curriculum but to create a picture of the current discourse associated with the term investigation, considering the historical development of this discourse to the extent that it throws useful light on the present situation. As one of the most significant factors in this context is the institutionalised nature of the investigation as part of the examination system it seems appropriate to take the publication of the Cockcroft report in 1982 as a critical starting point. Prior to the publication of this report, only a small minority of students sat examinations at 16+ which included any investigational element² and, where they did, this was considered to be experimental or appropriate only for lower attaining students (Cockcroft, 1982: p.162). Since then, however, 'investigation' has become part of the official mathematics curriculum, incorporated into all 16+ examinations in the form of GCSE coursework and eventually into the National Curriculum in the form of Attainment Target 1.

'Investigation' is not, however, a simple or uncontested term. There is clearly an 'official' discourse found in the Cockcroft report itself and in the publications of the examination boards. When considering how 'investigation' is experienced in schools by teachers and by students, this 'official' discourse is only one influence; there is also a 'practical' discourse, addressed to teachers and to the students themselves through the medium of text books, teachers' guides, students' guides to doing coursework, journal articles describing particular investigations or recommending particular forms of classroom practice, etc. In addition, there is a 'professional' discourse, to be found in professional journals and in books intended for pre-service or in-service education of teachers, which discusses both theoretical and practical issues related to 'investigation'. Although the overt intentions, contents and styles of these three discourses are very different, they all contribute towards the construction³ of the 'investigation' as a phenomenon in mathematics education. In considering these three types of source, the fundamental questions remain the same: what are the properties of 'investigation', and what are the desired properties of students' work on tasks within this domain? These questions will be addressed through examination of a number of key texts in each category.

¹A keyword search of the *Mathematics Teaching* Index reveals articles in the ATM's journal about 'investigation' or its cognates dating from 1959, including six separate articles in 1968 (compared to seven in 1982 and six in 1990). This does not mean that teachers and pupils in 1959 or 1968 thought of themselves as 'doing investigations'; rather, the compilers of the index saw similarities between the activities described and their own contemporary (1992) concept of investigation.

²There were a number of Mode 3 CSE examinations including coursework components, prepared by individual schools or small groups. There was also a GCE syllabus initiated by the ATM.

³Here and elsewhere I refer to the role of texts in the construction of meaning. This should not be taken as an assertion that texts determine the meanings taken from them by readers. In analysing these texts, however, I am attempting to discern the meanings that would be constructed by the "ideal reader" for whom the text is unproblematic and "natural" (Kress, 1989: p.36).

The distinction between these three public aspects of the discourse of 'investigation' is not as simple as this categorisation might suggest; there are overlaps in authorship and in content of the various publications. The distinction between them lies essentially in the authority of each type of text in relation to its subject matter and its constructed readers and in the rhetorical nature of the text itself. In the case of the 'official' discourse, the subject matter of the text is unquestionable and the reader is constructed explicitly or implicitly as an instrument of the text, bringing to it no opinions or possibility of challenging the message; such texts are statements of what is. The 'practical' discourse, while presenting the nature of 'investigation' as fixed, allows that there may be doubt about the reader's position in relation to it. In particular, it is expected that the reader may experience difficulty in its implementation and possibly uncertainty (arising from ignorance rather than opposition) about its value. The text thus attempts to persuade and advise its readers as well as to instruct them. In the 'professional' discourse, the subject matter itself is contestable and the text tends to be structured as an argument; the reader is constructed as a colleague, or at least as someone who might in the future have that status, whose possibly opposing opinions are to be taken seriously. This discourse includes some texts which are critical of the inclusion of 'investigation' in the mathematics curriculum or of some of the characteristics of its concrete manifestations.

1 'Official' discourse

'Investigation' and ideas associated with it may be found in a large number of government sponsored publications (e.g. HMI, 1985; DES, 1985; Low Attainers in Mathematics Project, 1987; DES/WO, 1988a; NCC, 1989; SEAC, 1992). In this section, however, I intend to examine only a selection of the official texts which have been most influential in the introduction of 'investigations' into secondary school mathematics classrooms and, especially, into the formal system of public examination. Other official publications do not differ significantly in the concepts they contain in relation to 'investigation'.

1.1 The Cockcroft report

The widespread institutionalisation of the idea that something called 'investigation' has a legitimate role within mathematics education may be traced back to the report of the Cockcroft Committee and its often- (and possibly over-) quoted paragraph 243. Six elements of mathematics teaching, including "investigational work" are presented in a format that lends itself to a reading of the list as exhaustive and of the elements as mutually exclusive. However, the report, in attempting to define investigational work, attacks the idea of 'the investigation' as a substantial and separate piece of work:

Investigations need be neither lengthy nor difficult. At the most fundamental level, and perhaps most frequently, they should start in response to pupils' questions, perhaps during exposition by the teacher . . . (pp.73-4)

Nevertheless, the use of the nominalisation *investigations* here and elsewhere in the report further reinforces the interpretation that there is a clearly defined object to be labelled 'an investigation'. The discussion of 'investigation' in the report is inherently ambiguous in its construction of its central concept.

The report claims that

The idea of investigation is fundamental both to the study of mathematics itself and also to an understanding of the ways in which mathematics can be used to extend knowledge and to solve problems in very many fields. (p.73)

but does not attempt to justify the statement by any further reference to the nature of mathematical activity. The nature of investigation itself is also defined only implicitly, largely through the use of a small number of examples of types of questions which might be pursued. Two of these questions, "could we have done the same thing with three other numbers?" and "what would happen if . . . ?",

suggest the notion of *extension*, which has become of importance in the assessment of coursework. Three further properties of an investigative way of working may be deduced from the text:

- there may be a variety of equally valid results;
- the method which has been used ought to be discussed;
- there is value in following and subsequently discussing "false trails".

The Cockcroft report thus establishes the idea that there is a desirable and clearly distinguishable type of activity that may be labelled 'investigation', and hints at some of this activity's properties. The ways in which these properties have been elaborated and transformed into practice will be a theme throughout this monograph.

Significantly, a later section of the report makes a connection between this discussion of teaching styles and the form of examination at 16+. Having castigated timed written examinations for causing a state of affairs in which "practical and investigational work finds no place in day-by-day work in mathematics" (p.161), the conclusion is drawn that:

Because, in our view, assessment procedures in public examinations should be such as to encourage good classroom practice, **we believe that provision should be made for an element of teacher assessment to be included in the examination of pupils of all levels of attainment.** (p.162, original emphasis)

A clear identification is being made between teacher assessment in public examinations and "good classroom practice", including investigational work. This part of the report, in particular the recommendation emphasised by the use of bold type in the extract above, may be seen as instrumental in the eventual institutionalisation of the 'investigation' as part of GCSE coursework.

1.2 GCSE

In order to address the two questions stated at the beginning of this monograph - what are the properties of 'investigation', and what are the desired properties of students' work on tasks within this domain? - two main examination board sources will be examined: the coursework tasks set by the London and East Anglian Group (LEAG)⁴, and the assessment criteria issued for teachers to use in assessing them. Although the various examination groups vary in their interpretation of what constitutes coursework, particularly in the level of prescription of the type and number of tasks to be undertaken by candidates, the assessment criteria developed by each group have, from the beginning, been fairly similar - varying in the detail of their formal application and conversion into marks or grades rather than in the type of criterion included, although there have been differences in the importance ascribed to various aspects of the criteria. A full analysis of the similarities and differences between the coursework practices of all the examination groups is beyond the scope of the present study which aims, rather, to describe how their publications contribute to the construction of the discourse of investigation and coursework within which teachers and pupils operate in schools.

The introduction of coursework into GCSE examinations was not uncontroversial and the concession that the examination groups might continue to offer syllabuses without teacher assessment until 1991 clearly marked coursework in mathematics as officially problematic in some sense.⁵ Nevertheless, the National Criteria for GCSE Mathematics which eventually emerged (DES, 1985) prescribed that, from

⁴Since 1993 the University of London Examinations and Assessment Council (ULEAC) has superseded LEAG as the body responsible for the examinations discussed here.

⁵From 1995, examination boards may once again set syllabuses which include no teacher-assessed coursework, but these syllabuses must still assess the investigative processes specified in Attainment Target 1 of the National Curriculum.

1991, "all schemes of assessment must include a coursework element", which "may take a variety of forms including practical and investigational work" (p.5). Such work is marked as different and separate from those aspects that may be assessed in traditional ways.

Assessment objective 3.17 states that candidates should:

carry out practical and investigative work and undertake extended pieces of work

(DES, 1985: p.2)

It is not clear whether 'practical', 'investigative' and 'extended' are aspects of the same phenomenon or are being listed in opposition to one another. In the first two years of the GCSE examination (1988 and 1989), LEAG attempted to distinguish between different types of task, which it labelled "Investigation", "Problem", and "Practical". The distinction between these categories is very unclear, while the subject matter of those tasks labelled "investigation" is, on the whole, 'pure' in that little or no attempt is made to relate either the original problem or its solution to any 'real world' context, the same may also be said of most of those tasks labelled "problem" or "practical". Analyses of the coursework tasks set in 1991 reveal the uniform nature of the tasks, all of which require inductive generalisation based on patterns 'spotted' in data generated by the candidate early on in the task. In effect, the problem of distinguishing between practical and investigational work has been resolved by subsuming the 'practical' within a unified type of 'investigational' task.

LEAG has dealt with the requirement to assess 'extended' work by providing the opportunity for candidates to create an 'extension' to any of the tasks they undertake. Through the specifications of the nature of these extensions we may see that the term *extension* has come to signify the undertaking of a repetition of the original problem with some minor variation. Thus, for example, having investigated the number of routes between opposite vertices on various polyhedra, the candidate is invited to extend

for example, by changing one or more of the rules, or by combining solids.

(LEAG, 1991: p.13)

Although in other cases the nature of the extension is not specified to the same extent, a genre has been established in which the idea of extending a task has become routine and algorithmic. Moreover, the contribution that the quality of work undertaken in the extension makes to the evaluation of the student's work as a whole is negligible; its existence appears almost sufficient to fulfil the assessment criterion. Thus, the advice provided for teachers assessing this task suggests that one of the indicators for awarding a grade A is that the candidate should have made "a reasonable attempt at the extension" (p.14), without quantifying or elaborating "reasonable".

Thus the 'practical' and 'extended' aspects of coursework specified by the GCSE criteria have both, in practice, been absorbed into a single type of task which may be labelled 'investigation'. In what follows it may be assumed that 'coursework' refers to an 'investigation' set in the context of the GCSE examination.

As well as providing the tasks and performance indicators related to each of the tasks set, LEAG included general 'grade descriptions' in their GCSE syllabuses to help teachers assign grades to coursework (LEAG, 1989). In spite of the presence of an expression of caution about the possibility of matching all the descriptors to any one task, the grade descriptions must be seen to prescribe the type of task to be undertaken as well as the nature of students' work on the tasks. In particular, the following selection of descriptors is easy to recognise in the context of the sort of inductive generalisation task set by LEAG but might be less applicable to other types of task:

Orders the information systematically and controls the variables.
Recognises patterns.
Makes conjectures about patterns, etc. and tests them.

Devises simple formulae when generalising.
Where appropriate, makes use of symbols when generalising
Attempts to verify and justify results.

While it has long been part of 'common knowledge' that the presence of algebraic symbols is to be used as a necessary, if not sufficient, criterion for awarding a grade C (Wolf, 1990), it was only in 1993 that this was made explicit by the examination board:

It is expected that the use of such algebra (or symbolism) will be seen in work appropriate for the award of a top grade C. (ULEAC, 1993: p.23)

As 'algebra' is the only item from the GCSE 'content' syllabus to be included in the general assessment criteria, it plays an important role in determining what types of task might be acceptable and in restricting the degree of choice available to students in deciding their route through the task.

The same page of ULEAC's guidance on coursework assessment sounds "A Word of Caution":

- With investigative work it is always possible for a candidate to take an unexpected or unusual direction. We can never legislate for this and to try to do so might interfere with the whole creative spirit of coursework.

but the relatively low status of this general statement (indicated by its heading and by the qualified modality of its expression) seems unlikely to encourage teacher-assessors to deviate from the course prescribed by the absolute statement above. The tension between the ideal of creativity and the desire for clear assessment standards is unlikely to be resolved in favour of the "creative spirit".

As was seen above, the Cockcroft report contained some ambiguity about whether the 'investigation' was a separate activity or whether 'investigative' ways of working were integral to everyday classroom activity. Although the setting of distinct tasks labelled as 'coursework tasks' and the self-contained nature of the tasks themselves clearly characterises coursework as something separate from everyday classroom activity, the examination board nevertheless officially endorses the opposite (integrationist) point of view.

- 1.2.1 Coursework should encourage good practice. The elements of such, as defined in Cockcroft paragraph 243, should be in evidence whilst students are undertaking coursework tasks.

- 1.2.2 Coursework should be an integral part of the Mathematics curriculum and not simply a bolt on exercise aimed at satisfying new assessment criteria.

(ULEAC, 1993: p.3)

The tension between rhetoric and practice is a strong indicator of the problematic nature of the Cockcroft paragraph 243 definition of 'good practice' (note that this attempt to encourage curriculum development was published more than ten years later), and in particular of the attempt to develop it through the imposition of new assessment methods.

1.3 The National Curriculum

A detailed consideration of 'investigation' in the context of the National Curriculum is beyond the scope of this study. There is, however, a discernible degree of continuity between the characteristics of the official discourse of 'investigation' that have been identified above and the publications which elaborated the introduction of the mathematics National Curriculum: the report of the Mathematics Working Group (DES/WO, 1988a) and the Non-Statutory Guidance (NCC, 1989). It is worth remarking that, in spite of the continued insistence that schools:

must . . . ensure that aspects of using, applying and investigating are integrated and embedded into the ways in which mathematics is taught and learnt (NCC, 1989: p.D6)

the structure of the statutory orders, by including a separate Attainment Target entitled "Using and Applying Mathematics", strengthens the separation of process from content.

1.4 Summary of 'investigation' in the 'official' discourse

Some properties of 'investigation' emerge unambiguously from this analysis of official documents:

- it is essentially mathematical in some way that, by implication, other types of school mathematics are not;
- its content is to do with pattern, relationships, generalisation;
- its learning objectives are predominately related to 'process' rather than 'content';
- it is exploratory and creative and may have multiple valid outcomes;
- it is part of 'good classroom practice', and hence
- it ought to be assessed.

A further property that has developed, particularly since the introduction of GCSE and the National Curriculum, is the identification of 'investigation' with 'pure' mathematics.

There are, however, a number of areas of uncertainty and tension within the discourse. One of the most important of these areas relates to the difference between, on the one hand, using terms such as *investigational work* or *investigating* and, on the other, referring to *an investigation*. Is investigation a general strategy which "permeates" (NCC, 1989) the curriculum, or is it a particular type of identifiable task? The official discourse slips between these two uses; at the level of general principles and justificatory rhetoric the former interpretation appears to be favoured, but when practical examples are called for, particularly in the context of assessment, separate and usually substantial tasks are identified. Similarly, the initial claim in the Cockcroft report that a mathematical investigation does not have to be "an extensive piece of work that will take a long time to complete" (1982: p.73) is in tension with the value placed upon undertaking 'extended' work and creating 'extensions' to tasks. Again, assessment requirements favour the lengthy task, particularly as creating an 'extension' is one of the indicators of high student attainment.

In considering the desired properties of students' work in the domain of 'investigation' there is a further tension between the value placed on multiple methods and outcomes and on creativity when general principles are expressed and the requirement for standardisation and comparability within the assessment context of GCSE coursework. In the manifestation of coursework governed by the London examination board this tension appears to have led to high value being placed on inductive algebraic generalisations arising from a stereotypical investigation task.

2 'Practical' discourse

It is not my intention here to review the field of publications offering practical advice on 'doing investigations' to teachers and students, but to identify the main issues within this discourse which are of relevance to the current study. As my focus is on the discourse of GCSE coursework, it is relevant to consider publications which concentrate on the preparation, presentation and assessment of investigations for examination purposes. Of particular interest is the advice related specifically to the written form of coursework texts. In this section, I shall analyse the desired properties of students' coursework as constructed by a guide for teachers (Pirie, 1988) and a guide for students (Bull, 1990). The choice of these two publications is, to some extent, arbitrary; they are not representative in any formal sense. On the other hand, neither may be considered to be 'maverick' as they are each published as part of a series of similar guides in other subject areas by a well established publishing

house. Both authors have credentials⁶ which contribute to the authority of their texts in relation to their intended audiences of teachers and students. These texts may thus be seen to be part of the mainstream of the discourse.

2.1 Advice for teachers

GCSE Coursework Mathematics: A teachers' guide to organisation and assessment (Pirie, 1988) reiterates many of the themes identified in the discourse of the Cockcroft report and the official GCSE publications. This includes reference to the idea that students are to be "encouraged to create their own mathematics" (p.7) and repeated stress on the idea of the curriculum development aim of the introduction of coursework being to change the emphasis from content to process. The focus of the guide is on coursework as examination, rather than on investigative ways of working in general (although all the examples of students' work provided are of an investigative nature).

The chapter entitled "Preparing pupils for assessed tasks" concentrates largely on communication skills. These include "personal recording" and "writing up"; Pirie identifies both these as problematic. In particular, there is a need to legitimise "personal recording" both for students and (implied by the degree of effort devoted to the argument) for teachers. Notes accompanying an example of a student's work (including both write-up and personal recordings) provide an indication of the significance of such personal (and allegedly private) writing to the assessment process. The student's 'write-up' contains a table of results, followed by a series of generalisations expressed in algebraic notation. The author of the guide annotates this with the comment:

The write-up raised some questions in the teacher's mind which were answered to her satisfaction when she looked at the pupil's recordings. Although the recording is usually for the pupil's benefit, it can be, as here, legitimate to draw positive conclusions from these recordings, with the pupil's permission. (p.47)

Personal recordings may thus play a part in the assessment process, although only in order to have a positive influence. At the same time, their private nature is stressed by the suggestion that they may only be so used with the student's permission. There is a tension here between the idea of rough jottings as private tools to be used by the student without fear that they will be evaluated, and the teacher's need for evidence to support her evaluation of the student's work. While Pirie attempts to resolve this tension by emphasising the students' ownership of their rough work, others have resolved it in the opposite direction. In particular, the London examination board denies the private nature of such work, insisting that it should be presented instead of a write-up as these "have a tendency to omit all the good maths which went into the work" (LEAG, 1991: inside front cover).

Difficulties with the write-up are presented as being related to the nature of the writing itself and a classroom activity is suggested to help students with developing the necessary skills. It is suggested that the teacher and students should:

Collect together also a list of ways of presenting results: tables, matrices, graphs, drawings, models and so on. (p.15)

In spite of the repeated claim throughout the book that it is process rather than content that is to be assessed, the only suggestions about appropriate forms of writing refer to the presentation of results. Methods of communication of processes are not explicitly addressed, possibly because the participants in the discourse share no explicit language to describe such aspects. The desired properties of

⁶Pirie as an established academic in Mathematics Education and Bull as Assistant Senior Moderator with one of the examination groups.

students' write-ups are, however, communicated implicitly in a section of the guide containing annotated examples of students' coursework.

There are three main issues related to the form in which students present their coursework which arise from these examples: the explicit display of processes, the incorporation of algebraic notation, and the interpersonal aspects of the writing.

explicit display of process Two contrasting pieces of work on the same problem are presented, each arriving at essentially the same generalised conclusions. The first piece of work is structured by a narrative of the mental processes gone through by its author. Pirie remarks:

A clear write-up presenting thinking as well as results. Brief, but revealing high mathematical ability. (p.28)

The second piece, in contrast, is structured as a list of results. The annotation comments:

No indication of how the pupil was thinking. . . . This write-up is an example of a situation where it is not possible to say much, either positively or negatively, about the pupils' ability. (p.29)

The implicit message to be read by teachers is that achieving a valid conclusion is not sufficient as evidence of mathematical thinking; it is necessary to make the thinking visible through the use of explicit verbal forms.

incorporating algebraic notation The 'good' student in the above example had introduced algebraic notation as headings in her table and then commented

When labelling the columns of the table, I realised the obvious relationship (p.28)

Again she has provided the reader with a narrative which explains how she obtained her result. Another student's symbolic generalisation appeared without any such preamble. In this case the annotation reads:

Sudden algebraic leap. Where did this come from? Was it his own work? (p.48)

Although there are several other examples criticised because the student has not shown how he was thinking, this is the only case of a suggestion that the student's work might have been copied rather than belonging to the student himself. It appears that algebraic notation, perhaps because of its particularly high status and its "rare, powerful economy which is the essence of higher mathematics" (p.28), must be suspect.

interpersonal aspects Most of the examples provided use a personal narrative style, using the first person singular or, where work had been done in groups, using "we" to refer to the members of the group. The last example, however, uses the passive mood instead, obscuring the student's own agency in tackling the problem. Pirie comments on this style:

This pupil was seen to be working on her own, although the curious impersonal style of write-up might lead one to think otherwise. (p.63)

Although the impersonal style is not explicitly condemned, the fact that it is seen to be "curious" strongly suggests that it does not conform to the author's ideas of what is appropriate in the context. As coursework is to be used to assess the individual student, a clear indication of ownership appears to be important for the teacher-reader.

A further issue about which there appears to be some difficulty is that of accuracy of results and the level of sophistication of the mathematical content. At several points in the guide it is stressed that process is more important than content and that students need to be persuaded that "getting it right" is not the main aim of investigative activity. Nevertheless, the annotated examples of student work are accompanied by several positive comments on the accuracy of results and one example of a student

said to be “floundering” as a result of an “erroneous theoretical solution” (p.35). This tension between accepting error and valuing accuracy reflects the impossibility of separating process from content and of attempting to define a decontextualised hierarchy of processes. Even if a student fulfils ‘process’ criteria related to, for example, working systematically, forming and testing hypotheses, using a range of mathematical language and forms of representation, a lack of technical accuracy or an ‘inappropriate’ choice of mathematical tools will lower the value attached to the work. While the discourse of ‘investigation’ places high value on ‘process’, within the practice of assessment there is a tension between this and the simultaneous requirement for students to display sophisticated mathematical skills and content knowledge.

2.2 Advice for students

Mathematics Coursework: A students guide to success (Bull, 1990) includes advice and examples of students’ work on both ‘investigation’ type coursework, involving primarily pure mathematics, and what the author refers to as “projects”. Although there is some separate discussion of these two categories, most of the advice given does not distinguish between the two types.

The key concepts related to investigation and coursework are very similar to those found in publications addressing teachers. Students are advised, for example, not to look for a single correct answer but to “get involved in the problem” (p.2). From the examples provided of students’ work annotated with comments on their degree of “involvement”, it seems that this is closely related to the idea of *extension*; the posing of supplementary problems appears to be particularly desirable. In distinguishing coursework from answers to examination questions Bull too invokes the ‘process rather than content’ theme, stressing the importance of “the quality of reasoning” (p.8). This is, however, in tension with a simultaneous emphasis on accuracy and on the importance of using sophisticated mathematical techniques.

Once again, algebra is presented as an important way of discriminating between students at different levels. In a sub-section entitled “Has the task been carried out satisfactorily?”, satisfactory completion of a task appears to be defined as achieving an algebraic generalisation. From the comments on examples it is clear that a “generalisation in symbols” is more highly valued than a “valid (but low level) *generalisation in words*” (p.34, original italics). It is stressed, however, that not all students will be able to achieve a symbolic generalisation and the “majority of students” are even advised to avoid the attempt because:

It is very easy to spot someone who has tried to generalise without understanding what is involved. (p.35)

The message to students is thus contradictory: if you use only ‘low level’ techniques, particularly if you do not use algebra, you will be unable to achieve high grades; on the other hand, if you use algebra you run the risk of being condemned for lacking ‘understanding’.

At a number of points throughout the book, students are advised to consider their potential readers. The main aspect of this advice identifies the reader as an examiner who has to read a large number of coursework texts and is thus likely to become bored. This is taken to imply that students should make their own texts more interesting, both in terms of the mathematical content and in terms of presentation. The reader is also invoked in a plea for ‘clarity’ because:

Failure to be clear in the meaning of what you are trying to pass on to the reader cannot be tolerated. . . . *It is not up to the reader to have to work out what you mean.* (p.37; original italics)

A ‘common sense’ view of communication as potentially transparent reinforces the view of the teacher-reader as an unsympathetic adversary who must be persuaded (forced?) to understand. This contrasts

with the picture of the teacher presented by Pirie (see section 2.1 above) who actively seeks to understand a student's methods by calling on oral and personal recording resources as well as the formal 'write-up'.⁷

Apart from these reader characteristics, a section on improving communication advises the student to imagine a reader who

is intelligent but knows nothing about your assignment and needs to know what you have done
and why you have done it. (p.105)

There are obvious difficulties with this act of imagining for students who have been working in a class in which all students have worked on the same assignment set by the teacher and indeed advised by the very teacher who is then going to read and assess the task. Moreover, unless they already share an understanding of what the implications of being "intelligent" might be, this advice seems unlikely to help them to achieve it.

Similarly, in spite of the stated importance of clarity in communication, little explicit help is provided to illuminate what characteristics it might have. The most explicit description of the features of good communication is a list of structural components of a piece of coursework. Ways of effectively achieving clarity in each of these components are not discussed. A number of examples of what is labelled "good communication" are provided but without annotation, so it is not possible for a reader to determine unambiguously what the author intends to indicate is "good" about them.

One of the listed components of good communication is "the use of more than one form of presentation" (p.36) and among the examples of good communication there are extracts of students' work containing diagrams, tables, graphs, calculations, as well as paragraphs of verbal text. There is, however, an acknowledgement that this advice is not unproblematic.

Think about the form of presentation of each item in the assignment. Use a variety of forms of
presentation. Consider which form will communicate the facts best. (p.105)

The advice is potentially contradictory in that, having considered which form of presentation would communicate the facts best, it might be possible to decide that a single form would be most appropriate. There is a tension between the demands of the particular task being undertaken and the need to display one's mathematical knowledge and skills, including communication skills.

There is an ambivalence in the text towards 'presentation'. Students are repeatedly advised that:

The time taken [on presentation] must not be at the expense of the completion of the task. (p.36; original italics)

Nevertheless, many of the examples of students' work provided throughout the book include presentation features such as elaborate covers, illustrations and word processed text and there are many positive comments on these. Similarly, it is suggested that poor handwriting, spelling, and grammar and "unconventional mathematical notation . . . can be tolerated if they are not the result of carelessness or lack of effort" but will not be found in "high-grade work" (p.37). As in the case of the use of algebra, discussed above, Bull appears to be attempting to convey different expectations to groups of students perceived to be of different abilities: algebra, typing, good spelling, etc. are presented as being beyond the capabilities of many students, who will therefore be prevented from attaining high grades however hard they try.

⁷ It is also interesting to note that, just as the guide for teachers suggests that communication failure in a coursework text may be blamed on the teacher, the guide for pupils lays the blame squarely on the pupil.

2.3 Summary of 'Investigation' In the 'practical' discourse

While the differences between the intended audiences are reflected in different emphases in the content of these two guides, on the whole they construct similar pictures of the characteristics of coursework. In particular, both contrast the answer-oriented nature of other school mathematics activities with the importance of 'process' in coursework. At the same time, both texts contain some tension between the value placed on 'process' and the simultaneous valuing of accuracy and other mathematical 'content', reflecting the inherent difficulty in separating 'process' from 'content'

Algebra is identified for both teachers and students as an area of particular significance in distinguishing the 'best' students. However, while the use of algebraic notation is highly valued, it may simultaneously be read as a sign of lack of understanding or even cheating if it is not accompanied by appropriate supporting evidence. The nature of such evidence is not made explicit.

One area of difference between the two guides is in the distinction made between 'personal recording' and 'writing-up'. While the guide for teachers makes much of this, it is absent from the guide for students. Bull focuses entirely on the 'write-up' and even suggests that this is all that is necessary. This is clearly a contested area; it is possible that other texts within the 'practical' discourse may take a range of different positions on this question.

The guides also differ in the related area of 'presentation'. While much is made of the importance of good presentation in the advice addressed to students, it is apparently not an issue for teachers. This difference is related to the different ways in which the teacher-reader of coursework is portrayed in the two texts. The guide for students constructs an adversarial role for the teacher as examiner, against whom the student must use all available weapons, including presentation. The idea that they might be influenced by presentation is, however, unlikely to be acceptable to teachers taking on the roles of pedagogue and advocate constructed for them by Pirie.

An issue that arises strongly from these analyses is the difficulty there appears to be in giving explicit advice about the writing of coursework. Both Pirie and Bull provide examples of students' work, annotated with comments which express their evaluation of the presentation and communication aspects. These comments do not, however, identify the features giving rise to these evaluations. Teachers and students reading these two books must construct their own interpretations of desirable features from the examples provided. It is likely that teachers, with access over time to a much larger number of such examples and with participation in a community engaging in activities which establish conventions and standards, will come to share enough 'common knowledge' to ensure that their evaluations are largely compatible; students do not, on the whole, have such opportunities. There is no explicit language available within the discourse of mathematics coursework to describe the desirable features of the communication of investigative activity. While it is possible to list ways of presenting results (tables, graphs, etc.), these represent only a relatively minor part of the writing that students must do. A focus on such easily identifiable features may even disadvantage students as Bull suggests:

Every item of coursework must have a purpose. There must be a reason for doing the work. Many candidates make the mistake of 'doing coursework'. They are obsessed with producing pages of writing, charts, diagrams, pretty pictures and eye-catching covers. (p.101; original italics)

The implicit form in which advice is provided is likely to assist mainly those students who already have access to the necessary forms of communication, without providing much help for those who do not.

3 'Professional' discourse

The professional journals for mathematics teachers, *Mathematics Teaching* and *Mathematics in School*, contain numerous articles describing examples of what may be labelled as investigations and investigative activities in classrooms. However, it is, on the whole, only since the institutionalisation of the investigation following Cockcroft and, in particular, the introduction of GCSE coursework that a more critical literature has emerged both in these journals and in other publications, debating the nature of investigation and its classroom incarnations. It is this literature that I intend to review in this section, identifying the main issues and areas of contention.

3.1 The nature of investigation

The assertion expressed by Cockcroft and others, that investigation is inherently mathematical in some way that other school mathematics may not be, is to be found once again in the professional discourse. For example, Fielker, responding to paragraph 243 of the Cockcroft report with a claim that the ATM had been advocating investigative work for many years, quotes in support of his argument:

We do not believe that a clear distinction can be drawn between the activities of the mathematician inventing new mathematics and the child learning mathematics that is new to him.
(Wheeler, 1969, cited by Fielker, 1982: p.2)

Although this quotation from an earlier ATM document does not itself use the term *investigation*, and does not, indeed, appear to be making a distinction between this and other types of learning activities, the context in which it is used in 1982 suggests an identity between 'investigating', learning "new" mathematics and "the activities of the mathematician". Later publications discussing the classroom implications of investigation, also take this identity between investigation and 'real' mathematical activity as given (e.g. Brown, 1990; McCafferty, 1989; Steward, 1989; Whitworth, 1988), while Ernest (1993b) draws an analogy between the culture of the 'progressive' mathematics classroom (including investigative activity) and that of the research mathematics community. A usually dissenting voice, Wells (1993) agrees that investigating and exploring are activities undertaken by research mathematicians. This does not, however, temper the virulence of his attack on the 'investigation' as a distinct object in the mathematics curriculum.

The development of *an investigation* as a distinct object was one of the themes identified in the official discourse (section 1 above). As Ernest points out, there has been a metonymic shift in meaning from *investigation* as a "process of inquiry" to *an investigation* as "the mathematical question or situation which serves as its starting point" (1991: p.284). This shift and the associated development of stereotypical starting points and subsequently stereotypical methods of solution are the basis of a substantial amount of criticism in the professional discourse (see, for example, Delaney (1986), Diffey et al (1988), Hewitt (1992), Perks & Prestage (1992), Wells (1993), Wiliam (1993)). Much of this criticism is aimed specifically at the type of investigation designated by Wells as "data-pattern-generalisation" (DPG) and by Hewitt as "train spotting" in which numerical data is generated, a pattern 'spotted' and an inductive generalisation formed. There appears to be a degree of consensus within the professional discourse that such stereotypical investigations fall short of the expressed ideal of the sort of mathematics done by mathematicians; moreover, they "may even inhibit mathematical thinking" (MacNamara & Roper, 1992a: p.27).

In spite of early suggestions that teacher assessment (i.e. coursework) would be effective for assessing "mathematical knowledge" in general (Love, 1981), the focus on process has become firmly established both in GCSE and in the assessment of the National Curriculum. There is still, however, some contestation of the relationship between 'investigation' and the process/content distinction. For

example, McCafferty (1989) bases his evaluation of "investigative materials" on the extent to which they allow the use of "specific strategies" irrespective of the content domain, while Diffey et al. (1988) argue for an investigative approach to teaching a "topic" in mathematics.

Another theme apparent in the official and practical discourse was the idea of lack of constraint, lack of a 'standard result' and variation in direction. The desirability of such 'openness' is stated more strongly in the professional discourse; indeed, Fielker (1982) specifically criticises the examples of investigations suggested by the Cockcroft report because of their lack of openness. Similarly, Ball & Ball (1990) condemn DPG investigations because they give rise to "right answers". The idea of a right answer or a right way of doing an investigation is taken to be inconsistent with the ideals of openness and lack of constraint. The distinction between right and wrong may even be brought into question:

It is often better to bite your lip and let students charge off in the 'wrong' direction because the more of this work you do the less you will be able to define 'wrong'. (Watson, 1986: p.18)

There are, nevertheless, problems with such a completely open approach. For example, Tall (1990) relates the story of a boy who 'succeeded' in finding a method for trisecting an angle. In this case, Tall suggests that the boy should not be penalised because he was working in an unfamiliar context and his work was valid within the framework of his existing experience. In other contexts it might be less easy to justify answers or methods which conflict with 'correct' mathematics. This issue is not explicitly addressed within the professional literature; the ideal of 'openness' is uncritically adopted, the only hesitation being related to the question of whether teachers are competent to cope with it.

3.2 Writing coursework

As was seen in section 2 above, the practical advice offered to both teachers and students lays great emphasis on the communication and presentation of coursework, reflecting the significance laid on written work in the assessment process. This significance is simultaneously recognised and criticised in the professional literature, the main theme being the mismatch between students' mathematical activity and the written product arising from it (e.g. Bloomfield, 1987; MacNamara & Roper, 1992a, 1992b; MacNamara, 1993). MacNamara (1993) suggests that, because the written texts "fix" the investigation, they come to represent the investigation itself. The teacher will thus assess the student's "level of engagement in the written task" rather than in the investigation itself.

There is, however, a recognition of the concern that teachers have in wishing their students to write things down, particularly as they feel they have to justify their own judgements (Ollerton & Hewitt, 1989). There is also some suggestion (Ollerton & Hewitt, 1989; Whitworth, 1988) that writing, even if difficult, may assist reflection and problem solving. No distinction is made, however, between the type of writing that might perform this function and the type of writing that is required in order to provide 'evidence' for assessment purposes. Indeed, the desirable characteristics of the writing produced by students are not addressed. Probably the most fully developed argument for the place of writing in investigative work is made by Mason et al. (1985) who recommend that recording of the problem solving process and subsequent 'writing up' for another person to read both play a role in developing mathematical thinking about the problem and its solution. The relationship between this and the role that writing may play in assessment is not discussed.

4 Coherence and tensions within the discourse of 'investigation'

There is explicit agreement within all three aspects of the discourse of 'investigation' about the 'ideal' characteristics of investigational work:

- it is 'real' mathematics;

- it is open, creative, 'empowering' for students;
- it should 'permeate' the curriculum.

The degree to which it is acknowledged that, in general, the practice does not (or cannot) live up to these ideals, however, varies considerably. The official discourse, although contrasting the ideal with other types of (not such 'good') practice, does not admit that there might be any problems in implementing it; the teacher-reader is constructed as an instrument who, once properly informed, will transfer the written description of the ideal directly into the classroom. The practical discourse, on the other hand, allows that such investigational work is likely to be unfamiliar to teachers and that they may lack experience and confidence. This acknowledgement of difficulty, however, does not problematise the ideal itself or the assumption that the guidance provided will ensure its practical implementation. The professional discourse explicitly deals with the mismatch between the rhetoric of the ideal and what it identifies as the dominant practice.

There is also agreement that, as investigational work is part of 'good practice' it ought to be assessed, the basis for this being the principle that 'What You Assess Is What You Get' and hence the idea that curriculum development can be 'led' by changes in assessment (Burkhardt, 1988). The official and practical discourse does not represent this as problematic, except in so far as teachers and students (being unfamiliar or lacking confidence with such ways of working and assessing) are likely to need extra support to do it effectively. There is no acknowledgement of any mismatch between the ideal and its operationalisation through GCSE coursework. The professional discourse, on the other hand, is concerned with a number of problems, ranging from the essentially practical problem of the inadequacy of using a student's written work as the sole measure of their mathematical activity to more fundamental criticisms of the effects of institutionalisation on the nature of investigational activity itself. Thus the professional discourse makes explicit some of the tensions and contradictions implicit within the official and practical discourse. In particular, the ideals of openness and creativity, once operationalised through the provision of examples, advice and assessment schemes, become predictable and even develop into prescribed ways of posing questions or 'extending' problems and rigid algorithms for 'doing investigations'.

Another implicit tension within the official and practical discourse concerns the focus of investigation on content or on process. This tension is again explicitly acknowledged and debated in the professional discourse. While the practical discourse in particular insists that the focus ought to be on process, there is a problem in separating process from content when it comes to the assessment of investigational work: how does the 'difficulty' of the mathematical content affect the value placed upon the mathematical processes used? Given that one of the purposes of public assessment at 16+ is to distinguish between 'successes' and 'failures', it may appear important that there should be at least some level of consistency between the distinctions made by coursework and those made by examination results. Such a desire for consistency would help explain the emphasis in the practical discourse on the need to include appropriate levels of content and the development of the use of an algebraic generalisation as an indicator of success.

A further issue of particular interest is the construction of the ideal characteristics of students' written reports of investigations. While there is implicit definition of the desired characteristics of students' texts through the use of examples, there appears to be no linguistic means available within the discourse to describe these characteristics explicitly. Again there is agreement that this is an area in which there is particular difficulty for students. The only advice available, however, is related to the presentation of results and the overall appearance of the text - neither of which (at least officially) has high status (although this is an area in which there are some contradictory messages, at least for students). No

support is provided for the communication of processes, which are supposedly the main objective of investigative work and assessment by coursework.

